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Weak approximation of the fractional Brownian sheet from random walks*

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Abstract

In this paper, we show an approximation in law of the fractional Brownian sheet by random walks. As an application, we consider a quasilinear stochastic heat equation with Dirichlet boundary conditions driven by an additive fractional noise.

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1 Introduction and main result

Given $\alpha, \beta \in (0,1)$, a fractional Brownian sheet on $\mathbb{R}$ is a two-parameter centered Gaussian process

$$W^{\alpha,\,\beta} = \{W^{\alpha,\,\beta}(t,s), (t,s) \in \mathbb{R}_+^2\}$$

such that

$$E\left[W^{\alpha,\,\beta}(t,s)W^{\alpha,\,\beta}(t',s')\right] = \frac{1}{2}\left[t^{2\alpha} + t'^{2\alpha} - |t'-t|^{2\alpha}\right] \cdot \frac{1}{2}\left[s^{2\beta} + s'^{2\beta} - |s'-s|^{2\beta}\right].$$

For $\alpha = \beta = \frac{1}{2}$, $W^{\alpha,\,\beta}$ coincides with the standard Brownian sheet. It is an extension of fractional Brownian motion $B^\alpha = \{B_t^\alpha, t \geq 0\}$ to two-parameter case. In this paper, we will be interested in the weak approximation of the fractional Brownian sheet with $\alpha, \beta \in (\frac{1}{2}, 1)$ from random walks in the plane and give an application.

Recently, Bardina *et al.* [6] (see also Tudor [16] for a similar approximation in the Besov space) proved that the family of processes

$$X_\varepsilon(t,s) := \frac{1}{\varepsilon^2}\int_0^1\int_0^1 K_\alpha(t,u)K_\beta(s,v)\sqrt{uv}(-1)^{N(\frac{u}{\varepsilon},\frac{v}{\varepsilon})}dudv$$

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with $\alpha, \beta \in (\frac{1}{2}, 1)$ converges in law, as ε tends to zero, to the fractional Brownian sheet $W^{\alpha,\,\beta}$, where $\{N(x,y),(x,y) \in \mathbb{R}_+^2\}$ is a standard Poisson process in the plane and the kernel K_H given by

$$K_H(t,s) = (H - \frac{1}{2})c_H s^{\frac{1}{2}-H} \int_s^t u^{H-\frac{1}{2}}(u-s)^{H-\frac{3}{2}} du \tag{1.1}$$

with $H \in (\frac{1}{2}, 1)$ and the normalizing constant $c_H > 0$ given by

$$c_H = \sqrt{\frac{2H\Gamma(\frac{3}{2}-H)}{\Gamma(H+\frac{1}{2})\Gamma(2-2H)}}.$$

The results of Bardina *et al.* [6] and Tudor [16] have been inspired by the following relationship between the standard one-parameter Poisson process and the standard Brownian motion proved by Stroock [15]: the family of processes

$$y_\varepsilon(t) := \frac{1}{\varepsilon}\int_0^t (-1)^{N(\frac{s}{\varepsilon})} ds,$$

where N is a standard Poisson process, converges in law, as ε tends to zero, to the standard Brownian motion W. More works concerning weak approximation for multi-dimensional parameter process have been studied by many authors (see, for examples, Bardina *et al.* [3, 5, 6]). In these references, the methods for obtaining the corresponding approximation sequences are Poisson processes due to their good properties such as independent increments and that if $Z \sim Poiss(\lambda)$ then $E[(-1)^Z] = \exp(-2\lambda)$.

Let now $\{\xi_i^{(n)},\ i = 1, 2, \ldots\}$ be a triangular array of i.i.d. random variables with $E\xi_i^{(n)} = 0$ and $E(\xi_i^{(n)})^2 = 1$. Then the sequence of stochastic processes

$$W_t^{(n)} := \frac{1}{\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \xi_i^{(n)}, \qquad t \in [0,T],\ n = 1, 2, \ldots$$

converges weakly to a standard Brownian motion W, where $\lfloor x \rfloor$ denotes the greatest integer not exceeding x. According to the next integral representation of the fractional Brownian motion B^H with Hurst index $H \in (\frac{1}{2}, 1)$:

$$B_t^H = \int_0^t K_H(t,s) dW_s, \qquad t \geq 0, \tag{1.2}$$

Sottinen [14] considered the family of processes $\{Z^{(n)}\}$

$$Z_t^{(n)} := \int_0^t K_H^{(n)}(t,s) dW_s^{(n)} = \sum_{i=1}^{\lfloor nt \rfloor} n \int_{\frac{i-1}{n}}^{\frac{i}{n}} K_H(\frac{\lfloor nt \rfloor}{n}, s) ds \frac{1}{\sqrt{n}} \xi_i^{(n)}, \qquad t \in [0,T]$$

for $n = 1, 2, \ldots$, and showed that the family converges weakly to B^H for $H \in (\frac{1}{2}, 1)$, where the sequence $\{K_H^{(n)}(t,\cdot),\ n = 1, 2, \ldots\}$ is an approximation to $K_H(t,\cdot)$ defined by

$$K_H^{(n)}(t,s) := n \int_{s-\frac{1}{n}}^{s} K_H(\frac{\lfloor nt \rfloor}{n}, u) du, \quad n = 1, 2, \ldots. \tag{1.3}$$

Motivated by this, in the present paper we consider the approximation of fractional Brownian sheet by random walks in the plane, and our main result is to explain and prove the following theorem.

