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Charged Particles in $\mathbb{Z}_2$ Gauge Theories

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Abstract. In the free charge phase of the $\mathbb{Z}_2$ gauge-Higgs model on a lattice charged particles are shown to exist.

1. Introduction

Charged particles in gauge theories cannot be created by applying local fields to the vacuum. In a massive theory, this fact leads to strong restrictions for charged particles to exist. In relativistic quantum field theory [1] as well as in a certain class of Hamiltonian lattice theories [2], it has been shown that in a particle state expectation values of local observables approach at spacelike infinity rapidly the vacuum expectation values. In a case where the total charge is the sum over the electric fluxes at spacelike infinity as in $U(1)$ gauge theories, it therefore must vanish (Swieca's theorem [1–4][1]). If however, the gauge symmetry is multiplicative like the triality in $SU(3)$ gauge theories, this conclusion is no longer valid, and charged particles may exist, provided the electric fluxes in different directions are strongly enough correlated. The absence of such correlations may be used as a criterium for confinement [5].

In [6] charged states of the $\mathbb{Z}_2$ gauge-Higgs model [7, 8] in the so-called free-charge phase (Fig. 1) have been constructed. This phase is massive, and the general discussion applies. It was left open in [6] whether there are particles in the charged sector. This gap will be closed by the present paper.

The starting point of our analysis is the euclidean Green's function in the charged sector:

$$G(x_0, \mathbf{x}) = (\Phi, U(\mathbf{x}) T^{|x_0|} \Phi) \ , \tag{1.1}$$

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[1] Note that the arguments in [2] and [1] do not use Lorentz covariance, in constrast to the original argument of Swieca [3] and the treatment in [4]

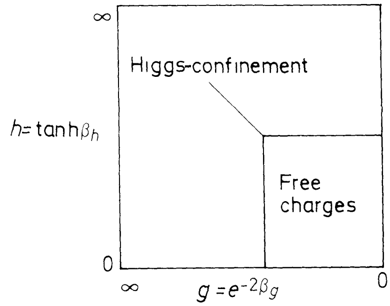

Fig. 1. Conjectured phase diagram for the $\mathbb{Z}_2$ gauge-Higgs model

where $(x_0, \mathbf{x}) \in \mathbb{Z} \times \mathbb{Z}^d$, $d \geqq 2$, Φ is the vector constructed in [6], which induces the charged state, U is the unitary representation of the lattice translations in $\mathbb{Z}^d$ and T is the transfer matrix in the charged sector. We show that in some part of the free charge phase the joint spectrum of T and U contains an isolated shell $\{(e^{-\omega(\mathbf{p})}, e^{i\mathbf{p}\cdot\mathbf{x}}), \mathbf{p} \in (-\pi, \pi]^d\}$ with an analytic function $\omega(\mathbf{p}) > 0$.

The existence of stable particle excitations in lattice gauge theories has first been proved by Schor [9] (see also [10, 11]. More references may be found in [12]). His analysis is based on the different exponential decay of G and its convolution inverse Γ in the presence of an isolated shell in the energy-momentum spectrum. Up to some modifications due to the non-local character of the particle creating fields, we can closely follow Schor's ideas in our proof. Another more recent approach to the particle structure in lattice gauge theories is due to Bricmont and Fröhlich [14–16]. Using their method it is also possible to prove the existence of charged particles in the $\mathbb{Z}_2$ gauge-Higgs model ([16] and private communication).

The $\mathbb{Z}_2$ gauge-Higgs model has the action

$$S = \sum_p \beta_g \delta\tau(p) + \sum_b \beta_h \tau(b) \delta\sigma(b) \tag{1.2}$$

with coupling constants β_g, $\beta_h > 0$, Ising spin fields σ and τ living on the sites and bonds, respectively, of the lattice $\mathbb{Z}^{d+1}$. The symbol δ denotes the lattice exterior derivative:

$$\delta\tau(p) = \prod_{b \in \partial p} \tau(b) \ , \qquad \delta\sigma(b) = \prod_{x \in \partial b} \sigma(x) \ , \tag{1.3}$$

where ∂p is the set of bonds contained in the plaquette p and ∂b the set of sites contained in the bond b. In this model σ represents the Higgs field and τ the gauge field.

All gauge invariant local functionals of the fields σ and τ are linear combinations of the functionals $\chi_L(\sigma, \tau)$, where L is a finite set of bonds and

$$\chi_L(\sigma, \tau) = \prod_{x \in \partial L} \sigma(x) \prod_{b \in L} \tau(b) \ , \tag{1.4}$$

∂L denoting the set of sites which are boundary points of an odd number of bonds in L.

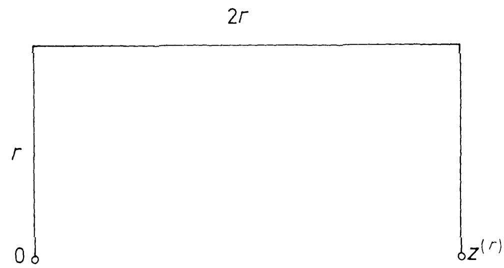

Fig. 2. The set N_r

The Green function G in (1.1) cannot be obtained as an expectation value of local fields in the Gibbs state defined by the action (1.2). The easiest way to describe G is as a square root of the Green function of two infinitely far separated charges,

$$G(x)=\lim_{r\to\infty}\{\langle\chi_{\theta N_r}\chi_{N_r+(|x_0|,\mathbf{x})}\rangle/\langle\chi_{\theta N_r}\chi_{N_r}\rangle\}^{1/2}\ . \tag{1.5}$$

Here N_r is the rectangular path from 0 to $z^{(r)}=(0,2r,0\ldots0)$ with corners in $(r,0\ldots0)$ and $(r,2r,0\ldots0)$ (Fig. 2) and θ is the reflection on the $(x_0=0)$-hyperplane.

Note that the transfer matrix T used in the definition of G differs from the transfer matrix given in [6] by a constant factor due to the automatic choice of the appropriate zero point of energy in the definition (1.5) (cf. [6, Theorem 6.2]).

