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Some Z_q-Equivariant Immersions

Dedicated to Professor Kiiti Morita on his 60th birthday

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§ 1. Introduction

Let q be an integer and Z_q be the cyclic group of order q. A C^∞-differentiable immersion / of a Z_q-manifold in another Z_q-manifold is called a *Z_q-equivariant immersion* (or simply a *Z $_q$-immer sion*) if f is a Z_q-equivariant map. The purpose of this note is to study the conditions for the existence of some Z_q-equivariant immersions.

Let m and k be non-negative integers, and R^{m+2k} be Euclidean $(m+2k)$-space. Let $R^{m,2k}$ be the Z_q-manifold (R^{m+2k}, Z_q) with the action

$$\mu: Z_q \times R^{m+2k} \longrightarrow R^{m+2k}$$

defined by

$$\mu(T, (t_1,\dots, t_m, z_{m+1},\dots, z_{m+k})) = (t_1,\dots, t_m, Tz_{m+1},\dots, Tz_{m+k}),$$

where $T(=e^{2\pi\sqrt{-1}/q})$ is the generator of Z_q, $t_1,\dots, t_m$ are real numbers $(\in R)$, and $z_{m+1},\dots, z_{m+k}$ are complex numbers (eC$=R^2$).

Let S^{2n+1} be the unit $(2n+1)$-sphere in complex $(n+1)$-space C^{n+1}. Let (S^{2n+1}, Z_q) be the Z_q-manifold defined by the action

$$v: Z_q \times S^{2n+1} \longrightarrow S^{2n+1};\ v(T, (z_0,\dots, z_n)) = (Tz_0,\dots, Tz_n),$$

where $z_0,\dots, z_n$ *arc* complex numbers with $\sum_{j=0}^{n}|z_j|^2=1$ The action v is free and differentiable of class C^∞. The orbit manifold S^{2n+1}/Z_q is the standard lens space $L^n(q) \bmod q$.

A. Jankowski obtained in [7] some non-existence theorems for Z_2-immersions. In this note we consider Z_q-immersions $(S^{n+1}, Z_q) \to R^{m,2k}$, and study the bounds of m for fixed k and n.

As is easily seen, there is a Z_q-immersion of (S^{2n+1}, Z_q) in $R^{m,0}$ if and only if there is an immersion of $L^n(q)$ in R^m.

If $k>n$, (S^{2n+1}, Z_q) is Z_q-immersible in $R^{m\ 2k}$ for any m, clearly. In case $k \leqq n$, we have the following results.

THEOREM 1. *Let q be an integer >1. Then (S^{2n+1}, Z_q) is not Z_q-immer-*

sible in $R^{2[n/2],2n}$.

THEOREM 2. *Let p be an odd prime and r be a positive integer. Let n and k be integers with* $0\leqq k\leqq n$. *Put*

$$L=\max\left\{i \mid 1\leqq i\leqq [n/2],\ \binom{n-k+i}{i}\not\equiv 0 \bmod p^{r+[(n-2i)/(p-1)]}\right\}.$$

Then there does not exist a Z_{p^r}*-immersion of* (S^{2n+1}, Z_{p^r}) *in* $(R^{2n+2L}, Z_{p^r}) = R^{2n+2L-2k,2k}$.

Iffc = 0, Theorem 2 is a consequence of Corollary 3.6 of T. Kawaguchi and M. Sugawara in [9]. A similar result for (S^{2n+1}, Z_4) is obtained from Corollary 6.7 in [12].

THEOREM 3. *Let p be an odd prime and r be a positive integer. Let n and k be integers with* $0\leqq k\leqq n$. *Assume that there is an integer m satisfying the following conditions:*

(i) $0<k+m\leqq [n/2]$,

(ii) $\binom{n+m}{n-k}\not\equiv (sp)^2 \bmod p$ *for any integer s with* $0\leqq s<p^{r-1}$,

(iii) $n+m+1\not\equiv 0 \bmod p^{[(n-m-k-1)/(p-1)]}$.