Theorem 1.1. *Let* $\alpha > \frac{1}{2},\ \beta > \frac{1}{2}$ *and let* $\left\{\xi_{i,j}^{(n)}, i,j = 1,2,\dots\right\}$ *be an independent family of identically distributed and centered random variables with* $E(\xi_{i,j}^{(n)})^2 = 1$*. For* $n \geq 1$*,* $(t,s) \in [0,T]\times[0,S]$*, we set*

$$B_n(t,s) := \frac{1}{n}\sum_{i=1}^{\lfloor nt\rfloor}\sum_{j=1}^{\lfloor ns\rfloor}\xi_{i,j}^{(n)} \tag{1.4}$$

and

$$\begin{aligned} Z_n(t,s) :&= \int_0^t\int_0^s K_\alpha^{(n)}(t,v)K_\beta^{(n)}(s,u)B_n(dv,du) \\ &= n\sum_{i=1}^{\lfloor nt\rfloor}\sum_{j=1}^{\lfloor ns\rfloor}\xi_{i,j}^{(n)}\int_{\frac{i-1}{n}}^{\frac{i}{n}}\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\alpha(\frac{\lfloor nt\rfloor}{n},v)K_\beta(\frac{\lfloor ns\rfloor}{n},u)dudv. \end{aligned} \tag{1.5}$$

where the kernel $K.$ *is given by* (1.1) *and the sequence* $\{K.^{(n)},\ n = 1,2,\dots\}$ *of approximation to* $K.$ *defined by* (1.3)*. Then,* $\{Z_n\}$ *converges weakly in the Skorohod space D(*$[0,T]\times[0,S]$*) to the fractional Brownian sheet* $W^{\alpha,\,\beta}$ *in the plane.*

This paper is organized as follows. In Section 2 we give the proof of Theorem 1.1. Clearly, when $\alpha > \frac{1}{2},\ \beta = \frac{1}{2}$, $W^{\alpha,\,\beta}$ is called a fractional noise with Hurst parameter α which is introduced in Nualart-Ouknine [12]. Thus, as an application of Theorem 1.1, in Section 3 we consider the approximation solution of a one-dimensional quasi-linear stochastic heat equation driven by fractional noise.

2 Proof of the Theorem 1.1

To prove Theorem 1.1, we first recall some facts. For a deeper discussion we refer the reader to see Ayache *et al.* [1], Cairoli-Walsh [8], Decreusefond-Üstünel [9], Kamont [10].

Let $(\Omega,\mathscr{F},P)$ be a complete probability space and let $\{\mathscr{F}_{t,s};(t,s)\in[0,T]\times[0,S]\}$ be a family of sub-σ-fields of $\mathscr{F}$ such that

- (C_1) $\mathscr{F}_{t,s}\subseteq\mathscr{F}_{t',s'}$ for any $t\leq t', s\leq s'$;
- (C_2) $\mathscr{F}_{0,0}$ contains all null sets of $\mathscr{F}$;
- (C_3) for each $z\in[0,T]\times[0,S]$, $\mathscr{F}_z = \cap_{z<z'}\mathscr{F}_{z'}$, where $z = (t,s) < z' = (t',s')$ denotes the partial order on $[0,T]\times[0,S]$, meaning that $t < t'$ and $s < s'$.

Given $(t,s) < (t',s')$, we denote by $\Delta_{t,s}X(t',s')$ the increment of the process X over the rectangle $((t,s),(t',s')]$, that is, $\Delta_{t,s}X(t',s') = X(t',s') - X(t,s') - X(t',s) + X(t,s)$.

Recall that a fractional Brownian sheet admits an integral representation of the form

$$W^{\alpha,\,\beta}(t,s) = \int_0^t\int_0^s K_\alpha(t,v)K_\beta(s,u)B(dv,du),\ (t,s)\in[0,T]\times[0,S], \tag{2.1}$$

where B is a standard Brownian sheet and K_H is the deterministic kernel given by (1.1). For the deterministic kernel given by (1.1) it is not difficult to see that

$$\int_{t_0}^{t_0'}(K_H(t',x) - K_H(t,x))^2dx \leq C_H(t_0' - t_0)^{2-2H}$$

for all $0 < t_0 < t_0'$ and $0 < t < t'$.

Let Λ be the group of all mappings $\lambda : [0,T]\times[0,S] \to [0,T]\times[0,S]$ of the form $\lambda(t,s) = (\lambda_1(t),\lambda_2(s))$, where each λ_i is continuous, strictly increasing and fixes zero and one. Denote by $D = D([0,T]\times[0,S])$ the Skorohod space of functions on $[0,T]\times[0,S]$ are continuous from above with limits from below and equip D with the metric

$$d(x, y) := \inf\{\min(\|x - y\lambda\|, \|\lambda\|) : \lambda \in \Lambda\},$$

where $\|x-y\lambda\| = \sup\{|x(t,s)-y(\lambda(t,s))| : (t,s) \in [0,T]\times[0,S]\}$ and $\|\lambda\| = \sup\{|\lambda(t,s) - (t,s)| : (t,s) \in [0,T]\times[0,S]\}$. Under this metric, D is a separable and complete metric space.

Now, we can prove Theorem 1.1, and we split the proof in several results. We first prove the tightness. Using the criterion given by Bickel-Wichura [7], and notice that our processes Z_n are null on the axes, it suffices to prove the following lemma.

Lemma 2.1. *Let $Z_n(t,s)$ be the family of processes defined by (1.5). Then for any $(t,s) < (t',s')$, we have*

$$\sup_n E[(\Delta_{t,s} Z_n(t',s'))^4] \le 16^{\alpha+\beta}(t'-t)^{4\alpha}(s'-s)^{4\beta}.$$

In order to prove Lemma 2.1 we need the next technical result.