G decays exponentially for $h\equiv\tanh\beta_h$ and $g\equiv e^{-2\beta_g}$ sufficiently small. One can therefore define, for $\mathbf{p}\in(-\pi,\pi]^d$

$$G_{\mathbf{p}}(t)=\begin{cases}(2\pi)^{-d/2}\sum\limits_{\mathbf{x}\in\mathbb{Z}^d}e^{i\mathbf{p}\cdot\mathbf{x}}G(t,\mathbf{x})\ , & t\geqq 0\\ 0\ , & t<0\end{cases}\ . \tag{1.6}$$

For small couplings $G_{\mathbf{p}}(t)$, $t\geqq 0$ is a nonzero analytic function of $\mathbf{p}$. For those $\mathbf{p}$ for which $G_{\mathbf{p}}(0)\neq 0$ the convolution inverse $\Gamma_{\mathbf{p}}$ of $-G_{\mathbf{p}}$ exists. It is given by the finite sum

$$\Gamma_{\mathbf{p}}(t)=-G_{\mathbf{p}}(0)^{-1}\sum_{n=0}^{t}(-1)^n\sum_{\substack{t_1,\ldots,t_n>0\\ \Sigma t_i=t}}\prod_{i=1}^{n}\{G_{\mathbf{p}}(t_i)/G_{\mathbf{p}}(0)\}\ . \tag{1.7}$$

Let $\mathscr{H}_1$ denote the Hilbert space which is spanned by the vectors $U(\mathbf{x})T^{(t)}\Phi$, $t\geqq 0$, $\mathbf{x}\in\mathbb{Z}^d$. $\mathscr{H}_1$ admits a representation as a direct integral with respect to the spectrum of U,

$$\mathscr{H}_1=\int^{\oplus}d^d\mathbf{p}\,\mathscr{H}_{(\mathbf{p})}\ . \tag{1.8}$$

T is a diagonal operator in $\mathscr{H}_1$,

$$T=\int^{\oplus}d^d\mathbf{p}\,T_{\mathbf{p}}\ . \tag{1.9}$$

With $\Phi = \int^{\oplus} d^d\mathbf{p}\,\Phi(\mathbf{p})$, $\Phi(\mathbf{p}) \in \mathscr{H}(\mathbf{p})$ we have

$$G_{\mathbf{p}}(t) = (\Phi(\mathbf{p}), T_{\mathbf{p}}^t \Phi(\mathbf{p})) \ , \qquad t \geqq 0 \ ,$$

and for $\mathbf{p}$ with $\|\Phi(\mathbf{p})\|^2 = G_{\mathbf{p}}(0) \neq 0$ we find

$$\Gamma_{\mathbf{p}}(t) = \begin{cases} (\Phi(\mathbf{p}), T_{\mathbf{p}}\{(1-E)T_{\mathbf{p}}\}^{t-1}\Phi(\mathbf{p}))\|\Phi(\mathbf{p})\|^{-4} \ , & t \geqq 1 \\ \|\Phi(\mathbf{p})\|^{-2} \ , & t = 0 \end{cases} , \tag{1.10}$$

where E is the projection onto $\{\lambda\Phi(\mathbf{p}), \lambda \in \mathbb{C}\}$.

Then

$$\omega(\mathbf{p}) = \lim_{t \to \infty} (-1/t \ln G_{\mathbf{p}}(t)) = \inf \operatorname{sp}(-\ln T_{\mathbf{p}}) \ , \tag{1.11}$$

and from the minimax theorem [18]

$$\hat{\omega}(\mathbf{p}) = \lim_{t \to \infty} (-1/t \ln \Gamma_{\mathbf{p}}(t)) \leqq \inf \{\operatorname{sp}(-\ln T_{\mathbf{p}}) \backslash \{\omega(\mathbf{p})\}\} \ . \tag{1.12}$$

In Sect. 3 we show that for h and g sufficiently small $G_{\mathbf{p}}(0) \neq 0$ and $\hat{\omega}(\mathbf{p}) > \omega(\mathbf{p})$ for all $\mathbf{p}$.

Thus $\omega(\mathbf{p})$ is an isolated eigenvalue of $-\ln T_{\mathbf{p}}$. Moreover, since the Fourier transform $\tilde{\Gamma}_{\mathbf{p}}(\omega)$ of $\Gamma_{\mathbf{p}}$ is an analytic function of $\mathbf{p}$ and ω for real $\mathbf{p}$ and $\operatorname{Im} \omega < \hat{\omega}(\mathbf{p})$ with a single zero at $\omega = i\omega(\mathbf{p})$, $\omega(\mathbf{p})$ is an analytic function of $\mathbf{p}$.

We conclude that there are normalizable states with energy-momentum spectrum contained in $\{(\omega(\mathbf{p}), \mathbf{p}), \mathbf{p} \in (-\pi, \pi]^d\}$. These states may be interpreted as particle states. Actually, one can even show by methods of Burnap [13, 9] that the spectrum of (T, U) in the orthogonal complement of $\mathscr{H}_1$ is contained in the set $\{(e^{-\lambda}, e^{i\mathbf{p}\cdot\mathbf{x}}), \lambda \geqq \hat{\omega}(\mathbf{p}), \mathbf{p} \in (-\pi, \pi]^d\}$. Thus the mass shell of these particles is isolated in the whole energy-momentum spectrum.

The existence of particle-like excitations in euclidean lattice gauge theories is a necessary condition for a particle interpretation of these models. In a next step it has to be investigated whether there are also states which can be interpreted as outgoing or incoming multiparticle states. In contrast to continuum quantum field theory where this property follows from locality by the Haag-Ruelle scattering theory ([19–21], for a generalization to gauge theories (massive case) see [1]) in euclidean lattice models no similar result is known.

2. The Polymer Expansion for G

In the free charge phase the $\mathbb{Z}_2$ gauge-Higgs model admits a convergent expansion which has first been described by Marra and Miracle-Sole [17]. We shall briefly review this expansion (for a more detailed exposition see [6]) and shall then give a representation of G which is meaningful also for space-time dependent couplings h and g. This will be important for the application of Schor's method for the proof of exponential decay of Γ/G.

The Marra and Miracle-Sole expansion is a polymer expansion where the polymers $\gamma \in \mathscr{G}_c$ are pairs $\gamma = \{P_\gamma, N_\gamma\}$ with P_γ being a coclosed set of plaquettes, N_γ a closed set of bonds, and where γ is connected as a graph in the following sense: The vertices are the co-connected components P_i of P_γ and the connected components N_j of N_γ, and the edges are the pairs $\{P_i, N_j\}$, where N_j winds an odd number of times $\omega(P_i, N_j)$ around P_i,

$$(P_i, N_j) = (-1)^{\omega(P_i, N_j)} = \chi_{N_j}(\sigma_i, \tau_{P_i}) = -1 \ , \tag{2.1}$$

where τ_{P_i} is a gauge field configuration with $\delta\tau_{P_i}(p) = -1$ iff $p \in P_i$.

