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On Lie algebras in which every subalgebra is a subideal

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Introduction

Heineken and Mohamed [4] have constructed a Fitting, metabelian group with trivial centre in which every subgroup is subnormal. In Lie theory, Unsin [10] has constructed a Fitting, metabelian Lie algebra with trivial centre in which every subalgebra is a subideal. As in group theory, the class $\mathfrak{D}$ of Lie algebras in which every subalgebra is a subideal is one of the typical classes of generalized nilpotent Lie algebras.

Recently Brookes [2] has proved that a hyperabelian group in which no non-trivial section is perfect and in which every subgroup is subnormal, is soluble ([2, Theorem A]), and he has concluded that a hypercentral group in which every subgroup is subnormal, is soluble ([2, Corollary A]). Subsequently, generalizing [2, Theorem A], Casolo [3] has proved that a group in which no non-trivial section is perfect and in which every subgroup is subnormal, is soluble ([3, Theorem]). The purpose of this paper is to present the Lie-theoretic analogues of [2, Theorem A and Corollary A] and [3, Theorem].

We shall first prove that $\mathfrak{D}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{A}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}\leqq\mathrm{E}\mathfrak{A}$ (Corollary 1), where $\acute{\mathrm{E}}(\lhd)\mathfrak{A}$ is the class of hyperabelian Lie algebras, $(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}$ is the largest Q-closed subclass of the class of hypoabelian Lie algebras and $\mathrm{E}\mathfrak{A}$ is the class of soluble Lie algebras. The group-theoretic analogue of this result is also true and is a slight generalization of [2, Theorem A]. We shall secondly prove that over any field $\mathfrak{k}$ of characteristic zero $\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}\leqq\mathrm{E}\mathfrak{A}$ (Theorem 2), where $(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$ is the largest Q-, S-closed subclass of the class of hypoabelian Lie algebras and is equal to the class of Lie algebras in which no non-trivial section is perfect.

1.

Throughout this paper we always consider not necessarily finite-dimensional Lie algebras over a field $\mathfrak{k}$ of arbitrary characteristic unless otherwise specified. Notations and terminology are based on [1]. But for the sake of convenience we list the terms that we use here.

Let L be a Lie algebra over a field $\mathfrak{k}$ and n be a non-negative integer. By $H\leqq L$ (resp. $H\lhd L$, $H \operatorname{ch} L$, $H\lhd^n L$, $H \operatorname{si} L$), we mean that H is a subalgebra (resp. an ideal, a characteristic ideal, an n-step subideal, a subideal) of L. If H si L, then there exists the least integer n with respect to $H \lhd^n L$, which we denote by si $(L:H)$ in [5]. For $H \leqq L$, H^L denotes the smallest ideal of L containing H. For a positive integer n, L^n denotes the n-th term of the lower central series of L. Angular brackets $\langle\ \rangle$ denote the subalgebra generated by their contents.

A class $\mathfrak{X}$ is a collection of Lie algebras together with their isomorphic copies and the 0-dimensional Lie algebras. $\mathfrak{A}$ (resp. E$\mathfrak{A}$, $\mathfrak{A}^n$, RE$\mathfrak{A}$, $\mathfrak{E}$, $\mathfrak{F}$, $\mathfrak{F}$t, $\mathfrak{N}$, $\mathfrak{N}_n$, $\mathfrak{Z}$) is the class of Lie algebras which are abelian (resp. soluble, soluble of derived length $\leqq n$, residually soluble, Engel, finite-dimensional, Fitting, nilpotent, nilpotent of class $\leqq n$, hypercentral). $\mathfrak{D}$ is the class of Lie algebras in which every subalgebra is a subideal. For a positive integer s, $\mathfrak{D}_{s,s}$ is the class of Lie algebras L such that $\langle x_1,\cdots, x_s\rangle \lhd^s L$ for all $x_i \in L$ $(1 \leqq i \leqq s)$.