Lemma 2.2. *Let $Z_n(t,s)$ be the family of processes defined by (1.5). Then for any $(t,s) < (t',s')$, we have*

$$E[(\Delta_{t,s} Z_n(t',s'))^2] \le 4^{\alpha+\beta}(t'-t)^{2\alpha}(s'-s)^{2\beta}.$$

Proof. First, we observe that

$$\begin{aligned}\Delta_{t,s} Z_n(t',s') &= \int_t^{t'}\int_s^{s'} \left(K_\alpha^{(n)}(\frac{\lfloor nt'\rfloor}{n}, v) - K_\alpha^{(n)}(\frac{\lfloor nt\rfloor}{n}, v)\right)\left(K_\beta^{(n)}(\frac{\lfloor ns'\rfloor}{n}, u)\right. \\ &\qquad \left. -K_\beta^{(n)}(\frac{\lfloor ns\rfloor}{n}, u)\right) B_n(dv, du) \\ &= \sum_{i=1}^{\lfloor nt'\rfloor}\sum_{j=1}^{\lfloor ns'\rfloor} n\int_{\frac{i-1}{n}}^{\frac{i}{n}}\int_{\frac{j-1}{n}}^{\frac{j}{n}} \left(K_\alpha^{(n)}(\frac{\lfloor nt'\rfloor}{n}, v) - K_\alpha^{(n)}(\frac{\lfloor nt\rfloor}{n}, v)\right) \\ &\qquad \cdot\left(K_\beta^{(n)}(\frac{\lfloor ns'\rfloor}{n}, u) - K_\beta^{(n)}(\frac{\lfloor ns\rfloor}{n}, u)\right) du dv \xi_{i,j}^{(n)}.\end{aligned}$$

Thus,

$$\begin{aligned}E[\Delta_{t,s} Z_n(t',s')]^2 &= \sum_{i=1}^{\lfloor nt'\rfloor} (\sqrt{n}\int_{\frac{i-1}{n}}^{\frac{i}{n}} (K_\alpha(\frac{\lfloor nt'\rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt\rfloor}{n}, v))dv)^2 \\ &\qquad \cdot . \sum_{j=1}^{\lfloor ns'\rfloor} (\sqrt{n}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\beta(\frac{\lfloor ns'\rfloor}{n}, u) - K_\beta(\frac{\lfloor ns\rfloor}{n}, u))du)^2.\end{aligned}$$

Then, applying the Cauchy-Schwarz inequality, the above term can be bounded by

$$\begin{aligned}&\sum_{i=1}^{\lfloor nt'\rfloor}\int_{\frac{i-1}{n}}^{\frac{i}{n}} (K_\alpha(\frac{\lfloor nt'\rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt\rfloor}{n}, v))^2 dv \sum_{j=1}^{\lfloor ns'\rfloor}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\beta(\frac{\lfloor ns'\rfloor}{n}, u) - K_\beta(\frac{\lfloor ns\rfloor}{n}, u))^2 du \\ &\qquad \le \int_0^{t'} (K_\alpha(\frac{\lfloor nt'\rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt\rfloor}{n}, v))^2 dv \int_0^{s'} (K_\beta(\frac{\lfloor ns'\rfloor}{n}, u) - K_\beta(\frac{\lfloor ns\rfloor}{n}, u))^2 du \\ &\qquad = \left|\frac{\lfloor nt'\rfloor - \lfloor nt\rfloor}{n}\right|^{2\alpha}\left|\frac{\lfloor ns'\rfloor - \lfloor ns\rfloor}{n}\right|^{2\beta}.\end{aligned}$$

Let now $0 < r < r'$ and $\frac{1}{2} < \nu < 1$. We then see that $nr' - nr \geq 1$ implies that $\left|\frac{\lfloor nr' \rfloor - \lfloor nr \rfloor}{n}\right|^{2\nu} \leq |2(r'-r)|^{2\nu}$. Conversely, $nr' - nr < 1$ implies that either r' and r belong to a same subinterval $[\frac{m}{n}, \frac{m+1}{n})$ for some integer m, and hence $\left|\frac{\lfloor nr' \rfloor - \lfloor nr \rfloor}{n}\right|^{2\nu} = 0$. It follows that

$$\left|\frac{\lfloor nr' \rfloor - \lfloor nr \rfloor}{n}\right|^{2\nu} \leq |2(r'-r)|^{2\nu}$$

for all $0 < r < r'$, $\nu \in (\frac{1}{2}, 1)$ and all $n \geq 1$. This completes the proof. □

We are now ready to prove Lemma 2.1.

Proof of Lemma 2.1. First, we observe that we can write

$$E[\Delta_{t,s} Z_n(t',s')]^4 = E\Bigg[\sum_{i=1}^{\lfloor nt' \rfloor}\sum_{j=1}^{\lfloor ns' \rfloor} n \int_{\frac{i-1}{n}}^{\frac{i}{n}}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\alpha^{(n)}(\frac{\lfloor nt' \rfloor}{n}, v) - K_\alpha^{(n)}(\frac{\lfloor nt \rfloor}{n}, v)) \cdot (K_\beta^{(n)}(\frac{\lfloor ns' \rfloor}{n}, u) - K_\beta^{(n)}(\frac{\lfloor ns \rfloor}{n}, u)) du dv \xi_{i,j}^{(n)}\Bigg]^4.$$

Notice that $E\xi^{(n)} = 0$ and $E^2\xi^{(n)} = 1$, therefore, the above expectation can be computed as

$$\begin{aligned}
&\sum_{i=1}^{\lfloor nt' \rfloor}\sum_{j=1}^{\lfloor ns' \rfloor}\sum_{k=1}^{\lfloor nt' \rfloor}\sum_{l=1}^{\lfloor ns' \rfloor} (\sqrt{n}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\beta(\frac{\lfloor ns' \rfloor}{n}, u) - K_\beta(\frac{\lfloor ns \rfloor}{n}, u)) du)^2 \\
&\qquad \cdot (\sqrt{n}\int_{\frac{i-1}{n}}^{\frac{i}{n}} (K_\alpha(\frac{\lfloor nt' \rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt \rfloor}{n}, v)) dv)^2 \\
&\qquad \cdot (\sqrt{n}\int_{\frac{l-1}{n}}^{\frac{l}{n}} (K_\beta(\frac{\lfloor ns' \rfloor}{n}, u) - K_\beta(\frac{\lfloor ns \rfloor}{n}, u)) du)^2 \\
&\qquad \cdot (\sqrt{n}\int_{\frac{k-1}{n}}^{\frac{k}{n}} (K_\alpha(\frac{\lfloor nt' \rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt \rfloor}{n}, v)) dv)^2 \\
&= \left(\sum_{i=1}^{\lfloor nt' \rfloor} (\sqrt{n}\int_{\frac{i-1}{n}}^{\frac{i}{n}} (K_\alpha(\frac{\lfloor nt' \rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt \rfloor}{n}, v)) dv)^2\right)^2 \\
&\qquad \cdot \left(\sum_{j=1}^{\lfloor ns' \rfloor} (\sqrt{n}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\beta(\frac{\lfloor ns' \rfloor}{n}, u) - K_\beta(\frac{\lfloor ns \rfloor}{n}, u)) du)^2\right)^2.
\end{aligned}$$

