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BOOK REVIEW: "Applied Singular Integral Equations" Published by Science Publishers, Enfield, NH CRC Press, Boca Raton, FL, 2011. x+264 pp. ISBN: 978-1-57808-710-5.

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This book is concerned with one-dimensional linear singular integral equations. Examples are Abel's equation, equations with Cauchy principal-value integrals such as

$$\alpha u(x) + \int_a^b \frac{u(t)}{t-x}\, dt = f(x), \quad a < x < b, \tag{1}$$

including the airfoil equation (put $\alpha = 0$ in (1)), hypersingular equations (replace $t-x$ in the kernel of (1) by $(t-x)^2$ and use a finite-part interpretation of the integral), equations with simple logarithmic kernels such as

$$\int_a^b u(t) \log \left| \frac{t+x}{t-x} \right| dt = f(x), \quad a < x < b, \tag{2}$$

and related integro-differential equations (replace $\alpha u(x)$ in (1) by $\alpha u'(x)$). The first 150 pages describe various analytical methods for finding exact solutions. Simple numerical methods are developed and used in the remaining 100 pages. There is a bibliography of 104 items, 45 of which are by the authors and their students.

After an introductory chapter, Chapter 2 begins by solving Abel's equation (stated incorrectly in (2.1.1)) and then mainly real-variable analytical techniques are used to solve (1) and (2). Chapter 3 gives an introduction to complex-variable methods, including the Poincaré-Bertrand formula for closed contours and reduction of (1), (2) and other integral equations to Riemann-Hilbert problems. Further "special methods" are given in Chapter 4, including methods in which u is expanded as a weight function multiplied by a series of Chebyshev polynomials, for example. Simple hypersingular equations are solved in Chapter 5.

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The impression gained at this stage is one of a collection of tricks. Of course, tricks are needed in many instances, and explicit examples are valuable, but there should be more. What can be said about solvability (existence and uniqueness) in general? When are supplementary conditions needed to secure exactly one solution? What happens if the kernel is not as simple as in (1) or (2)? The authors hope that their book will be useful "to researchers as well as graduate students" interested in singular integral equations, but they give no pointers to the wider literature beyond Muskhelishvili and Gakhov.

The last four chapters are mainly concerned with numerical methods. Typically, approximations of the form

$$u(x) \simeq w(x) \sum_{n=0}^{N} a_n \phi_n(x)$$

are used, with various choices for the weight function w and the expansion functions ϕ_n. Then, collocation or Galerkin methods are used to find the coefficients a_n. Examples of $\phi_n(x)$ are x^n, Chebyshev polynomials and Bernstein polynomials. Numerical results are given, but there is no guidance on how to make good choices when faced with a new integral equation.

The book contains numerous minor typographical errors. The mathematical typography is ugly, in my opinion.

In summary, the book is not a textbook (there are no exercises). It is not a handbook, collecting formulas for solving integral equations. It does not give an overview of the whole research area. It does not attempt to synthesize a body of work, not even the authors' own work. Nevertheless, some readers might find the book useful.

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