Then there does not exist a Z_{p^r}*-immersion of* (S^{2n+1}, Z_{p^r}) *in* $(R^{2n+2m+2k+1}, Z_{p^r}) = R^{2n+2m+1,2k}$.

If $k=0$, we have a corollary which establishes the non-existence of an immersion of $L^n(p^r)$ in $R^{2n+2m+1}$ for some n and m. This corollary gives a generalization of Theorem C in [10] and Theorem 7.9 in [12].

There is an example of a Z_q-immersion of S^{2n+1} in $R^{m,2k}$ when both m and $2k$ are less than $2n+1$. Let $(re^{a\sqrt{-1}}, se^{b\sqrt{-1}})$ be a point of S^3, where a, b, r and s are real numbers with $r^2+s^2=1$. Then the map f: $S^3\to R^4$, defined by the equality

$$f(re^{a\sqrt{-1}}, se^{b\sqrt{-1}})=((r+s+2)e^{qa\sqrt{-1}}, (r-s-2)e^{b\sqrt{-1}}),$$

is a Z_q-immersion of (S^3, Z_q) in $R^{2,2}$, where q is any integer >1.

In § 2, the proofs of Theorems 1 and 2 are carried out by making use of the γ-operation in KO-theory (cf. [3]) and the fact that the generator $\bar{\sigma}^i \in \widetilde{KO}(L^n(p^r))$ is of order $p^{r+[(n-2i)/(p-1)]}$ ([9, Theorem 1.1]). In §3, we prove Propositions 3.1 and 3.2 which show the existence of some relations between the immersions of lens spaces and the stable homotopy types of the stunted lens spaces. The proof of Theorem 3 is based on these propositions. In § 4, the method in § 3 is applied to Z_2-equivariant immersions, and some results (Theorems 4.1 and 4.2) are obtained.

§ 2. γ-operations in KO-theory

For a Z_q-space (X, Z_q), let $\Theta = \theta(X, Z_q)$ be a Z_q-vector bundle $(X \times R^2, X, p_1, R^2)$ defined as follows.

(1) $p_1 \colon X \times R^2 \to X$ is the projection onto the first factor.

(2) The action of Z_q on $X \times R^2$ is the diagonal action: $T(x, z) = (Tx, Tz)$, where $x \in X$, $z \in R^2$, and $T(=e^{2\pi\sqrt{-1}/q})$is the generator of Z_q.

Then we have immediately the following

LEMMA 2.1. *///: $X \to Y$ is a Z_q-map between Z_q-spaces X and Y, we have* $f^*\theta(Y, Z_q) = \theta(X, Z_q)$.

A G-vector bundle $E \to X$ determines naturally a vector bundle $E/G \to X/G$. This correspondence induces a homomorphism

$$\rho \colon KO_G(X) \longrightarrow KO(X/G).$$

It is well-known that p is an isomorphism if the G-action on X is free (cf. [4, Proposition 1.3.1]).

Let η be the real restriction of the canonical complex line bundle over $L^n(q)$. Considering the transition functions we see easily

LEMMA 2.2. $\rho(\theta(S^{2n+1}, Z_q)) = \eta$.

Define the action of Z_q on the total space $E_{m,2k}$ of the Whitney sum of the m-dimensional trivial bundle m over $R^{m,2k}$ and $k\theta(R^{m+2k}, Z_q)$ as follows:

$$T((u, t_1), \dots, (u, t_m), (u, z_{m+1}), \dots, (u, z_{m+k}))$$
$$= ((Tu, t_1), \dots, (Tu, t_m), (Tu, Tz_{m+1}), \dots, (Tu, Tz_{m+k})),$$

where $u \in R^{m\ 2k}$, $t_i \in R$ ($i = 1, \dots, m$), $z_{m+j} \in R^2$ ($j = 1, \dots, k$), and T is the generator of Z_q. Then the following holds.