Using the Cauchy-Schwarz inequality, we get that

$$\begin{aligned}
E[\Delta_{t,s} Z_n(t',s')]^4 &\leq \left(\sum_{i=1}^{\lfloor nt' \rfloor}\int_{\frac{i-1}{n}}^{\frac{i}{n}} (K_\alpha(\frac{\lfloor nt' \rfloor}{n}, v) - K_\alpha(\frac{\lfloor nt \rfloor}{n}, v))^2 dv\right)^2 \\
&\qquad \cdot \left(\sum_{j=1}^{\lfloor ns' \rfloor}\int_{\frac{j-1}{n}}^{\frac{j}{n}} (K_\beta(\frac{\lfloor ns' \rfloor}{n}, u) - K_\beta(\frac{\lfloor ns \rfloor}{n}, u))^2 du\right)^2 \\
&\leq \left|\frac{\lfloor nt' \rfloor - \lfloor nt \rfloor}{n}\right|^{4\alpha}\left|\frac{\lfloor ns' \rfloor - \lfloor ns \rfloor}{n}\right|^{4\beta} \leq 16^{\alpha+\beta}(t'-t)^{4\alpha}(s'-s)^{4\beta}
\end{aligned}$$

by Lemma 2.2 and the lemma follows. □

Now, it suffices to show that the law of all possible weak limits is the law of a fractional Brownian sheet.

Theorem 2.3. *The family of processes $Z_n(t,s)$ defined by (1.5) converge, as n tends to infinity, to the fractional Brownian sheet in the sense of finite-dimensional distribution.*

Proof. For any $a_1,\dots,a_d \in \mathbb{R}$ and $(t_1,s_1),\dots,(t_d,s_d) \in [0,T]\times[0,S]$. We claim that

$$Y_n := \sum_{k=1}^{d} a_k Z_n(t_k,s_k)$$

converges in distribution to a normal random variable with zero mean and variance

$$E\left(\sum_{k=1}^{d} a_k W^{\alpha,\,\beta}(t_k,s_k)\right)^2. \tag{2.2}$$

In fact, the zero mean is trivial. Let us now calculate the limiting variance of Y_n. We have

$$\begin{aligned}(\sigma^{(n)})^2 := E(Y_n)^2 &= \sum_{k,l=1}^{d} a_k a_l n^2 \sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor} \int_{\frac{i-1}{n}}^{\frac{i}{n}}\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\alpha(\frac{\lfloor nt_k\rfloor}{n},v)K_\beta(\frac{\lfloor ns_k\rfloor}{n},u)dudv \\ &\qquad \cdot \int_{\frac{i-1}{n}}^{\frac{i}{n}}\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\alpha(\frac{\lfloor nt_l\rfloor}{n},v)K_\beta(\frac{\lfloor ns_l\rfloor}{n},u)dudv \\ &= \sum_{k,l=1}^{d} a_k a_l \sum_{i=1}^{\lfloor nT\rfloor} n\int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha(\frac{\lfloor nt_k\rfloor}{n},v)dv \int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha(\frac{\lfloor nt_l\rfloor}{n},v)dv \\ &\qquad \cdot \sum_{j=1}^{\lfloor nS\rfloor} n\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\beta(\frac{\lfloor ns_k\rfloor}{n},u)du \int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\alpha(\frac{\lfloor ns_l\rfloor}{n},u)du.\end{aligned}$$

By the mean value theorem the above equation is equal to

$$\sum_{k,l=1}^{d} a_k a_l \frac{1}{n}\sum_{i=1}^{\lfloor nT\rfloor} K_\alpha(\frac{\lfloor nt_k\rfloor}{n}, s_{i,k}^{(n)})K_\alpha(\frac{\lfloor nt_l\rfloor}{n}, s_{i,l}^{(n)})\frac{1}{n}\sum_{j=1}^{\lfloor nS\rfloor} K_\beta(\frac{\lfloor ns_k\rfloor}{n}, s_{j,k}^{(n)})K_\beta(\frac{\lfloor ns_l\rfloor}{n}, s_{j,l}^{(n)}) \tag{2.3}$$

for some $s_{i,k}^{(n)}, s_{i,l}^{(n)} \in (\frac{i-1}{n},\frac{i}{n}]$ and $s_{j,k}^{(n)}, s_{j,l}^{(n)} \in (\frac{j-1}{n},\frac{j}{n}]$. Since the kernel $K_\cdot(t,\cdot)$ is continuous and decreasing we get the inner sum in (2.3) is equal to

$$\frac{1}{n}\sum_{i=1}^{\lfloor nT\rfloor} K_\alpha(\frac{\lfloor nt_k\rfloor}{n}, v_i^{(n)})K_\alpha(\frac{\lfloor nt_l\rfloor}{n}, v_i^{(n)})\frac{1}{n}\sum_{j=1}^{\lfloor nS\rfloor} K_\beta(\frac{\lfloor ns_k\rfloor}{n}, u_j^{(n)})K_\beta(\frac{\lfloor ns_l\rfloor}{n}, u_j^{(n)}) \tag{2.4}$$

for some

$$v_i^{(n)} \in \left[\min(s_{i,k}^{(n)}, s_{i,l}^{(n)})\right] \subseteq \left(\frac{i-1}{n},\frac{i}{n}\right];\; u_j^{(n)} \in \left[\min(s_{j,k}^{(n)}, s_{j,l}^{(n)})\right] \subseteq \left(\frac{j-1}{n},\frac{j}{n}\right].$$