Let $\mathfrak{X}$ be a class of Lie algebras. L is called an $\mathfrak{X}$-algebra if $L \in \mathfrak{X}$. An ascending $\mathfrak{X}$-series $\{L_\alpha\colon \alpha \leqq \rho\}$ of L is a family of subalgebras of L such that

(a) $L_0 = \{0\}$ and $L_\rho = L$;

(b) $L_\alpha \lhd L_{\alpha+1}$ and $L_{\alpha+1}/L_\alpha \in \mathfrak{X}$ for any ordinal $\alpha < \rho$;

(c) $L_\mu = \bigcup_{\alpha<\mu} L_\alpha$ for any limit ordinal $\mu \leqq \rho$.

L is called a hyper $\mathfrak{X}$-algebra if L has an ascending $\mathfrak{X}$-series $\{L_\alpha\colon \alpha \leqq \rho\}$ such that $L_\alpha \lhd L$ for all $\alpha \leqq \rho$. The class of hyper $\mathfrak{X}$-algebras is denoted by $\acute{\text{E}}(\lhd)\mathfrak{X}$. In particular, $\acute{\text{E}}(\lhd)\mathfrak{A}$ is the class of hyperabelian Lie algebras. For an ordinal α, $L^{(\alpha)}$ denotes the α-th term of the transfinite derived series of L. We use $L^{(*)}$ to denote the intersection of all the $L^{(\alpha)}$'s. L is said to be hypoabelian if $L^{(*)} = \{0\}$. $\grave{\text{E}}\mathfrak{A}$ is the class of hypoabelian Lie algebras. $L \in \text{RE}\mathfrak{A}$ iff $L^{(\omega)} = \{0\}$. It follows that $\text{RE}\mathfrak{A} \leqq \grave{\text{E}}\mathfrak{A}$. $\mathfrak{X}$ is s-closed (resp. Q-closed) if $H \in \mathfrak{X}$ (resp. $L/H \in \mathfrak{X}$) whenever $H \leqq L$ (resp. $H \lhd L$) and $L \in \mathfrak{X}$. We use $\mathfrak{X}^{\text{Q}}$ (resp. $\mathfrak{X}^{\text{QS}}$) to denote the largest Q-closed (resp. Q-, S-closed) subclass of $\mathfrak{X}$.

As in group theory, we say that H/K is a section of L if $K \lhd H \leqq L$. L is said to be perfect if $L^2 = L$. Then we have

LEMMA 1. *$L \in (\grave{\text{E}}\mathfrak{A})^{\text{QS}}$ if and only if no non-trivial section of L is perfect.*

PROOF. Let $\mathfrak{X}$ be the class of Lie algebras in which no non-trivial section is perfect. Since perfect hypoabelian Lie algebras must be 0-dimensional, we have $(\grave{\text{E}}\mathfrak{A})^{\text{QS}} \leqq \mathfrak{X}$. Let $L \in \mathfrak{X}$ and suppose that $L^{(*)} \neq \{0\}$. Since $L^{(*)}$ is a non-trivial section of L, $L^{(*)} = (L^{(*)})^2 < L^{(*)}$, a contradiction. It follows that $\mathfrak{X} \leqq \grave{\text{E}}\mathfrak{A}$. Since $\mathfrak{X}$ is Q-, S-closed, we have $\mathfrak{X} \leqq (\grave{\text{E}}\mathfrak{A})^{\text{QS}}$.

2.

In this section we shall present the Lie-theoretic analogues of [2, Theorem A and Corollary A].

We begin with the following

THEOREM 1. *Let* $L\in\mathfrak{D}$. *If* L *has an ascending* $\mathfrak{A}$*-series* $\{L_\alpha\colon \alpha\leqq\rho\}$ *such that* $L_\alpha \lhd L$ *and* $L/L_\alpha \in \acute{\mathrm{E}}\mathfrak{A}$ *for all ordinals* $\alpha\leqq\rho$, *then* $L\in\mathrm{E}\mathfrak{A}$.