LEMMA 2.3. *There is a Z_q-bundle isomorphism of the tangent Z_q-bundle $\tau(R^{m,2k})$ onto the Z_q-bundle $m \oplus k\theta(R^{m+2k}, Z_q)$.*

PROOF. Carrying a point $(u, (t_1, \dots, t_m, z_{m+1}, \dots, z_{m+k}))$ of the total space of $\tau(R^{m\,2k})$ into a point $((u, t_1), \dots, (M, t_m), (u, z_{m+1}), \dots, (M, z_{m+k})) \in E_{m,2k}$, we obtain a desired Z_q-bundle isomorphism. q. e. d.

Now we are in a position to prove

PROPOSITION 2.4. *Let q be an integer >1, and n and k be integers with $0 \leqq k \leqq n$. Put*

$$L = \max\left\{j \mid \binom{n-k+j}{j}\bar{\sigma}^j \neq 0\right\}, \quad \textit{where} \quad \sigma = \eta - 2 \in \widetilde{KO}(L^n(q)).$$

Then there does not exist 0 *Z_q-immersion of* (S^{2n+1}, Z_q) *into* $(R^{2n+2L}, Z_q) = R^{2n+2L-2k, 2k}$.

PROOF. Suppose that there exists a Z_q-immersion /: $(S^{2n+1}, Z_q) \to (R^{m+2k}, Z_q) = R^{m,2k}$. Let ν_f be the normal Z_q-bundle of f. Then we have

$$\tau(S^{2n+1}, \ Z_q) \oplus \nu_f = f^*\tau(R^{m,2k}).$$

Applying p to the equation, noting that $\rho(\tau(S^{2n+1}, Z_q)) = \tau(L^n(q))$, and using Lemmas 2.1-2.3, we get

$$\tau(L^n(q)) \oplus \rho\nu_f = \rho f^*(m \oplus k\theta(R^{m+2k}, Z_q)) = m \oplus k\rho\theta(S^{2n+1}, Z_q) = m \oplus k\eta.$$

Since $\tau(L^n(q)) \oplus 1 = (n+1)\eta$, we have

$$\rho\nu_f = m + 1 + (k-n-1)\eta = m + 2k - 2n - 1 + (k-n-1)\bar{\sigma} \quad \text{in} \quad KO(L^n(q)).$$

Let $\nu_0 = (k-n-1)\bar{\sigma}$ be the stable class of $\rho\nu_f$. Then it follows that

$$g.\dim \nu_0 \leqq m + 2k - 2n - 1,$$

where $g.\dim \nu_0$ denotes the geometric dimension of ν_0. According to [3, Proposition (2.3)], $\gamma^j(\nu_0) = 0$ for $j > g.\dim \nu_0$, where $\gamma^j : KO(X) \to KO(X)$ is the Grothendieck γ-operation. Hence we see

$$\gamma^j(\nu_0) = 0 \qquad \text{for} \quad j > m + 2k - 2n - 1.$$

On the other hand,

$$\begin{aligned} \textstyle\sum_{j=0}^{\infty} \gamma^j(\nu_0) t^j &= \gamma_t(\nu_0) = \gamma_t((k-n-1)\bar{\sigma}) = \gamma_t(\bar{\sigma})^{k-n-1} \\ &= (1 + \bar{\sigma}t - \bar{\sigma}t^2)^{k-n-1} = \textstyle\sum_{j=0}^{\infty} \binom{k-n-1}{j} \bar{\sigma}^j (t-t^2)^j \\ &= \textstyle\sum_{j=0}^{\infty} (-1)^j \binom{n-k+j}{j} \bar{\sigma}^j (t-t^2)^j. \end{aligned}$$

(Here we use the fact that fcrgn.) For the given L, we have

$$\gamma_t(\nu_0) = \textstyle\sum_{j=0}^{L} (-1)^j \binom{n-k+j}{j} \bar{\sigma}^j (t-t^2)^j = \textstyle\sum_{j=0}^{2L} \gamma^j(\nu_0) t^j.$$