By using the following facts:

- The kernel K_H with $\frac{1}{2} < H < 1$ is continuous with respect to both arguments;
- The maps $t \mapsto \frac{\lfloor nt\rfloor}{n}$, $s \mapsto \frac{\lfloor ns\rfloor}{n}$ converge uniformly to the identity map in $[0,T]\times[0,S]$,

we see that (2.4) is a Riemann type sum. It follows that (2.3) converges to

$$\sum_{k,l=1}^{d} a_k a_l \int_0^T K_\alpha(t_k,s)K_\alpha(t_l,s)ds \int_0^S K_\beta(s_k,s)K_\beta(s_l,s)ds = E(\sum_{k=1}^{d} a_k W^{\alpha,\,\beta}(t_k,s_k))^2.$$

Decompose Y_n as follows

$$\begin{aligned} Y_n &= \sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor} n\xi_{i,j}^{(n)} \sum_{k=1}^{d} a_k \int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha(\frac{\lfloor t_k\rfloor}{n},v)dv \int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\beta(\frac{\lfloor s_k\rfloor}{n},u)du \\ &:= \sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor} Y_{i,j}^{(n)}. \end{aligned} \tag{2.5}$$

Now, in order to end the proof we need to obtain the following Lindeberg condition:

$$\lim_{n\to\infty}\frac{1}{(\sigma^{(n)})^2}\sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor} E\left[(Y_{i,j}^{(n)})^2 1_{\{|Y_{i,j}^{(n)}|>\varepsilon\sigma^{(n)}\}}\right]=0 \tag{2.6}$$

for all $\varepsilon>0$. To see that, let us consider the set

$$\left\{|Y_{i,j}^{(n)}|>\varepsilon\right\}=\left\{(Y_{i,j}^{(n)})^2>\varepsilon^2\right\}.$$

Noticing that the kernel $K_H(t,s)$ with $\frac{1}{2}<H<1$ is increasing in t and decreasing in s, we get

$$\begin{aligned} (Y_{i,j}^{(n)})^2 &= n^2(\xi_{i,j}^{(n)})^2\left(\sum_{k=1}^{d} a_k \int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha(\frac{\lfloor t_k\rfloor}{n},v)dv \int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\beta(\frac{\lfloor s_k\rfloor}{n},u)du\right)^2 \\ &\le n^2(\xi_{i,j}^{(n)})^2 A\left(\int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha(T,v)dv\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\beta(S,u)du\right)^2 \\ &\le (\xi_{i,j}^{(n)})^2 A\int_{\frac{i-1}{n}}^{\frac{i}{n}} K_\alpha^2(T,v)dv\int_{\frac{j-1}{n}}^{\frac{j}{n}} K_\beta^2(S,u)du \\ &\le (\xi_{i,j}^{(n)})^2 A\int_0^{\frac{1}{n}} K_\alpha^2(T,v)dv\int_0^{\frac{1}{n}} K_\beta^2(S,u)du = (\xi_{i,j}^{(n)})^2 A\delta^{(n)}, \end{aligned}$$

where $A:=(\sum_{k=1}^d a_k)^2$ and $\delta^{(n)}:=\int_0^{\frac{1}{n}} K_\alpha^2(T,v)dv\int_0^{\frac{1}{n}} K_\beta^2(S,u)du$, which deduces

$$\left\{|Y_{i,j}^{(n)}|>\varepsilon\sigma^{(n)}\right\}\subseteq\left\{(\xi_{i,j}^{(n)})^2A\delta^{(n)}>\varepsilon^2(\sigma^{(n)})^2\right\}. \tag{2.7}$$

It follows that

$$E\left[(Y_{i,j}^{(n)})^2 1_{\{|Y_{i,j}^{(n)}|>\varepsilon\sigma^{(n)}\}}\right]\le E\left[(\xi_{i,j}^{(n)})^2A\delta^{(n)}1_{\{(\xi_{i,j}^{(n)})^2A\delta^{(n)}>\varepsilon^2(\sigma^{(n)})^2\}}\right]$$

for all $i,j=1,2,\dots,n$, and that

$$\begin{aligned} &\frac{1}{(\sigma^{(n)})^2}\sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor}E\left[(Y_{i,j}^{(n)})^2 1_{\{|Y_{i,j}^{(n)}|>\varepsilon\sigma^{(n)}\}}\right] \\ &\qquad\le\frac{1}{(\sigma^{(n)})^2}\sum_{i=1}^{\lfloor nT\rfloor}\sum_{j=1}^{\lfloor nS\rfloor}E\left[(\xi_{i,j}^{(n)})^2A\delta^{(n)}1_{\{(\xi_{i,j}^{(n)})^2A\delta^{(n)}>\varepsilon^2(\sigma^{(n)})^2\}}\right] \\ &\qquad\le E\left[(\xi^{(n)})^2 1_{\{(\xi_{i,j}^{(n)})^2A\delta^{(n)}>\varepsilon^2(\sigma^{(n)})^2\}}\right]\to 0\quad(n\to\infty) \end{aligned}$$

because $\delta^{(n)}\to 0$. Thus, the Lindeberg condition (2.6) holds and the theorem follows. □

3 An application

It is well-known that a fractional Brownian sheet $W^{\alpha,\,\beta}$ with $\beta = \frac{1}{2}$ and $\alpha > \frac{1}{2}$ is called the fractional noise with Hurst parameter α, denoted by W^{α}, which is first introduced in Nualart-Ouknine [12]. Obviously, it is a zero mean Gaussian process with the covariance function

$$E\left[W^{\alpha}(t,x)W^{\alpha}(s,y)\right] = \frac{1}{2}\left[t^{2\alpha} + s^{2\alpha} - |t-s|^{2\alpha}\right](x \wedge y).$$

That is, W^{α} is a Brownian motion in the space variable and a fractional Brownian motion with Hurst parameter $\alpha \in (\frac{1}{2},\ 1)$ in the time variable.