PROOF. Assume that $L\notin\mathrm{E}\mathfrak{A}$. Then there is the least ordinal $\mu\leqq\rho$ with respect to $L_\mu\notin\mathrm{E}\mathfrak{A}$. Clearly $\mu>0$. Since $L_\alpha\in\mathrm{E}\mathfrak{A}$ for all $\alpha<\mu$, μ is a limit ordinal. The method of proof is essentially that used by Brookes in proving [2, Theorem A]. We aim to construct a sequence $\{H_i\}_{i=1}^\infty$ of subalgebras of L_μ, strictly ascending sequences $\{n(i)\}_{i=1}^\infty$ and $\{s(i)\}_{i=1}^\infty$ of positive integers and a sequence $\{\alpha(i)\}_{i=1}^\infty$ of ordinals $<\mu$, which satisfy the following conditions:

(i) for each $i>1$, H_i is a finitely generated subalgebra of $L_\mu^{(n(i-1))}$;

(ii) for each $i>1$, $s(i)=\mathrm{si}\,(K_{i,n(i-1)}/K_{i,n(i)}\colon (H_i+K_{i,n(i)})/K_{i,n(i)})$, where $K_{i,j}=L_\mu^{(j)}+L_{\alpha(i-1)}$ $(j=1,2,\cdots)$;

(iii) for each $i\geqq 1$, $\langle H_1,\cdots,H_i\rangle\leqq L_{\alpha(i)}$.

We set $H_1=\{0\}$, $n(1)=s(1)=1$ and $\alpha(1)=1$. Let $i>1$ and suppose that those have been constructed up to the $(i-1)$-th terms. For convenience sake, we set $n=n(i-1)$, $s=s(i-1)$ and $\alpha=\alpha(i-1)$. Clearly $K_{i,1}\geqq K_{i,2}\geqq\cdots$. Suppose that $K_{i,j}=K_{i,j+1}$ for some $j\geqq1$. Then $(L_\mu/L_\alpha)^{(j)}=K_{i,j}/L_\alpha=K_{i,j+1}/L_\alpha=(L_\mu/L_\alpha)^{(j+1)}$. It follows that $(L_\mu/L_\alpha)^{(j)}=(L_\mu/L_\alpha)^{(*)}\leqq(L/L_\alpha)^{(*)}=\{0\}$. Hence $L_\mu/L_\alpha\in\mathrm{E}\mathfrak{A}$. Since $\alpha<\mu$, $L_\alpha\in\mathrm{E}\mathfrak{A}$. Therefore we have $L_\mu\in\mathrm{E}\mathfrak{A}$, a contradiction. Thus we obtain $K_{i,1}>K_{i,2}>\cdots$.

By using [1, Theorem 7.2.5], we can find a positive integer m such that $\mathfrak{D}_{s,s}\leqq\mathfrak{N}_m$. Define $n(i)=n+m+1$. Let ψ_i denote the natural map $K_{i,n}\to K_{i,n}/K_{i,n(i)}$. Suppose that $\mathrm{si}\,(\psi_i(K_{i,n})\colon\psi_i(X))\leqq s$ for all finitely generated subalgebras X of $L_\mu^{(n)}$. Then we have $\psi_i(L_\mu^{(n)})=\psi_i(K_{i,n})\in\mathfrak{D}_{s,s}\leqq\mathfrak{N}_m$. Hence $\psi_i(K_{i,n(i)-1})=\psi_i(L_\mu^{(n+m)})=\psi_i(L_\mu^{(n)})^{(m)}\leqq\psi_i(L_\mu^{(n)})^{m+1}=\{0\}$ and therefore $K_{i,n(i)-1}=K_{i,n(i)}$. This is a contradiction. Thus there exists a finitely generated subalgebra H_i of $L_\mu^{(n)}$ such that $\mathrm{si}\,(\psi_i(K_{i,n})\colon\psi_i(H_i))>s$. Define $s(i)=\mathrm{si}\,(\psi_i(K_{i,n})\colon\psi_i(H_i))$. It is clear that $\langle H_1,\cdots,H_i\rangle$ is a finitely generated subalgebra of L_μ. Since μ is a limit ordinal, there exists an ordinal $\alpha(i)<\mu$ such that $\langle H_1,\cdots,H_i\rangle\leqq L_{\alpha(i)}$. Therefore the i-th terms have been defined. Thus $\{H_i\}$, $\{n(i)\}$, $\{s(i)\}$ and $\{\alpha(i)\}$ can be inductively constructed.

