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Corrections to "On the Vector Bundles $m\xi_n$ over Real Projective Spaces"

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Here, we shall give corrections to §4 of [3].

p. 11, line 23 and footnote: "$H^{n-1}(X; Z_2)$" should be "$H^{n-1}(X; Z)$".

p. 12, line 29 and *p.* 13, line 1: "$H^{k-2}(RP^k; Z_2)$" should be "$H^{k-2}(RP^k; Z)$".

P. 13, line 12-line 32: The proof of Theorem 4.4 should be replaced as follows:

PROOF. Case (a). By 2.2, we can write $n\xi_k=(n-k-1)\oplus\eta_1$, where η_1 is the $(k+1)$-dimensional vector bundle over RP^k. We consider the obstructions for η_1 to have three linearly independent cross-sections.

The primary obstruction is $w_{k-1}(\eta_1)$, which is zero sinbe $\binom{n}{k-1}$ is even.

The secondary one belongs to $H^k(RP^k; \pi_{k-1}(V_{k+1,3}))$, and $\pi_{k-1}(V_{k+1,3})=0$ if $k\equiv 1 \pmod 4$ by [1].

Therefore, we have span $\eta_1\geqq 3$ and so span $(n\xi_k)\geqq n-k+2$, which is the first result. Assume $k\geqq 8$, and write $n\xi_k=(n-k+2)\oplus\eta_2$, where η_2 is the $(k-2)$-dimensional vector bundle over RP^k. We consider the obstructions for η_2 to have a non-zero cross-section.

The primary obstruction is the Euler class $X(\eta_2)$ of η_2, which is zero because $H^{k-2}(RP^k; Z)=0$ for odd k. So, η_2 has a non-zero cross-section over the $(k-2)$-skeleton of RP^k.

The sacondary one is a coset of

$$(w_2\otimes 1+1\otimes Sq^2)\cdot H^{k-3}(RP^k; Z)$$

by 4.1 with the above corrections, where the dot operates by η_2. This group is equal to $H^{k-1}(RP^k; Z_2)$ since $n\equiv 0$, $k\equiv 1 \pmod 4$. So, η_2 has a non-zero cross-section over the $(k-1)$-skeleton of RP^k.

Finally, the third one is a coset of

$$(w_2\otimes 1+1\otimes Sq^2)\cdot H^{k-2}(RP^k; Z_2)$$

by 4.2, where the dot operates by η_2, and this group is equal to $H^k(RP^k; Z_2)$ since $n\equiv 0$, $k\equiv 1 \pmod 4$.

Therefore, η_2 has a non-zero cross-section over RP^k and the proof is completed.

Case (b). By 2.2, we can write $n\xi_k=(n-k)\oplus\eta$, where η is the k-

dimensional vector bundle over RP^k. Then, we have

$$Sq^1 H^{k-1}(RP^k; Z_2)=0, \quad w_{k-1}(\eta)=0, \quad w_{k-3}(\eta)=0.$$

The first equality holds since k is odd, and the second since $\binom{n}{k-1}$ is even. For the third, it is easy to see that $w_{k-3}(\eta)=w_{k-3}(n\xi_k)=\binom{n}{k-3}x^{k-3}$, where x is the generator of $H^1(RP^k; Z_2)\cong Z_2$. Since $n\equiv 2$, $k\equiv 3 \pmod 4$ and $\binom{n}{k-1}$ is even by the assumptions, we see that $\binom{n}{k-3}$ is even and so $w_{k-3}(\eta)=0$.

Therefore, we have $span\ \eta \geqq 2$ by [2. Theorem 6.4] and the above three equalities. So $span\,(n\xi_k)\geqq n-k+2$.

The proof of $span\,(n\xi_k)\geqq n-k+3$ for $k\geqq 8$ follows by the same methods as (a). Thus the proof is completed. *q.e.d.*

References

[1] G.F. Paechter: *The groups $\pi_r(V_{n,m})$ (I)*, Quart. J. Math., Oxford Ser. (2), **7** (1956), 249-268.

[2] E. Thomas: *Postnikov invariants and higher order cohomology operations*, Ann. of Math., **85** (1967), 184-217.

[3] T. Yoshida: *On the Vector Bundles $m\xi_n$ over Real Projective Spaces*, J. Sci. Hiroshima Univ. Ser. **A-I, 32** (1968), 5-16.

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