Two polymers γ_1, γ_2 are compatible, $\gamma_1 \sim \gamma_2$, if no elementary 3-cube has plaquettes in P_{γ_1} and P_{γ_2} as faces, if no point is a boundary point of bonds in N_{γ_1} and N_{γ_2} and if no co-connected component of P_{γ_1} has an odd winding number with a connected component of N_{γ_2} and vice-versa.

The activity of a polymer γ is

$$\mu(\gamma) = h^{N_\gamma} g^{P_\gamma}(P_\gamma, N_\gamma) \tag{2.2}$$

with $h^{N_\gamma} = \prod_{b \in N_\gamma} h(b)$, $g^{P_\gamma} = \prod_{p \in P_\gamma} g(p)$. [We use space-time dependent couplings $h(b) = \tanh \beta_h(b)$, $g(p) = e^{-2\beta_g(p)}$.]

The partition function of the model is

$$Z = \sum_{n=0}^{\infty} (1/n!) \sum_{\substack{\gamma_1, \ldots, \gamma_n \\ \gamma_i \sim \gamma_j, i \neq j}} \mu(\gamma_1) \ldots \mu(\gamma_n) \ , \tag{2.3}$$

and its logarithm is

$$\ln Z = \sum_{\Gamma \in \mathscr{G}} c_\Gamma \mu^\Gamma \ , \tag{2.4}$$

where $\mathscr{G}$ is the set of nonnegative, integer valued functions on $\mathscr{G}_c$ with finite support (called clusters) and $\mu^\Gamma = \prod_{\gamma \in \mathscr{G}_c} \mu(\gamma)^{\Gamma(\gamma)}$. The coefficients c_Γ (the so-called Ursell functions) are of a purely combinatorial nature. We shall often exploit the fact that $c_\Gamma = 0$ if $\Gamma = \Gamma_1 + \Gamma_2$ with $\gamma_1 \sim \gamma_2$ for all $\gamma_1 \in \operatorname{supp} \Gamma_1$, $\gamma_2 \in \operatorname{supp} \Gamma_2$, as well as the estimate

$$\sum_{\Gamma \not\sim \gamma} |c_\Gamma| \, |\mu^\Gamma| \leqq F_1(\beta) |\gamma| \ , \tag{2.5}$$

where $\beta = -\ln(\sup\{|h(b)|, |g(p)|\})$ and F_1 is a monotonically decreasing function related to the generating function of the polymers. For each $\beta_0 < \beta$ with $F_1(\beta_0) < \infty$, (2.5) implies the following bound for the contribution of large clusters:

$$\sum_{\substack{\Gamma \not\sim \gamma \\ \|\Gamma\| \geqq n}} |c_\Gamma| \, |\mu^\Gamma| \leqq e^{-(\beta - \beta_0)n} F_1(\beta_0) |\gamma| \ , \tag{2.6}$$

where $\|\Gamma\| = \sum \Gamma(\gamma)|\gamma|$ and $|\gamma| = |P_\gamma| + |N_\gamma|$.

The expectation value of χ_L has the following expansion. Let S be a finite even family of lattice sites, and let $\hat{S}$ be the set of sites occurring an odd number of times in S. Denote by Conn (S) the set of all sets of bonds M with $\partial M = \hat{S}$ such that each connected component of M meets at least one point in S. Furthermore, for

$\partial M = \partial L = \hat{S}$, set

$$a_{L,M,S}(\gamma) = \begin{cases} 0 & \text{if } N_\gamma \text{ is connected with } M \text{ or } S, \\ (P_\gamma, L \triangle M) & \text{otherwise} \end{cases} \tag{2.7}$$

($\triangle$ denotes the symmetric difference) and $a_L \equiv a_{L,\emptyset,\emptyset}$ if L is closed. Then for all S with $\hat{S} = \partial L$ one gets

$$\langle \chi_L \rangle = \sum_{M \in \mathrm{Conn}(S)} h^M \exp\left\{ \sum_{\Gamma \in \mathscr{G}} c_\Gamma \mu^\Gamma (a^\Gamma_{L,M,S} - 1) \right\} . \tag{2.8}$$

This expression is a generalization of the expansion for $\langle \chi_L \rangle$ given in [6] [expressions (4.6) and (4.16)] where $S = \partial L$ was chosen. Different choices of S (with $\hat{S} = \partial L$) correspond to resummations in (2.8). The freedom in the choice of S allows a more symmetric treatment of open and closed paths L which turns out to be convenient in the proof of Theorem 3.2 below. The convergence of (2.8) follows from the estimate (2.5) for the exponent together with the combinatorial estimate $|\{M : M \in \mathrm{Conn}(S), |M| \leqq n\}| \leqq |S|(2d+1)^n$.

To obtain an expansion of the Green function in the case of variable couplings, one has to take the dependence of the normalization factors on the couplings properly into account. Let the symbol k denote the dependence of the expectation values on the configuration of couplings: $k = \{h(b), g(p)\}_{b,p}$. Let $k^{(r)}$ be periodic with respect to translations by $z^{(r)}$ and coincide with k on bonds and plaquettes which belong to the set $\{x \in \mathbb{Z}^{d+1}, -r \leqq x_1 < r\}$; let $k_{t,-}$ (respectively $k_{t,+}$) be symmetric with respect to reflections on the $x_0 = t$ plane and coincide with k on bonds and plaquettes in the half space $x_0 \leqq t$ ($x_0 \geqq t$).

Let us denote

$$\langle \chi_{\theta N_r + x} \chi_{N_r + y} \rangle_k \equiv H_r(x, y | k) . \tag{2.9}$$

Then we define the Green function G for space-time dependent couplings by ($x_0 \leqq y_0$):

$$G(x,y)_k = \lim_{r \to \infty} \{ H_r(x,y|k^{(r)}) / [H_r(x,x|k^{(r)}_{x_0,-}) H_r(y,y|k^{(r)}_{y_0,+})]^{1/2} \}^{1/2} \tag{2.10}$$

with the square root functions being uniquely fixed by the condition $\langle \,.\, \rangle^{1/2} \geqq 0$ for real positive couplings. Now we apply (2.8) and choose $S = \{x, y, x + z^{(r)}, y + z^{(r)}\}$ for the numerator and $S = \emptyset$ for the expectation values in the denominator in (2.10).

We identify bonds, plaquettes, 3-cubes, etc., of the lattice with their geometrical central points (which are points in $(1/2\mathbb{Z})^{d+1}$). For $a < b$, define $\mathscr{G}_{a,b}$ as the set of all clusters $\Gamma \in \mathscr{G}$ such that all bonds in N_γ and all plaquettes in P_γ are contained in the time-slice $a < x_0 < b$ for all $\gamma \in \Gamma$.