We now set $H=\langle H_i\colon i=1,2,\cdots\rangle$ and $r=\mathrm{si}\,(L_\mu\colon H)$. Since the sequence $\{s(i)\}$ is strictly ascending, there is a positive integer t such that $r<s(t)$. Let ψ denote the natural map $L_\mu\to L_\mu/K_{t,n(t)}$. Then evidently $\psi|_{K_{t,n(t-1)}}=\psi_t$. Let i be a positive integer. If $i\leqq t-1$, then $\psi(H_i)=\{0\}$ since $\langle H_1,\cdots,H_i,\cdots,H_{t-1}\rangle\leqq L_{\alpha(t-1)}\leqq K_{t,n(t)}$. If $i\geqq t+1$, then $\psi(H_i)=\{0\}$ since $H_i\leqq L_\mu^{(n(i-1))}\leqq L_\mu^{(n(t))}\leqq K_{t,n(t)}$. Hence we have $\psi(H)=\langle\psi(H_i)\colon \mathrm{i}=1,2,\cdots\rangle=\psi_t(H_t)\leqq\psi_t(K_{t,n(t-1)})$. Since $\psi(H)\lhd^r\psi(L_\mu)$, $\psi(H)=\psi_t(H_t)\lhd^r\psi_t(K_{t,n(t-1)})$. Thus $s(t)=\mathrm{si}\,(\psi_t(K_{t,n(t-1)})\colon\psi_t(H_t))\leqq r<s(t)$. This is the final contradiction. Therefore we have $L\in\mathrm{E}\mathfrak{A}$.

COROLLARRY 1. $\mathfrak{D}\cap \acute{\mathrm{E}}(\lhd)\mathfrak{A}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}\leq \mathrm{E}\mathfrak{A}$.

REMARK. The proof of Theorem 1 can carry over in group theory without difficulties. Therefore the group-theoretic analogues of Theorem 1 and Corollary 1, which are slight generalizations of [2, Theorem A], are also true.

COROLLARY 2. *Let $\mathfrak{X}$ be a class of Lie algebras. If $\mathfrak{Z}\leq\mathfrak{X}\leq\acute{\mathrm{E}}(\lhd)\mathfrak{F}$, then $\mathfrak{D}\cap\mathfrak{X}\leq\mathrm{E}\mathfrak{A}$.*

PROOF. By [1, Lemma 8.1.1] we have $\mathfrak{Z}\leq(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}$. It follows from Corollary 1 that $\mathfrak{D}\cap\mathfrak{Z}\leq\mathrm{E}\mathfrak{A}$. Since $\mathfrak{D}\leq\mathfrak{E}$, by [6, Theorem 8] we have $\mathfrak{D}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{F}=\mathfrak{D}\cap\mathfrak{E}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{F}=\mathfrak{D}\cap\mathfrak{Z}$.

In Theorem 1 the assumption that $L\in\mathfrak{D}$ is essential. In fact, the following proposition shows that in Theorem 1 we cannot replace the assumption that $L\in\mathfrak{D}$ by the assumption that $L\in\mathfrak{F}\mathrm{t}$.