Therefore $\gamma^{2L}(\nu_0) \neq 0$. This implies that $2L \neq m + 2k - 2n$, as desired. **q.e.d.**

PROOF OF THEOREM 1. If fc $= n$, we have $L = [n/2]$ by [9, Proposition 2.6]. Then Theorem 1 follows from Proposition 2.4. **q.e.d.**

PROOF OF THEOREM 2. If $q = p^r$, the order of $\bar{\sigma}^j$ is equal to $p^{r+[(n-2j)/(p-1)]}$ ([9, Theorem 1.1]). Hence Theorem 2 also follows from Proposition 2.4. q.e.d.

§3. Mod p^r S-relation

For a space X and an integer $k \geqq 0$, we denote by $S^k X$ the fc-fold suspension of X. Two spaces X and Y are said to be mod q *S-related,* if there are non-negative integers a and b, and a map f: $S^a X \to S^b Y$ which induces isomorphisms of all homology groups with Z_q-coefficients. We prepare the following two propositions for the proof of Theorem 3.

PROPOSITION 3.1. *Let p be an odd prime, r be a positive integer, and l and n be integers with $0 < l \leqq [n/2]$. Suppose that there is a positive integer t satisfying the following conditions:*

(i) *$(l+t)\eta$ has linearly independent $2t$ cross-sections, where η is the real restriction of the canonical complex line bundle over $L^n(p^r)$.*

(ii) *$\binom{[illegible]}{[illegible]} \not\equiv (sp)^2 \bmod p^r$ for any integer s with $0 \leqq s < p^{r-1}$.*

Then the stunted lens spaces $L^n(p^r)/L^{l-1}(p^r)$ and $L^{n+t}(p^r)/L^{l-1+t}(p^r)$ are mod p^r *S-related.*

PROPOSITION 3.2. *Let p be a prime, r be a positive integer with $p^r \neq 2$, and l and n be integers with $0 < l < n$. // the stunted lens spaces $L^n(p^r)/L^{l-1}(p^r)$ and $L^{n+t}(p^r)/L^{l-1+t}(p^r)$ are* mod p^r *S-related, we have*

$$t = 0 \mod p^{[(n-l-1)/(p-1)]}.$$

PROOF OF PROPOSITION 3.1. Let $q = p^r$ and $L^n = L^n(p^r)$ It follows from the property (i) that there is a 2/-dimensional vector bundle α over L^n satisfying

$$(1) \qquad (l+t)\eta = 2t \oplus \alpha.$$

Let $\chi(\alpha)$ and $p_j(\alpha)$ be the Euler class and the j-th Pontrjagin class of α, respectively. Then it is well-known that

$$\chi(\alpha)^2 = p_l(\alpha) = \binom{l+t}{l} x^{2l},$$

where x is the generator of $H^2(L^n; Z)$. Now, our assumptions imply that

$$(2) \qquad \chi(\alpha)_q = u(x^l)_q,$$

where z_q denotes the image of z by the mod q reduction, and u is some integer such that $u \not\equiv 0 \bmod p$.

Let D and E denote the total spaces of the disk and the sphere bundles associated with the bundle α, respectively. Consider the commutative diagram (cf. [14])

$$\begin{array}{ccc} H^k(D, E; Z_q) & \xrightarrow{j*} & H^k(D; Z_q) \\ \uparrow \phi & & \uparrow \pi^* \\ H^{k-2l}(L^n; Z_q) & \xrightarrow{h} & H^k(L^n; Z_q), \end{array}$$

where j^* is the **homomorphism** induced by the inclusion j: $D \rightarrow (D,$ E), π* is the isomorphism induced by the projection π: $D \rightarrow L^n$, ϕ is the **Thom** isomorphism, and h is the homomorphism defined by

$$\text{AGO} = y \text{ U } \chi(\alpha)_q, \quad \text{for } y \in H^{k-2l}(L^n; Z_q).$$