In the sequel, as an application to Theorem 1.1 we consider the approximation solution (in law) of the stochastic heat equation

$$\frac{\partial U}{\partial t} - \frac{\partial^2 U}{\partial x^2} = b(U) + \frac{\partial^2 W^{\alpha}}{\partial t \partial x}, \tag{3.1}$$

with Dirichlet boundary conditions

$$U(t,0) = U(t,1) = 0,\ t \in [0,T]$$

and initial condition $U(0,x) = u_0(x), x \in [0,1]$, where u_0 is a continuous function and W^{α} is the fractional noise with $\frac{1}{2} < \alpha < 1$. This is a one-dimensional quasi-linear stochastic heat equation on $[0,1]$ which was first studied by Nualart-Ouknine [12].

For each $t \in [0,T]$, let $\mathscr{F}_t^W$ be the $\sigma-$ field generated by the random variables $\{W^{\alpha}(t,A),\ t \in [0,T], A \in \mathcal{B}[0,1]\}$ and the sets of probability zero, $\mathcal{P}$ be the $\sigma-$ field of progressively measurable subsets of $[0,T] \times \Omega$. We denote by $\mathscr{E}$ the set of step functions on $[0,T] \times [0,1]$. Let $\mathcal{H}$ be the Hilbert space defined as the closure of $\mathscr{E}$ with respect to the scalar product

$$\langle 1_{[0,t]\times A},\ 1_{[0,s]\times B}\rangle_{\mathcal{H}} = E\left[W^{\alpha}(t,A)W^{\alpha}(s,B)\right].$$

According to Nualart-Ouknine [12], the mapping $1_{[0,t]\times A} \to W^{\alpha}(t,A)$ can be extended to an isometry between $\mathcal{H}$ and the Gaussian space $H_1(W^{\alpha})$ associated with W^{α} and denoted by

$$\varphi \mapsto W^{\alpha}(\varphi) := \int_{[0,t]\times A} \varphi(s,y) W^{\alpha}(ds,dy).$$

Consider the linear operator K_{α}^{*} from $\mathscr{E}$ to $L^2([0,T])$ defined by

$$K_{\alpha}^{*}(\varphi) = K_{\alpha}(T,s)\varphi(s,x) + \int_s^T (\varphi(r,x) - \varphi(s,x)) \frac{\partial K_{\alpha}}{\partial r}(r,s)dr,$$

where K_{α} is the square integrable kernel given by (1.1). Moreover, for any pair of step functions φ and ψ in $\mathscr{E}$ we have

$$\langle K_{\alpha}^{*}(\varphi), K_{\alpha}^{*}(\psi)\rangle_{L^2([0,T]\times[0,1])} = \langle \varphi, \psi\rangle_{\mathcal{H}},$$

because

$$(K_{\alpha}^{*} 1_{[0,t]\times A})(s,x) = K_{\alpha}(t,s) 1_{[0,t]\times A}(s,x).$$

As a consequence, the operator K_{α}^{*} provides an isometry between the Hilbert space $\mathcal{H}$ and $L^2([0,T]\times[0,1])$. Hence, the Gaussian family $\{B(t,A), t \in [0,T], A \in \mathcal{B}[0,1]\}$ defined by

$$B(t,A) = W^{\alpha}((K_{\alpha}^{*})^{-1}(1_{[0,t]\times A})),$$

is a space-time white noise, and the process W^α has an integral representation of the form

$$W^\alpha(t,x) = \int_0^t \int_0^x K_\alpha(t,s) B(ds,dy).$$

Denote by

$$G_t(x,y) = \frac{1}{\sqrt{2\pi t}} \sum_{n=-\infty}^{\infty} \left(e^{-\frac{(y-x-2n)^2}{4t}} + e^{-\frac{(y+x-2n)^2}{4t}} \right) = 2\sum_{n=1}^{\infty} sin(n\pi x) sin(n\pi y) e^{-n^2\pi^2 t},$$

$(t,x,y) \in [0,T] \times [0,1]^2$, the Green function associated to the heat equation in $[0,1]$ with Dirichlet boundary conditions. We have

$$0 \le G_t(x,y) \le \frac{1}{\sqrt{2\pi t}} e^{\frac{-(y-x)^2}{4t}}, \quad t > 0, \quad (x,y) \in [0,1]^2.$$

Assume that b is bounded, then a $\mathcal{P} \otimes \mathcal{B}([0,1])$-measurable and continuous random field $U = \{U(t,x), (t,x) \in [0,T] \times [0,1]\}$ is a solution to (3.1) if and only if

$$\begin{aligned} U(t,x) = \int_0^1 G_t(x,y) u_0(y) dy + \int_0^t \int_0^1 G_{t-s}(x,y) b(U(s,y)) dy ds \\ + \int_0^t \int_0^1 G_{t-s}(x,y) W^\alpha(ds,dy), \end{aligned} \tag{3.2}$$

where the last term is equal to

$$W^\alpha(1_{[0,t]}(\cdot) G_{t-\cdot}(x,\cdot)) = \int_0^t \int_0^1 K_\alpha^* G_{t-s}(x,y) B(ds,dy).$$

It follows from Nualart-Ouknine [12] that (3.1) admits a unique solution satisfying (3.2), provided $\alpha \in (\frac{1}{2}, 1)$ and b is Lipschitz function with the linear growth.

Remark 3.1. We should notice that the mild solution to (3.1), given by (3.2), is understood in the generalized sense defined by Walsh [17] in the case of a space-time white noise.