PROPOSITION 1. *Over any field $\mathfrak{k}$, there exists a non-soluble, Fitting Lie algebra L having an ascending $\mathfrak{A}$-series $\{L_n: n\leq\omega\}$ such that $L_n\lhd L$ and $L/L_n\in\mathrm{RE}\mathfrak{A}$ for all $n\leq\omega$.*

PROOF. We here consider the McLain Lie algebra $L=\mathscr{L}t(N)$ over $\mathfrak{k}$ (cf. [1, p. 111]), where N is the set of positive integers with natural ordering. Then L has basis $\{a_{ij}: i,j\in N, i<j\}$ with multiplications $[a_{ij}, a_{kl}]=\delta_{jk}a_{il}-\delta_{li}a_{kj}$. It is well known (cf. [1, p.119]) that $L\in\mathfrak{F}\mathrm{t}$. We can easily verify that $L^{(n)}=\langle a_{ij}: j-i\geq 2^n\rangle\neq\{0\}$ $(n=0, 1,\cdots)$ and $L^{(\omega)}=\bigcap_{n<\omega}L^{(n)}=\{0\}$. Therefore $L\in\mathrm{RE}\mathfrak{A}\backslash\mathrm{E}\mathfrak{A}$. For each positive integer n, we set $L_n=\langle a_{ij}: i\leq n\rangle$ and $K_n=\langle a_{ij}: n<i\rangle$. Then it is not hard to see that $L_n\lhd L=L_n+K_n$ and $L_n\cap K_n=\{0\}$. Set $L_0=\{0\}$ and $L_\omega=L$. For any positive integer n, we have $L_n/L_{n-1}=\langle a_{nj}+L_{n-1}: n<j\rangle\in\mathfrak{A}$. Since $L=\bigcup_{n<\omega}L_n$, $\{L_n: n\leq\omega\}$ is an ascending $\mathfrak{A}$-series of ideals of L. Furthermore, it can be easily seen that for any positive integer n, $L/L_n\cong K_n\cong L\in\mathrm{RE}\mathfrak{A}$.

3.

In this section we shall consider $\mathfrak{D}$-algebras over a field $\mathfrak{k}$ of characteristic zero and present the Lie-theoretic analogue of [3, Theorem]. The method of proof is essentially that used by Casolo in proving [3, Theorem].

We need the following

LEMMA 2. *Let L be a Lie algebra over a field $\mathfrak{k}$ of characteristic zero. If $\{0\}\neq L\in\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$, then L has a non-trivial abelian ideal.*

PROOF. We denote by $n(L)$ a minimal member of $\{\mathrm{si}(L:\langle x\rangle): 0\neq x\in L\}$ and show the result by using induction on $n(L)$. If $n(L)=0$, then L is 1-dimensional and so the result is true. Let $n(L)\geqq 1$. There is a non-zero element x of L such that $n(L)=\mathrm{si}(L\colon \langle x\rangle)$. Set $H=\langle x\rangle^L$. Then $\{0\}\neq H\in\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$. Since $\mathrm{si}(H\colon\langle x\rangle)=n(L)-1$, we have $n(H)=n(L)-1$. By inductive hypothesis, H has a non-trivial abelian ideal A. Let F be the Fitting radical of H. Since $A\leqq F$, $F\neq\{0\}$. By [1, Corollary 6.3.2] we have $F\,\mathrm{ch}\,H\lhd L$, so that $F\lhd L$. As in the proof of [9, Lemma 4.2], we can show that $F\in\acute{\mathrm{E}}(\lhd)\mathfrak{A}$. It follows from Corollary 1 that $F\in\mathfrak{D}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{A}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}\leqq\mathrm{E}\mathfrak{A}$. Since $\{0\}\neq F\in\mathrm{E}\mathfrak{A}$, there is a positive integer m such that $F^{(m-1)}\neq\{0\}$ and $F^{(m)}=\{0\}$. Since $F^{(m-1)}\mathrm{ch}\,F\lhd L$, $F^{(m-1)}$ is a non-trivial abelian ideal of L.

THEOREM 2. *Over any field* $\mathfrak{k}$ *of characteristic zero,* $\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}\leqq\mathrm{E}\mathfrak{A}$.

PROOF. Let $L\in\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$ and let M be any non-zero homomorphic image of L. Since $\{0\}\neq M\in\mathfrak{D}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{QS}}$, by Lemma 2 M has a non-trivial abelian ideal. Owing to [7, Lemma 1.1], we have $L\in\acute{\mathrm{E}}(\lhd)\mathfrak{A}$. Thus by Corollary 1 we obtain $L\in\mathfrak{D}\cap\acute{\mathrm{E}}(\lhd)\mathfrak{A}\cap(\grave{\mathrm{E}}\mathfrak{A})^{\mathrm{Q}}\leqq\mathrm{E}\mathfrak{A}$.