To control the convergence of the right-hand side of (2.10) we divide the set of clusters into several subsets $\mathscr{G}_{t,+\infty}$ and $\mathscr{G}_{-\infty,t}$. Using the representation (2.8) with the abbreviations

$$\begin{aligned} a^r_{x,y;M}(\gamma) &= a_{(\theta N_r + x) \triangle (N_r + y), M, \{x, y, x + z^{(r)}, y + z^{(r)}\}}(\gamma) , \\ a^r_z(\gamma) &= a_{(\theta N_r \cup N_r) + z, \emptyset, \emptyset}(\gamma) , \end{aligned} \tag{2.11}$$

and defining (for $z_0 \leqq w_0$) $\mathscr{G}^*_{z_0,w_0} \equiv \mathscr{G} \backslash (\mathscr{G}_{-\infty,z_0} \cup \mathscr{G}_{w_0,\infty})$ we may write $G(x,y)^2$ as the limit $r \to \infty$ of

$$\sum_{M \in \mathrm{Conn}(x,y,x_r,y_r)} h^M \exp \left\{ \sum_{\Gamma \in \mathscr{G}_{y_0,\infty}} c_\Gamma \mu^\Gamma ((a^r_{x,y;M})^\Gamma - (a^r_y)^\Gamma) \right.$$
$$+ \sum_{\Gamma \in \mathscr{G}_{-\infty,x_0}} c_\Gamma \mu^\Gamma ((a^r_{x,y;M})^\Gamma - (a^r_x)^\Gamma) + \sum_{\Gamma \in \mathscr{G}^*_{x_0,y_0}} c_\Gamma \mu^\Gamma ((a^r_{x,y;M})^\Gamma - 1)$$
$$-1/2 \sum_{\Gamma \in \mathscr{G}^*_{x_0,x_0}} c_\Gamma \mu^\Gamma_{x_0,-} ((a^r_x)^\Gamma - 1)$$
$$\left. -1/2 \sum_{\Gamma \in \mathscr{G}^*_{y_0,y_0}} c_\Gamma \mu^\Gamma_{y_0,+} ((a^r_y)^\Gamma - 1) \right\} . \tag{2.12}$$

The subscript $z_{0,\pm}$ on $\mu^\Gamma_{z_0,\pm}$ indicates that the activity is defined on $k_{z_0,\pm}$, μ and h being defined on k.

By straightforward application of the estimates (2.5) and (2.6) we may control the limit $r \to \infty$ of the expression above and get

$$G(x,y)_k = \sum_{M \in \mathrm{Conn}(x,y)} h^M \varrho(M;x,y) \ , \tag{2.13}$$

$$\varrho(M;x,y) = \exp(D_{M;x,y}) \ , \tag{2.14}$$

with

$$D_{M;x,y} = A_+(M;x,y) + A_-(M;x,y) + B(M;x,y) - C_+(y)/2 - C_-(x)/2 \ , \tag{2.15}$$

where

$$A_+(M;x,y) = \sum_{\Gamma \in \mathscr{G}_{y_0,\infty}} c_\Gamma \mu^\Gamma (a^\Gamma_{x,y;M} - a^\Gamma_y) \ , \tag{2.16}$$

$$A_-(M;x,y) = \sum_{\Gamma \in \mathscr{G}_{-\infty,x_0}} c_\Gamma \mu^\Gamma (a^\Gamma_{x,y;M} - a^\Gamma_x) \ , \tag{2.17}$$

$$B(M;x,y) = \sum_{\Gamma \in \mathscr{G}^*_{x_0,y_0}} c_\Gamma \mu^\Gamma (a^\Gamma_{x,y;M} - 1) \ , \tag{2.18}$$

and

$$C_\pm(x) = \sum_{\Gamma \in \mathscr{G}^*_{x_0,x_0}} c_\Gamma \mu^\Gamma_{x_0,\pm} (a^\Gamma_x - 1) \tag{2.19}$$

with $a_x(\gamma) = \lim_{r\to\infty} a^r_x(\gamma)$ and for $M \in \mathrm{Conn}(x,y)$, $a_{x,y;M}(\gamma) = \lim_{r\to\infty} a^r_{x,y;M\,\Delta(M'+z^{(r)})}$ for some $M' \in \mathrm{Conn}(x,y)$.

The representation (2.13) converges provided $|h(b)| < h_c$, $|g(p)| < g_c$ for some $h_c, g_c > 0$. It will be the basis for the analysis of the energy-momentum spectrum in the charged sector.

Using the estimates (2.5) and (2.6) we can easily derive the following bound for $D_{M;x,y}$:

$$|D_{M;x,y}| \leqq k_1 |M| + k_2 \ , \tag{2.20}$$

k_1 and k_2 being constants.

The term $k_1|M|$ is a bound for $A_+ + A_- + B$ and k_2 is a bound for $C_+ + C_-$. The first bound is an easy consequence of (2.5). The second one follows simply from the estimates

$$\left|\sum_{\Gamma\in\mathscr{G}^*_{x_0,x_0}} c_\Gamma \mu^\Gamma_{x_0}(a^\Gamma_x-1)\right| \leqq \sum_{\substack{y:y\in Z^{d+1}\\ y_0=x_0}} \sum_{\Gamma:\Gamma\not\perp y} |c_\Gamma|\,|\mu^\Gamma_{x_0}|\,|a^\Gamma_x-1|$$
$$\leqq 2\sum_{\substack{y:y\in Z^{d+1}\\ y_0=x_0}} \sum_{\substack{\Gamma:\Gamma\not\perp y\\ \|\Gamma\|\geqq|\mathbf{y}-\mathbf{x}|}} |c_\Gamma|\,|\mu^\Gamma_{x_0}|$$
$$\leqq 2\sum_{\substack{y:y\in Z^{d+1}\\ y_0=x_0}} F_1(\beta_0)e^{-(\beta-\beta_0)|\mathbf{y}-\mathbf{x}|}\,, \tag{2.21}$$

where the last sum is a finite constant. Above we used improperly the symbol $\Gamma\not\perp y$ to indicate that the clusters Γ touch the point y. □

Inserting (2.20) into (2.13)–(2.14) one obtains the following bound:

$$|G(x,y)|\leqq c_1(c_2h_0)^{|x-y|} \tag{2.22}$$

for $|h(b)|\leqq h_0$, $|g(p)|\leqq g_0$; $0<h_0<h_c$; $0<g_0<g_c$ and constants $c_1,c_2>0$, where h_c and g_c are sufficiently small.