To study the approximation solution of (3.1) in the space $C([0,T] \times [0,1])$ we consider the triangular array $\{\xi_i^{(n)}, \ i = 1,2,\ldots\}$ of i.i.d. random variables with $E\xi_i^{(n)} = 0$ and $E(\xi_i^{(n)})^2 = 1$, as in Theorem 1.1, and define the processes

$$\begin{aligned} U_n(t,x) = \int_0^1 G_t(x,y) u_0(y) dy + \int_0^t \int_0^1 G_{t-s}(x,y) b(U_n(s,y)) dy ds \\ + \int_0^t \int_0^1 K_\alpha^* G_{t-s}(x,y) \theta_n(s,y) dy ds, \quad n = 1,2,\ldots \end{aligned} \tag{3.3}$$

where $\theta_n(t,x), (t,x) \in [0,T] \times [0,1]$ stands for the Donsker kernel given by

$$\theta_n(t,x) = n \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \xi_{i,j}^{(n)} 1_{[\frac{i-1}{n}, \frac{i}{n}) \times [\frac{j-1}{n}, \frac{j}{n})}(t,x). \tag{3.4}$$

Observe that since θ_n have square integrable paths, the integrals in (3.3) are well defined. Standard arguments yield existence and uniqueness of solution for (3.3). Notice that

$$\int_0^t \int_0^x \theta_n(s,y) dy ds = \frac{1}{n} \sum_{i=1}^{\lfloor nt \rfloor} \sum_{j=1}^{\lfloor nx \rfloor} \xi_{i,j}^{(n)} = B_n(t,x), \quad n = 1,2,\ldots. \tag{3.5}$$

We see, as an application of Theorem 1.1, that

$$W_n^\alpha(t,x) := \int_0^t \int_0^x K_\alpha(t,s)B_n(ds,dy) = \int_0^t \int_0^x K_\alpha(t,s)\theta_n(s,y)dyds, \tag{3.6}$$

converges in law to fractional noise W^α. Our main object of this section is to explain and prove the following theorem.

Theorem 3.2. *Let $\{\theta_n(t,x),(t,x) \in [0,T]\times[0,1]\}$, $n = 1,2,\dots$ be the Donsker kernel given in (3.4). Assume that $u_0 : [0,1] \to \mathbb{R}$ is a continuous function and $b : \mathbb{R} \to \mathbb{R}$ is Lipschitz. Then, the family $\{U_n, n = 1,2,\dots\}$ defined by* (3.3) *converges in law, as n tends to infinity, in the space $C([0,T]\times[0,1])$, to the mild solution U of* (3.1)*, given by* (3.2).

In order to prove Theorem 3.2, we first consider the linear problem, which is amount to establish the convergence in law, in $C([0,T]\times[0,1])$, of the solutions of

$$\frac{\partial X_n}{\partial t} - \frac{\partial^2 X_n}{\partial x^2} = \frac{\partial^2 W_n^\alpha}{\partial t \partial x}, \tag{3.7}$$

with vanishing initial data and Dirichlet boundary conditions $U(t,0) = U(t,1) = 0,\ t \in [0,T]$, towards the solution of

$$\frac{\partial X}{\partial t} - \frac{\partial^2 X}{\partial x^2} = \frac{\partial^2 W^\alpha}{\partial t \partial x}, \tag{3.8}$$

where the solutions of (3.7) and (3.8) are respectively given by

$$X_n(t,x) = \int_0^t \int_0^1 K_\alpha^* G_{t-s}(x,y)\theta_n(s,y)dyds \tag{3.9}$$

and

$$X(t,x) = \int_0^t \int_0^1 K_\alpha^* G_{t-s}(x,y)B(ds,dy). \tag{3.10}$$

We will make use of the following results, which is a quotation of Theorem 2.1 and Lemma 2.2 in Mellali-Ouknine [11] (see, also Theorem 2.2 and Lemma 2.3 in Bardina *et al.* [4]).

Lemma 3.3. *Let $\{X_n, n = 1,2,\dots\}$ be a family of random variables taking values in $C([0,T]\times[0,1])$. The family of the laws of $\{X_n, n = 1,2,\dots\}$ is tight, if there exist $p,p' > 0, \delta > 2$ and a constant $C > 0$ such that*

$$\sup_{n\geq 1} E|X_n(0,0)|^{p'} < \infty$$

and

$$\sup_{n\geq 1} E|X_n(t',x') - X_n(t,x)|^p < C(|x'-x| + |t'-t|)^\delta$$

for all $t,t' \in [0,T]$, $x,x' \in [0,1]$.

Lemma 3.4. *Let $(F,\|\cdot\|)$ be a normed space and let $J, J_n, n = 1,2,\dots$ be linear maps defined on F with their values in the space $L^0(\Omega)$ of almost surely finite random variables. Assume that there exists a positive constant C such that, for any $f \in F$,*

$$\sup_{n\geq 1} E|J_n(f)| \leq C\|f\|, \qquad E|J(f)| \leq C\|f\|,$$

and that, for some dense subspace D of F, it holds that $J_n(f)$ converges in law to $J(f)$, as n tends to infinity, for all $f \in D$. Then, the sequence of random variables $\{J_n(f), n = 1,2,\dots\}$ converges in law to $J(f)$, for any $f \in F$.

We also will use the following lemmas which are given in Bardina *et al.* [4] and Bally *et al.* [2].

Lemma 3.5. *Let* $\{\theta_n(t,x),(t,x)\in[0,T]\times[0,1]\}$, $n=1,2,\ldots$ *be the Donsker kernels and let* $m\geq 10$ *be some even number. Then, there exists a positive constant* C_m *such that*

$$E\left(\int_0^T\int_0^1 f(t,x)\theta_n(t,x)dxdt\right)^m\leq C_m\left(\int_0^T\int_0^1 f^2(t,x)dxdt\right)^{\frac{m}{2}},\tag{3.11}$$

for all $n\geq 1$ *and all* $f\in L^2([0,T]\times[0,1])$.