It can be easily deduced from Theorem 2 and Lemma 1 that over any field $\mathfrak{k}$ of characteristic zero, if no non-trivial $\mathfrak{D}$-algebra is perfect, then every $\mathfrak{D}$-algebra is soluble.

4.

In group theory Smith [8] has constructed a non-nilpotent, hypercentral, metabelian group in which every subgroup is subnormal. In Lie theory, however, it is still an open question whether every hypercentral $\mathfrak{D}$-algebra is nilpotent. In this section we shall show that in order to give the answer to this question it is sufficient to consider whether every hypercentral, Fitting, metabelian $\mathfrak{D}$-algebra is nilpotent.

LEMMA 3. *Let* $L\in\mathfrak{D}\cap\mathfrak{A}^2$ *and* $H, K\leqq L$. *Then*:
(1) *If* $H\in\mathfrak{N}$, *then* $H^L\in\mathfrak{N}$.
(2) *If* $H, K\in\mathfrak{N}$, *then* $\langle H, K\rangle\in\mathfrak{N}$.

PROOF. (1) Since $L\in\mathfrak{D}$, H si L. There are non-negative integers r and s such that $H^{r+1}=\{0\}$ and $H\lhd^s L$. Set $n=r+s$. Then it is clear that $[L,_n H]=H^{n+1}=\{0\}$. Set $A=L^2$. Since $H\leqq H+A\lhd L$, we have $H^L\leqq H+A$. By modular law $H^L=H+(H^L\cap A)$. Since A is an abelian ideal of L, by using induction on k we can easily see that for all non-negative integers k, $(H^L)^{k+1}=H^{k+1}+[H^L\cap A,_k H]$. It follows that $(H^L)^{n+1}\leqq H^{n+1}+[L,_n H]=\{0\}$. Hence $H^L\in\mathfrak{N}$.

(2) By (1) $H^L, K^L\in\mathfrak{N}$. Therefore by Fitting's theorem (cf. [1, Theorem 1.2.5]) we have $H^L+K^L\in\mathfrak{N}$. Since $\langle H, K\rangle\leqq H^L+K^L$, $\langle H, K\rangle\in\mathfrak{N}$.

PROPOSITION 2. $\mathfrak{D}\cap\mathfrak{Z}\cap\mathfrak{Fi}\cap\mathfrak{A}^2\leqq\mathfrak{N}$ *if and only if* $\mathfrak{D}\cap\mathfrak{Z}\leqq\mathfrak{N}$.

PROOF. Assume that $\mathfrak{D}\cap\mathfrak{Z}\cap\mathfrak{Fi}\cap\mathfrak{A}^2\leqq\mathfrak{N}$ and let $L\in\mathfrak{D}\cap\mathfrak{Z}\cap\mathfrak{A}^2$. Then by

Lemma 3 (1) we have $L=\sum_{x\in L}\langle x\rangle^{L}\in\mathfrak{F}\mathfrak{t}$. Therefore $L\in\mathfrak{N}$. It follows that $\mathfrak{D}\cap\mathfrak{Z}\cap\mathfrak{A}^{2}\leqq\mathfrak{N}$. Since the class $\mathfrak{D}\cap\mathfrak{Z}$ is s-, Q-closed, by using [1, Proposition 7.1.1 (d)] we see that $\mathfrak{D}\cap\mathfrak{Z}\cap\mathfrak{A}^{n}\leqq\mathfrak{N}$ for all positive integers n. Hence $\mathfrak{D}\cap\mathfrak{Z}\cap\mathrm{E}\mathfrak{A}\leqq\mathfrak{N}$. Therefore, by using Corollary 2, we have $\mathfrak{D}\cap\mathfrak{Z}\leqq\mathfrak{N}$. The converse is trivial.

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