It follows from (2) that A is isomorphic for each fc with $2l \leqq k \leqq 2n+1$. By $(L^n)^\alpha$ we mean the **Thom** complex D/E of α. Then we see from the above commutative diagram that the natural inclusion λ: $L^n \rightarrow (L^n)^\alpha$ induces isomorphisms

$$\lambda^* : H^k((L^n)^\alpha ; Z_q) \longrightarrow H^k(L^n; Z_q) \text{for } 2l \leqq k \leqq 2n + l.$$

Since $(L^n)^\alpha$ is $(2l-1)$**-connected,** there is a map /: $L^n/L^{l-1} \rightarrow (L^n)^\alpha$ such that λ is homotopic to fp, where p: $L^n \rightarrow L^n/L^{l-1}$ is the projection. It is easy to see that / induces isomorphisms

$$f^* : H^k((L^n)^\alpha Z_q) \longrightarrow H^k(L^n/L^{l-1}; Z_q) \text{for } 0 \leqq k \leqq 2n + 1.$$

According to [2, Lemma (2.4)] and [8, Theorem 1], there are natural homeomorphisms

$$S^{2t}(L^n)^\alpha \sim (L^n)^{2t\oplus\alpha}, \ (L^n)^{(l+t)\eta} \sim L^{n+l+t}/L^{l-1+t}.$$

Thus we obtain, by (1), a natural homeomorphism

$$\psi : S^{2t}(L^n)^\alpha \longrightarrow L^{n+l+t}/L^{l-1+t}.$$

Then there is a map

$$\text{flf}: S^{2t}(L^n/L^{l-1}) \longrightarrow L^{n+t}/L^{l-1+t}$$

such that $\psi S^{2t}f$ is homotopic to ig, where $S^{2t}f$is the $2t$**-fold** suspension **of** f and ί: $L^{n+t}/L^{l-1+t} \rightarrow L^{n+l+t}/L^{l-1+t}$ is the natural inclusion. We see that the map g induces isomorphisms of all cohomology groups with Z_q**-coefficients.** Hence g induces isomorphisms of all homology groups with Z_q**-coefficients.** q. e. d.

PROOF OF PROPOSITION 3.2. Put $q = p^r$ and $L^n = L^n(p^r)$. By the assumption there are non-negative integers a and b, and a map

$$g: S^a(L^n/L^{l-1}) \longrightarrow S^b(L^{n+t}/L^{l-1+t})$$

which induces isomorphisms of all homology groups with Z_q-coefficients. We may assume that g is cellular. It is not difficult to prove that g induces a map

$$g_0: S^a(L_0^n/L_0^l) \longrightarrow S^b(L_0^{n+t}/L_0^{l+t})$$

which induces isomorphisms of all homology groups with Z_q-coefficients, where L_0^n denotes the $2n$-skeleton of L^n. Obviously g_0 is a homotopy equivalence, and the result follows from [13, Theorem 1.1]. q. e. d.

PROOF OF THEOREM 3. Put $q=p^r$. Suppose that there exists a Z_q-immersion

$$f: (S^{2n+1}, Z_q) \longrightarrow R^{2n+2m+1,2k}.$$

Let ν_f be the normal Z_q-bundle of f. Then we have

$$\tau(L^n(q)) \oplus \rho\nu_f = (2n+2m+1) \oplus k\eta,$$

as in the proof of Proposition 2.4. Therefore

$$(n+1-k)\eta \oplus \rho\nu_f = 2n+2m+2.$$

Take a sufficiently large positive integer a so that $A = ap^{r+[(n-2)/(p-1)]}$ satisfies $2A+2k-2n-2>2n+1$. Then $A\tilde{\sigma}=0$, because $\sigma\,(=\eta-2 \in \widetilde{KO}(L^n(q)))$ is of order $p^{r+[(n-2)/(p-1)]}$ ([9, Theorem 1.1]), and hence we have