We also need a lower bound for $G_{\mathbf{p}}(t)$, $t\geqq 0$, in the case of constant positive couplings. Since

$$\frac{G_{\mathbf{p}}(t)}{G_{\mathbf{p}}(0)}=\frac{(\Phi(\mathbf{p}),T^t_{(\mathbf{p})}\Phi(\mathbf{p}))}{\|\Phi(\mathbf{p})\|^2}\geqq\left[\frac{(\Phi(\mathbf{p}),T_{(\mathbf{p})}\Phi(\mathbf{p}))}{\|\Phi(\mathbf{p})\|^2}\right]^t=\left[\frac{G_{\mathbf{p}}(1)}{G_{\mathbf{p}}(0)}\right]^t, \tag{2.23}$$

a lower bound on $G_{\mathbf{p}}(1)$ provides a lower exponential estimate on $G_{\mathbf{p}}(t)$. A lower bound for $G_{\mathbf{p}}(1)$ is easy to find. According to (2.13) and (2.14), $G(x_0=1,\mathbf{x}=0)$ is a sum of positive terms and using (2.20) we find immediately

$$G(x_0=1,\mathbf{x}=0)\geqq he^{-k_1-k_2}\,. \tag{2.24}$$

Now

$$G_{\mathbf{p}}(1)=(2\pi)^{-d/2}[G(x_0=1,\mathbf{x}=0)+\sum_{\mathbf{x}\neq 0}G(x_0=1,\mathbf{x})e^{-i\mathbf{p}\cdot\mathbf{x}}]\,. \tag{2.25}$$

So using (2.21) we find c_3, c_4 such that

$$G_{\mathbf{p}}(1)\geqq(2\pi)^{-d/2}h[e^{-k_1-k_2}-c_1c_2\sum_{\mathbf{x}\neq 0}(c_2h)^{|\mathbf{x}|}]\geqq c_3h(1-c_4h)\,, \tag{2.26}$$

which is positive for h small enough.

3. Decay Properties of $\Gamma_{\mathbf{p}}$

We now want to show that the exponential decay of $\Gamma_{\mathbf{p}}(x_0)$ is faster than that of $G_{\mathbf{p}}(x_0)$ for sufficiently small couplings. The reason for this behavior is that the leading term in the expansion of $G_{\mathbf{p}}(x_0)$ is absent in the expansion of $\Gamma_{\mathbf{p}}(x_0)$. Schor's method of proof relies on a decoupling property of $G_{\mathbf{p}}(x_0,y_0)$ for space-independent but time-dependent couplings. We will consider $g(p)=g_t$ for all plaquettes in the hyperplane $x_0=t$, $h(b)=h_t$ for all time-like bonds connecting the hyperplanes $x_0=t$ and $x_0=t+1$. For all time-like plaquettes or space-like bonds we have $g(p)=g$ and $h(b)=h$.

In order to write down an expression for $\Gamma_{\mathbf{p}}$ we need to prove that $G_{\mathbf{p}}(t_2, t_2)$ is non-zero. We have

$$G_{\mathbf{p}}(t_1, t_1) = (2\pi)^{-d/2}\left[G(t_1, t_1; \mathbf{x}=0) + \sum_{\mathbf{x}\neq 0} G(t_1, t_1; \mathbf{x})e^{-i\mathbf{p}\cdot\mathbf{x}}\right]. \tag{3.1}$$

But, according to (2.13), $G(t_1, t_1; \mathbf{x}=0) = \varrho(\emptyset; (t_1, \mathbf{0}), (t_1, \mathbf{0}))$ whose modulus, according to (2.20) is bounded from below by e^{-k_2}.

So

$$\begin{aligned} |G_{\mathbf{p}}(t_1, t_1)| &\geqq (2\pi)^{-d/2}\left[e^{-k_2} - \sum_{\mathbf{x}\neq 0} |G(t_1, t_1; \mathbf{x})|\right] \\ &\geqq (2\pi)^{-d/2}\left[e^{-k_2} - c_1 \sum_{\mathbf{x}\neq 0} (c_2 h_0)^{|\mathbf{x}|}\right], \end{aligned} \tag{3.2}$$

according to (2.22). Choosing h_0 small enough we get a non-zero $G_{\mathbf{p}}(t_1, t_1)$ for all $\mathbf{p}$'s, which enables us to write

$$\Gamma_{\mathbf{p}} = -P_{\mathbf{p}} \sum_{n\geqq 0} (-1)^n Q_{\mathbf{p}}^n, \tag{3.3}$$

where $\Gamma_{\mathbf{p}}$, $P_{\mathbf{p}}$, and $Q_{\mathbf{p}}$ are understood as operators on $l^2(\mathbb{Z})$, the matrix elements of the two latter being

$$P_{\mathbf{p}}(t_1, t_2) = (G_{\mathbf{p}}(t_1, t_1))^{-1}\delta_{t_1, t_2} \tag{3.4}$$

and

$$Q_{\mathbf{p}}(t_1, t_2) = (G_{\mathbf{p}}(t_1, t_2)/G_{\mathbf{p}}(t_2, t_2))(1 - \delta_{t_1, t_2}). \tag{3.5}$$

For each matrix element $\Gamma_{\mathbf{p}}(t_1, t_2)$ the sum in (3.3) is finite since $Q_{\mathbf{p}}^n(t_1, t_2) = 0$ if $n > |t_1 - t_2|$. This establishes the existence of an inverse for $G_{\mathbf{p}}$ on $l^2(\mathbb{Z})$.

Now we start with the analysis of the exponential decay of $\Gamma_{\mathbf{p}}$. We will study the dependence of $\Gamma_{\mathbf{p}}$ on the variables h_t and g_t. In order to determine the leading contributions we investigate the derivatives of $G_{\mathbf{p}}$ and $\Gamma_{\mathbf{p}}$ with respect to h_t and g_t for various t's.

Lemma 3.1. i) $G_{\mathbf{p}}(x_0, y_0)|_{h_t=0} = 0$ *for* $x_0 \leqq t < y_0$.
ii) $\Gamma_{\mathbf{p}}(x_0, y_0)|_{h_t=0} = 0$ *if* $x_0 \leqq t < y_0$.