Lemma 3.6. *(i) Let* $\alpha\in(\frac{3}{2},3)$. *Then, for all* $t\in[0,T]$ *and* $x,\ y\in[0,1]$,

$$\int_0^t\int_0^1|G_{t-s}(x,z)-G_{t-s}(y,z)|^\alpha dzds\leq C|x-y|^{3-\alpha}.$$

(ii) Let $\alpha\in(1,3)$. *Then, for all* $s,\ t\in[0,T]$ *such that* $s\leq t$ *and* $x\in[0,1]$,

$$\int_0^s\int_0^1|G_{t-r}(x,y)-G_{s-r}(x,y)|^\alpha dydr\leq C|t-s|^{\frac{3-\alpha}{2}}.$$

(iii) Under the same hypothesis as (ii), we have

$$\int_s^t\int_0^1|G_{t-r}(x,y)|^\alpha dydr\leq C|t-s|^{\frac{3-\alpha}{2}}.$$

Proposition 3.7. *The family* $\{X_n,n=1,2,\ldots\}$ *given by* (3.9) *is tight in* $C([0,T]\times[0,1])$.

Proof. First, we observe that

$$\begin{aligned}X_n(t',x')-X_n(t,x)&=\int_0^t\int_0^1 K_\alpha^*(G_{t'-s}(x',y)-G_{t'-s}(x,y))\theta_n(s,y)dyds\\&\quad+\int_0^t\int_0^1 K_\alpha^*(G_{t'-s}(x,y)-G_{t-s}(x,y))\theta_n(s,y)dyds\\&\quad+\int_t^{t'}\int_0^1 K_\alpha^*G_{t'-s}(x',y)\theta_n(s,y)dyds\\&\equiv I_1+I_2+I_3\end{aligned}$$

for all $t<t'$. It follows from Lemma 3.5 that

$$\begin{aligned}\sup_{n\geq1}E|X_n(t',x')-X_n(t,x)|^m&\leq C_m\left[\int_0^t\int_0^1[K_\alpha^*(G_{t'-s}(x',y)-G_{t'-s}(x,y))]^2dyds\right]^{\frac{m}{2}}\\&\quad+C_m\left[\int_0^t\int_0^1[K_\alpha^*(G_{t'-s}(x,y)-G_{t-s}(x,y))]^2dyds\right]^{\frac{m}{2}}\\&\quad+C_m\left[\int_0^t\int_0^1(K_\alpha^*G_{t'-s}(x',y))^2dyds\right]^{\frac{m}{2}}\equiv C_m(J_1+J_2+J_3).\end{aligned}$$

Using the continuous embedding established in Pipiras-Taqqu [13]

$$L^{\frac{1}{\alpha}}([0,T]\times[0,1])\subset\mathcal{H},$$

and the inequality in Lemma 3.6, we obtain

$$J_1\leq C_\alpha\left[\int_0^t\int_0^1(G_{t'-s}(x',y)-G_{t'-s}(x,y)^{\frac{1}{\alpha}}dyds\right]^{m\alpha}\leq C_\alpha|x'-x|^{(3\alpha-1)m},$$

and similarly, we also have $J_2, J_3 \le C_\alpha |t'-t|^{(3\alpha-1)\frac{m}{2}}$. Consequently, we have

$$\sup_{n\ge1} E|X_n(t',x') - X_n(t,x)|^m \le C_{\alpha,m}[|t'-t|^{(3\alpha-1)\frac{m}{2}} + |x'-x|^{(3\alpha-1)m}],$$

and the proposition follows from Lemma 3.3. □

Proposition 3.8. *The family* $\{X_n, n=1,2,\ldots\}$ *defined by (3.9) converges to the process* X *given by (3.10), in the sense of finite-dimensional distributions, as* n *tends to infinity, in the space* $C([0,T]\times[0,1])$.

Proof. Let us consider the normed space $(F := L^2([0,T]\times[0,1]), \|\cdot\|_2)$. Set

$$J^n(f) := \int_0^t \int_0^1 f(s,y)\theta_n(s,y)dyds, \qquad J(f) := \int_0^t \int_0^1 f(s,y)B(ds,dy),$$

where

$$f(s,y) = \sum_{j=1}^m a_j 1_{[0,s_j]}(s) K_\alpha^* G_{s_j-s}(y_j,y).$$

Then, J^n and J define two linear applications on F. By Lemma 3.5 and the continuous embedding as above, we obtain

$$\sup_{n\ge1} E|J^n(f)| \le C\|f\|_2.$$

Notice that from the computations of the proof of (Nualart-Ouknine [12], Lemma 5), and applying Lemma 3.6 we obtain

$$\int_0^t \int_0^1 (K_\alpha^* G_{t-r}(x,y))^2 dydr \le C_\alpha (\int_0^t \int_0^1 (G_{t-s}(x,y))^{\frac{1}{\alpha}} dydr)^{2\alpha} \le C_\alpha t^{3\alpha-1} < \infty,$$

which implies

$$E|J(f)| \le C\|f\|_2.$$

Combining this with Lemma 3.4, we complete the proof. □

As a consequence of the above two propositions, we can see that the family $\{X_n, n\ge 1\}$ defined by (3.9) converges in law to the Gaussian process X defined by (3.10).

Finally, in a similar way as Theorem 4.5 in Mellali-Ouknine [11] we can obtain the next theorem, and Theorem 3.2 follows as a direct consequence.

Theorem 3.9. *Let* $\{\theta_n(t,x), (t,x) \in [0,T]\times[0,1]\}$, $n = 1,2,\ldots$ *be the Donsker kernel given in (3.4). Assume that* $u_0 : [0,1] \to \mathbb{R}$ *is a continuous function and* $b : \mathbb{R} \to \mathbb{R}$ *is Lipschitz. If the family* $\{X_n, n \ge 1\}$ *defined by (3.9) converges in law, as* n *tends to infinity, to the Gaussian process* X *defined by* (3.10). *Then, the family* $\{U_n, n \in \mathbb{N}\}$ *defined by* (3.3) *converges in law, as* n *tends to infinity, in the space* $C([0,T]\times[0,1])$, *to the mild solution* U *of* (3.1).

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