$$(A-n-1+k)\eta = (2A-2n-2m-2)+\rho\nu_f = (2A-2n-2m-2)\oplus\rho\nu_f.$$

Putting $t=A-n-m-1$ and $l=k+m$, we see that the conditions (i) and (ii) of Proposition 3.1 are satisfied. (Notice that a can be taken sufficiently large.) Hence by Propositions 3.1 and 3.2 we get

$$t=A-n-m-1 = -n-m-1 = 0 \bmod p^{[(n-m-k-1)/(p-1)]}.$$

But this contradicts the assumption (iii). q. e. d.

§ 4. Z_2-equivariant immersions

Let (S^n, Z_2) be the Z_2-manifold whose Z_2-action on S^n is the antipodal involution, and $R^{m,k}$ be the Z_2-manifold (R^{m+k}, Z_2) whose Z_2-action on R^{m+k} is given by the involution

$$\Gamma: R^{m+k} \longrightarrow R^{m+k}$$

$$T(t_1,\dots,t_m,\ t_{m+1},\dots,\ t_{m+k}) = (t_1,\dots,\ t_m,\ -t_{m+1},\dots,\ -t_{m+k}),$$

where T is the generator of Z_2 and $t_1,\dots, t_{m+k}$ are real numbers.

For integers m and n with $0 \leqq m < n$, denote by $\phi(n, m)$ the number of integers s such that $m < s \leqq n$ and $s \equiv 0$, 1, 2 or 4 mod 8. Let $\psi(n, m)$ be the integer given by

$$\psi(n, m) = \begin{cases} \phi(n, m), & \text{if } m = 0 \bmod 4, \\ \phi(n, m-1), & \text{if } m \not\equiv 0 \bmod 4. \end{cases}$$

THEOREM 4.1. *Let k and n be integers such that $0 \leqq k \leqq n$ and*

(o) $n+1-k \equiv 0 \bmod 8$.

Assume that there is a non-negative integer m satisfying the following conditions:

(i) $n < m+k \leqq 2n$,

(ii) $\binom{m+1}{n+1-k} \not\equiv 0 \bmod 2$,

(iii) $n+1-k \not\equiv 0 \bmod 2^{\psi(n,m+k-n)-1}$.

Then there does not exist a Z_2-immersion of (S^n, Z_2) in $R^{m,k} = (R^{m+k}, Z_2)$.

THEOREM 4.2. *Suppose that the conditions* (o) *and* (iii) *in Theorem* 4.1 *are replaced by the following condition:*

$n+1 \equiv 0 \bmod 2^{\phi(2n-m-k,0)}$ *if and only if* $2n+2-k \not\equiv 0 \bmod 2^{\phi(2n-m-k,0)}$.

Then the same conclusion as Theorem 4.1 *holds.*

PROOFS OF THEOREMS 4.1 AND 4.2. Assume that there is a Z_2-immersion f: $(S^n, Z_2) \rightarrow R^{m,k}$. Let ν_f be the normal Z_2-bundle of f. Then it holds that

$$(n+1)\xi \oplus \rho\nu_f = (m+1) \oplus k\xi \text{ (cf. the proof of Theorem 3.4 in [7])},$$

where ξ is the canonical line bundle over the real projective space RP^n. Since $\xi \otimes \xi = 1$, we have

$$(m+1)\xi = (n+1-k) \oplus \rho\nu_f \otimes \xi. \tag{1}$$

Applying Lemmas (4.1) and (4.2) in [11] to the equality (1), we obtain Theorem 4.1.

Applying Lemma (4.1) in [11] to the equality (1), and using the fact that RP^n/RP^{m-1} is S-reducible if and only if $n+1 \equiv 0 \bmod 2^{\phi(n-m,0)}$ (cf. [6, (3.1)], [5, (2.7)] and [1]), we have Theorem 5.2. q. e. d.

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