Proof. i) is obvious from (2.13). For ii) we note that i) means that $G_{\mathbf{p}}|_{h_t=0}$, seen as an operator acting on $l^2(\mathbb{Z})$, reduces the sub-spaces $l^2(\mathbb{Z}_{t+})$ and $l^2(\mathbb{Z}_{t-})$, ($\mathbb{Z}_{t\pm} = \{x_0 \in \mathbb{Z} | x_0 > t$ for $+$ or $x_0 \leqq t$ for $-\}$), and $\Gamma_{\mathbf{p}}|_{h_t=0}$, being its inverse, does the same. Alternatively, ii) is a consequence of (3.3) and i) since at least one of the matrix elements $Q_{\mathbf{p}}$ in the expansion of $Q_{\mathbf{p}}^n(x_0, y_0)$ is zero at $h_t = 0$, $x_0 \leqq t < y_0$. □

Theorem 3.2.

$$\partial_{h_t} G_{\mathbf{p}}|_{h_t=g_t=0} = G_{\mathbf{p}} A_t G_{\mathbf{p}}|_{h_t=g_t=0} \tag{3.6}$$

with $A_t(x_0, y_0) = (2\pi)^{d/2} \exp[f_{t+1}]\delta_{x_0,t}\delta_{y_0,t+1}$, *where* f_{t+1} *is a holomorphic function of* h, g, *and* g_{t+1} *for* $|h| < a$, $|g|$, $|g_{t+1}| < b$ *for certain* a, $b > 0$.

This is the central theorem of this work. Its proof is somewhat technical and will be given in the Appendix. The function f_{t+1} will also be defined in the Appendix. $(-f_{t+1})$ can be interpreted as the contribution of the gauge field to the mass of the particle. It corresponds to $-\ln a$ in the notation of [6, Theorem 6.2].

Corollary 3.3.

$$\partial_{h_t} \Gamma_{\mathbf{p}}|_{h_t = g_t = 0} = A_t \ , \tag{3.7}$$

hence $\partial_{h_t} \Gamma_{\mathbf{p}}(x_0, y_0)|_{h_t = g_t = 0} = 0$ *for* $y_0 - x_0 \geqq 2$.

Proof. From the relation $\Gamma_{\mathbf{p}} G_{\mathbf{p}} = -1$ we get $\partial_{h_t} \Gamma_{\mathbf{p}} = \Gamma_{\mathbf{p}} (\partial_{h_t} G_{\mathbf{p}}) \Gamma_{\mathbf{p}}$. Inserting now the relation (3.6) we obtain the desired result. □

Comparing Corollary 3.3 with Theorem 3.2 we understand the essential reason why $\Gamma_{\mathbf{p}}$ has a faster decay than $G_{\mathbf{p}}$: the dominant dependence of $G_{\mathbf{p}}$ on h_t, $x_0 \leqq t < y_0$, is of first order, but the one of $\Gamma_{\mathbf{p}}$ is of higher order. Corollary 3.3 is the crucial point of the proof of the existence of isolated singularities of $G_{\mathbf{p}}$ as we shall see below. Nevertheless, informations about the other derivatives of $\Gamma_{\mathbf{p}}$ are also useful since they provide information about the spectrum of the model beyond the one-particle singularity.

Lemma 3.4.

$$\begin{aligned}
&\text{i) } \partial_{g_t}^m G(x, y)|_{g_t = 0} = 0 && \forall t, 1 \leqq m < 2(d-1), (d \geqq 2), \forall (x, y) \ , \\
&\text{ii) } \partial_{g_t}^m G(x, y)|_{g_t = 0} = 0 && \textit{if } m \textit{ is odd} \ , \\
&\text{iii) } \partial_{h_t}^m G_{\mathbf{p}}(x_0, y_0)|_{h_t = 0} = 0 && \textit{if } x_0 \leqq t < y_0 \textit{ and } m \textit{ is even} \ .
\end{aligned} \tag{3.8}$$

Proof. Part i). If $d > 2$ the smallest co-closed set of plaquettes in the hyperplane $x_0 = t$ is formed by $2(d-1)$ plaquettes. These sets correspond to co-boundaries of isolated bonds in $\mathbb{Z}^d$. So, if a polymer contains plaquettes on the hyperplane $x_0 = 0$ its activity contains at least one factor $g_t^{2(d-1)}$. If $d = 2$ such a polymer contains also at least one factor g_t^2 in its activity since in order to be co-closed it cannot have only one plaquette on the hyperplane.

Part ii) is a consequence of the fact that all polymers contain an even number of plaquettes on the hyperplane $x_0 = t$.

Part iii) comes from the observation that the lines $M \in \mathrm{Conn}\,(x, y)$ for $x_0 \leqq t < y_0$ contain an odd number of variables h_t, but the polymers γ contain an even number. □

Corollary 3.5.

$$\begin{aligned}
&\text{i) } \partial_{g_t}^m \Gamma_{\mathbf{p}}|_{g_t = 0} = 0 && \forall t, \ 1 \leqq m < 2(d-1) \ , \\
&\text{ii) } \partial_{g_t}^m \Gamma_{\mathbf{p}}|_{g_t = 0} = 0 && \textit{if } m \textit{ is odd} \ , \\
&\text{iii) } \partial_{h_t}^2 \Gamma_{\mathbf{p}}(x_0, y_0)|_{h_t = g_t = 0} = 0 && \textit{if } x_0 \leqq t < y_0 \ .
\end{aligned} \tag{3.9}$$

Proof. The above relations are simple consequences of the Leibniz formula,

$$\partial^m \Gamma_{\mathbf{p}} = \sum_{s=0}^{m-1} \binom{m}{s} (\partial^s \Gamma_{\mathbf{p}}) (\partial^{m-s} G_{\mathbf{p}}) \Gamma_{\mathbf{p}} \ , \tag{3.10}$$

with the use of the Lemmas 3.1 and 3.4, of the Theorem 3.2, Corollary 3.3 and induction.

We show explicitly the proof of iii), the others can be done in an analogous way. We have at $h_t = g_t = 0$,

$$\begin{aligned}\partial_{h_t}^2 \Gamma_{\mathbf{p}} &= \Gamma_{\mathbf{p}}(\partial_{h_t}^2 G_{\mathbf{p}})\Gamma_{\mathbf{p}} + 2(\partial_{h_t}\Gamma_{\mathbf{p}})(\partial_{h_t} G_{\mathbf{p}})\Gamma_{\mathbf{p}} = \Gamma_{\mathbf{p}}(\partial_{h_t}^2 G_{\mathbf{p}})\Gamma_{\mathbf{p}} + 2A_t G_{\mathbf{p}} A_t G_{\mathbf{p}} \Gamma_{\mathbf{p}} \\ &= \Gamma_{\mathbf{p}}(\partial_{h_t}^2 G_{\mathbf{p}})\Gamma_{\mathbf{p}} \ . \end{aligned} \tag{3.11}$$

Now, for $x_0 \leqq t < y_0$ we conclude from the last equality that $\partial_{h_t}^2 \Gamma_{\mathbf{p}}(x_0, y_0)|_{h_t = g_t = 0} = 0$, since $\Gamma_{\mathbf{p}}$ reduces $l^2(\mathbb{Z}_{t+})$ and $l^2(\mathbb{Z}_{t-})$ and, according with Lemma 3.4, iii), $\partial_{h_t}^2 G_{\mathbf{p}}$ does the same. □

It suffices to consider the function $\Gamma_{\mathbf{p}}(0, y_0)$, $y_0 > 1$. We have proven the following facts about it:

$$\begin{array}{lll} 1) \ \Gamma_{\mathbf{p}}(0, y_0)|_{h_t = 0} = 0 & \text{if} \quad 0 \leqq t < y_0 \ , & \\ 2) \ \partial_{h_t} \Gamma_{\mathbf{p}}(0, y_0)|_{h_t = g_t = 0} = 0 & \forall t \, (\text{since } |y_0| > 1) \ , & \\ 3) \ \partial_{g_t}^m \Gamma_{\mathbf{p}}(0, y_0)|_{g_t = 0} = 0 & \forall t; \ 1 \leqq m < 2(d-1), \text{ or } m \text{ odd} \ , & (3.12) \\ 4) \ \partial_{h_t}^2 \Gamma_{\mathbf{p}}(0, y_0)|_{g_t = h_t = 0} = 0 & \text{if} \quad 0 \leqq t < y_0 \ . & \end{array}$$

We consider now $h_t = h$ and $g_t = g$ for all t outside of the interval $0 \leqq t < y_0$. Considering $\Gamma_{\mathbf{p}}(0, y_0)$ as a function of h_t and g_t for a t on $0 \leqq t < y_0$, $\Gamma_{\mathbf{p}}(0, y_0)$ is holomorphic in all the couplings and has an absolutely convergent power series expansion for $|h_t| < a$, $|g_t| < b$,

$$\Gamma_{\mathbf{p}}(0, y_0) = \sum_{n, m \geqq 0} a_{m,n} h_t^m g_t^n \ , \tag{3.13}$$

where the coefficients $a_{m,n}$ are holomorphic functions of all other variables. The relations (3.12) impose constraints on the coefficients $a_{m,n}$. An analysis of these constraints brings the following general structure for $\Gamma_{\mathbf{p}}(0, y_0)$ as function of h_t and g_t:

$$\Gamma_{\mathbf{p}}(0, y_0) = h_t u_t(h_t, g_t) \tag{3.14}$$

with $u_t(h_t, g_t) = h_t^2 \alpha_t(h_t) + g_t^{2(d-1)} \beta_t(h_t, g_t^2)$, where $\alpha_t(h_t)$ is a holomorphic function of h_t (for $|h_t| < a$) and of all other variables except g_t. β_t is also holomorphic in all variables. The dependence of β_t on g_t^2 comes from the fact that Γ depends on even powers of g_t.

$u_t(h_t, g_t)$ is a holomorphic function of h_t, g_t for $|h_t| \leqq a$, $|g_t| \leqq b$. We define

$$\varphi(\lambda) \equiv u_t(\lambda^{d-1} h_t, \lambda g_t) = \lambda^{2(d-1)} [h_t^2 \alpha_t(\lambda^{d-1} h_t) + g_t^{2(d-1)} \beta_t(\lambda^{d-1} h_t, \lambda^2 g_t^2)] \ . \tag{3.15}$$

$\varphi(\lambda)$ as function of λ is holomorphic for λ's satisfying $|\lambda^{d-1} h_t| \leqq a$ and $|\lambda g_t| \leqq b$. From Schwarz' lemma we have

$$|\varphi(\lambda)| \leqq |\lambda/\lambda_c|^{2(d-1)} \sup_{\substack{\lambda' \\ |\lambda'| \leqq \lambda_c}} |\varphi(\lambda')| \tag{3.16}$$

with $\lambda_c = \min(|a/h_t|^{1/(d-1)}; |b/g_t|)$.

So setting $\lambda=1$ we get

$$|u_t(h_t, g_t)| \leqq j_t \cdot \sup_{h_t', g_t'} |u_t(h_t', g_t')| \ , \tag{3.17}$$

where $j_t = \max(|h_t/a|^2; |g_t/b|^{2(d-1)})$.

Inserting (3.17) in (3.14) and using the maximum modulus theorem, we arrive finally at the estimate

$$|\Gamma_{\mathbf{p}}(0, y_0)| \leqq (|h_t|/a) j_t \left\{ \sup_{\substack{|h_t'|<a \\ |g_t'|<b}} |\Gamma_{\mathbf{p}}'(0, y_0)| \right\} , \tag{3.18}$$

where $\Gamma_{\mathbf{p}}'(0, y_0)$ is a short-hand notation for $\Gamma_{\mathbf{p}}(0, y_0)$ as a function of h_t' and g_t' for the specified t, the other variables being maintained. Iterating this inequality for all t with $0 \leqq t < y_0$, we find

$$|\Gamma_{\mathbf{p}}(0, y_0)| \leqq \prod_{t=1}^{y_0-1} (|h_t|/a) j_t \left\{ \sup_{\substack{|h_t'|<a, \, |g_t'|<b \\ 0 \leqq t < y_0}} |\Gamma_{\mathbf{p}}'(0, y_0)| \right\} . \tag{3.19}$$

The supremum of the right-hand side can be estimated by inserting into the expansion (3.3) for $\Gamma_{\mathbf{p}}$ the upper bound for $|G_{\mathbf{p}}(t_1, t_2)|$ implied by (2.22) and the lower bound for $|G_{\mathbf{p}}(t_1, t_1)|$ of (3.2). One obtains

$$\begin{aligned}\sup_{\substack{|h_t|<a, \, |g_t|<b \\ 0 \leqq t < y_0}} |\Gamma_{\mathbf{p}}(0, y_0)| &\leqq \sup \left\{ \sum_{k=1}^{y_0} \sum_{0=t_0<t_1 \ldots <t_k=y_0} \prod_{i=1}^{k} |G_{\mathbf{p}}(t_{i-1}, t_i)| \Big/ \prod_{i=0}^{k} |G_{\mathbf{p}}(t_i, t_i)| \right\} \\ &\leqq c_1 (c_2)^{y_0} \end{aligned} \tag{3.20}$$

for some $c_1, c_2 > 0$.

Taking finally $h_t = h$, $g_t = g \, \forall t$, we get

$$|\Gamma_{\mathbf{p}}(0, y_0)| \leqq c_1 (c_2)^{y_0} [|h/a| \cdot j]^{|y_0|} , \tag{3.21}$$

where

$$j \equiv \max [(h/a)^2, (g/b)^{2(d-1)}] . \tag{3.22}$$

Now, using (3.21) and the exponential bound on $G_{\mathbf{p}}(t)$ given by (2.22) and (2.23) combined with (2.26), it follows that, for h and g sufficiently small $\hat{\omega}(\mathbf{p}) = \lim_{t \to \infty} (-1/t \ln \Gamma_{\mathbf{p}}(t)) > \omega(\mathbf{p})$, which completes our proof for the existence of an isolated eigenvalue of $-\ln T_{\mathbf{p}}$ at $\omega(\mathbf{p})$. We note that $\omega(\mathbf{p}) \simeq -\ln h$ and $\hat{\omega}(\mathbf{p}) \simeq -\ln h - \ln j$. It is interesting to see that $-\ln j$ is just the mass gap in the vacuum sector in first order of approximation. We may interpret $-\ln j$ as the mass of a stable "photon" (case $j = g^{2(d-1)}$, $d > 2$) or as the energy of a scattering state of two charged particles (case $j = h^2$).

4. Appendix. Proof of Theorem 3.2

In this appendix we will present the proof of Theorem 3.2 above. According to (2.13) we have for $x_0 \leqq t < y_0$:

$$\partial_{h_t} G(x, y)|_{h_t=0} = \sum_{M \in \mathrm{Conn}(x, y)} (\partial_{h_t} h^M)|_{h_t=0} \, \varrho(M; x, y)|_{h_t=0} \ . \tag{4.1}$$

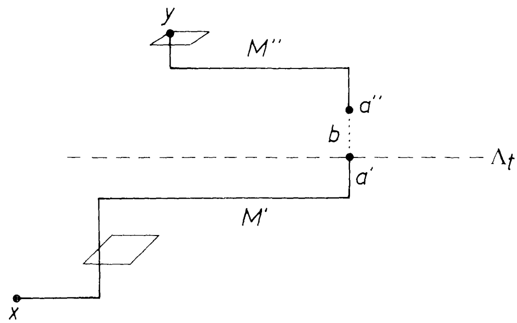

Fig. 3

For t outside of the interval $x_0 \leqq t < y_0$ the derivative vanishes at $h_t = 0$, since in this case the sets $M \in \mathrm{Conn}(x, y)$ contain an even number of timelike bonds in the time slice $t < z_0 < t+1$, $z \in \mathbb{Z}^{d+1}$.

The lines M which contribute to the sum (4.1) have in general the form found in Fig. 3. M is a disjoint union $M = M' \cup M'' \cup \{b\}$, where M' is the subset of M which lies below the hyperplane $\Lambda_t = \{x \in \mathbb{Z}^{d+1} : x_0 = t\}$, M'' the sub-set which lies above Λ_{t+1} and b is the (unique) bond in M connecting M' and M''. Let a', a'' be the boundary points of b, with a' in Λ_t and $a'' \equiv a' + \hat{e}_0$, $\hat{e}_0$ being a unit vector in positive time direction. The set M' belongs to the set $\mathrm{Conn}(x, a')$ and the set M'' belongs to $\mathrm{Conn}(a'', y)$. So, we are able to write (for $x_0 \leqq t < y_0$):

$$\partial_{h_t} G(x, y)|_{h_t=0} = \sum_{a' \in \Lambda_t} \sum_{\substack{M' \in \mathrm{Conn}(x, a') \\ M'' \in \mathrm{Conn}(a'', y)}} (h^{M'} h^{M''})|_{h_t=0}\, \varrho(M; x, y)|_{h_t=0} \,. \tag{4.2}$$

Now we have the following factorization formula:

$$\varrho(M; x, y)|_{h_t=g_t=0} = \exp[f_{t+1}] \cdot [\varrho(M'; x, a')\varrho(M''; a'', y)]|_{h_t=g_t=0} \,. \tag{4.3}$$

Here

$$f_{t+1} = 1/2 \sum_{\Gamma \in \mathcal{G}_{t,t+2}} c_\Gamma \mu^\Gamma_{t+1,-}(a^\Gamma_s - 1)|_{h_t=g_t=0} \,, \tag{4.4}$$

where $s \in \Lambda_t$. Note that f_{t+1} depends only on h, g, and g_{t+1}, in particular it does not depend on s.

Expression (4.3) follows from the relations (valid at $h_t = g_t = 0$)

$$A_+(M; x, y) = A_+(M''; a'', y) \,, \tag{4.5}$$

$$A_-(M; x, y) = A_-(M'; x, a') \,, \tag{4.6}$$

$$C_+(a') = A_+(M'; x, a') = A_-(M''; a'', y) = 0 \tag{4.7}$$

and

$$B(M; x, y) = B(M'; x, a') + B(M''; a'', y) + f_{t+1} - 1/2\, C_-(a'') \,. \tag{4.8}$$

The last relation follows from the fact that at $h_t = g_t = 0$ the clusters belonging to $\mathcal{G}^*_{x_0,y_0}$ for which $\mu^\Gamma \neq 0$ are contained either in the set $\mathcal{G}^*_{x_0,t}$, in the set $\mathcal{G}^*_{t+1,y_0}$ or in the set $\mathcal{G}_{t,t+1}$. Thus the sum in the definition of $B(M; x, y)$ splits into a sum over

$\mathscr{G}^*_{x_0,t}$ (where $a^\Gamma_{x,y;M} = a^\Gamma_{x,a';M'}$), a sum over $\mathscr{G}^*_{t+1,y_0}$ (where $a^\Gamma_{x,y;M} = a^\Gamma_{a'',y;M''}$) and a sum over $\mathscr{G}_{t,t+1}$ (where $a^\Gamma_{x,y;M} = a^\Gamma_{a'}$). The first sum coincides with $B(M', x, a')$, the second one with $B(M'', a'', y)$ and the third one yields (at $h_t = g_t = 0$)

$$\sum_{\Gamma \in \mathscr{G}_{t,t+1}} c_\Gamma \mu^\Gamma (a^\Gamma_{a'} - 1) = f_{t+1} - 1/2\, C_-(a'') \ . \tag{4.9}$$

So, for $h_t = g_t = 0$, $x_0 \leqq t < y_0$, we have

$$\partial_{h_t} G(x,y) = \exp\ [f_{t+1}] \sum_{a' \in \Lambda_t} G(x,a') G(a' + \hat{e}_0, y) \ . \tag{4.10}$$

By Fourier transform with respect to $\mathbf{x} - \mathbf{y}$ and $\mathbf{y} - \mathbf{a}'$ the statement of Theorem 3.2 follows. $\square$

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