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Details of the Non-Unitarity Proof for Highest Weight Representations of the Virasoro Algebra

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Abstract. We give an exposition of the details of the proof that all highest weight representations of the Virasoro algebra for $c<1$ which are not in the discrete series are non-unitary.

The Virasoro algebra is the infinite dimensional Lie algebra with generators L_n, $n \in \mathbb{Z}$, satisfying the commutation relations

$$[L_m, L_n] = (m-n)L_{m+n} + \tfrac{1}{12}c(m^3-m)\delta_{m+n,0}\,. \tag{1}$$

The number c is called the central charge. The Verma module $V(c,h)$ is the representation of the Virasoro algebra generated by a vector $|h\rangle$ satisfying

$$L_0|h\rangle = h|h\rangle\,, \quad L_n|h\rangle = 0\,, \quad n>0\,, \tag{2}$$

and spanned by the linearly independent vectors $|h\rangle$ and

$$L_{-k_1}L_{-k_2}\dots L_{-k_n}|h\rangle\,, \quad 1 \leqq k_1 \leqq k_2 \leqq \dots \leqq k_n\,. \tag{3}$$

We assume that both c and h are real. In this case, a hermitian inner product on $V(c,h)$ is defined by $\langle h|h\rangle = 1$, and $L_n^\dagger = L_{-n}$. Define, for p and q positive integers,

$$c(m) = 1 - \frac{6}{m(m+1)}\,, \quad h_{p,q}(m) = \frac{((m+1)p-mq)^2-1}{4m(m+1)}\,. \tag{4}$$

The non-unitary theorem [1] is

Theorem 1. *For $c<1$ there are negative metric states in $V(c,h)$ if (c,h) does not belong to the discrete list*

$$c = c(m)\,, \quad m = 2,3,4\dots\,, \quad h = h_{p,q}(m)\,, \quad p+q \leqq m\,. \tag{5}$$

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The proof of Theorem 1 was given in [1]. The present paper is an exposition of the details of that proof. We recommend the graphs in [1] as a visual aid.

There are analogous non-unitarity theorems for the $N=1$ supersymmetric extensions of the Virasoro algebra [1, 2]. The details of the proofs of the $N=1$ non-unitarity theorems are exactly parallel to the proof of the Virasoro theorem. Goddard et al. [3] proved that all representations in the discrete series allowed by the non-unitarity theorems for the Virasoro algebra and its $N=1$ extensions are in fact unitary. Boucher et al. [4] have given the non-unitarity theorems for the $N=2$ extensions. The $N=2$ proofs [5] are somewhat different from the $N<2$ proofs. Di Vecchia, Petersen, Yu, and Zheng have proved that the discrete series of representations allowed by the $N=2$ non-unitarity theorems are in fact unitary [6].

For N a nonnegative integer, define *level* N to be the eigenspace of the Verma module on which L_0 has eigenvalue $h+N$. Level 0 is spanned by $|h\rangle$, and level N, $N \geqq 1$, is spanned by the vectors listed in (3) which satisfy $\sum k_i = N$. Level N has dimension $P(N)$, the partition number of N. Clearly, the levels span $V(c,h)$ and are linearly independent. Since $L_0^\dagger = L_0$, levels N and N' are orthogonal if $N \neq N'$. Define the null subspace on level N to be the subspace of vectors in level N which are orthogonal to all of level N, and thus to all of $V(c,h)$.

The inner products of the states on level N listed in (3) form a $P(N) \times P(N)$ real symmetric matrix $M_N(c,h)$ whose entries are polynomials in c and h. An explicit formula for the determinant of this matrix was announced by Kac [7] and proved by Feigin and Fuchs [8]. Up to multiplication by a positive number independent of c and h,

$$\det M_N(c,h) = \prod_{\substack{p,q \geqq 1 \\ pq \leqq N}} (h - h_{p,q}(m))^{P(N-pq)}, \tag{6}$$

where $h_{p,q}(m)$ is given by Eq. (4). In Eq. (6) it does not matter which branch is chosen for m as a function of c. For $c<1$ we choose by convention the branch $0<m<\infty$. There is a nontrivial null subspace on level N if and only if $\det M_N(c,h)=0$.

Kac [9] showed that, for $c \geqq 1$, the metric on $V(c,h)$ is nonnegative if and only if $h \geqq 0$. Direct calculation gives the 1×1 matrix $M_1 = 2h$, so $h \geqq 0$ is necessary if the metric is to be nonnegative. It is straightforward to verify that, in the limit $h \to +\infty$, M_N goes to a diagonal matrix with positive entries. It is also straightforward to check that $\det M_N(c,h) \neq 0$ for $c>1$, $h>0$. Therefore $M_N(c,h)$ is nondegenerate and positive for $c>1$, $h>0$, and is non-negative for $c \geqq 1$, $h \geqq 0$. Since this is true for all levels N, the result follows.

The proof of Theorem 1 is entirely elementary. The strategy is to consider the matrices M_N, $N=1,2,\ldots$, one by one. For each N we find a subset G_N of the half-plane $c<1$ on which $M_N(c,h)$ has a negative eigenvalue. We then say that the subset G_N has been *eliminated*. Theorem 1 will follow from the fact that the discrete set (5) is the complement of $\bigcup_N G_N$ in the half-plane $c<1$.

Henceforth we write $h_{p,q}(c)$ in place of $h_{p,q}(m)$, with the understanding that, for $c<1$, we choose the branch of m with $0<m<\infty$. Write $C_{p,q}$ for the vanishing curve $h=h_{p,q}(c)$. Because $\det M_N(c,h)$ vanishes on the curve $C_{p,q}$ for $pq \leqq N$, we say that the vanishing curve $C_{p,q}$ *first appears* on level pq, and that the vanishing curves on level N are the $C_{p,q}$, $pq \leqq N$. The curve $C_{p,q}$ intersects the line $c=1$ at $h=h_{p,q}(1) = (p-q)^2/4$. Orient each vanishing curve so that $c=1$ is the initial point, and forward is the direction of decreasing c.

Proposition 1. *When the curve $C_{p,1}$ first appears on level $N=p$, it intersects no other vanishing curves in the half-plane $c<1$. When $C_{p,q}$, $q>1$, first appears on level $N=pq$, its first intersection, moving forward from $c=1$, is with $C_{q-1,p}$ at $m=p+q-1$.*

Proof. The proof is straightforward algebra. □

For $q=1$ define $C'_{p,1}$ to be all of $C_{p,q}$ in the half-plane $c<1$. For $q>1$ define $C'_{p,q}$ to be the part of $C_{p,q}$ for which $m>p+q-1$. That is, $C'_{p,q}$ is the open subset of $C_{p,q}$ between $c=1$ and the first intersection of $C_{p,q}$ on level $N=pq$. The first step in the proof of Theorem 1 is to eliminate all of the half-plane $c<1$ except the curves $C'_{p,q}$. For $N \geqq 1$ define

$$S_N = \bigcup_{q<p,\, pq \leqq N} \{(c,h): c<1,\ h_{q,p}(c) \leqq h \leqq h_{p,q}(c)\} \bigcup_{p^2 \leqq N} \{(c,h): c<1,\ h \leqq h_{p,p}(c)\}\,. \tag{7}$$

Proposition 2. $\lim\limits_{N\to\infty} S_N$ *is the half-plane $c<1$.*

Define a *first intersection F* on $C'_{p,q}$ to be an intersection of $C'_{p,q}$ and $C_{p',q'}$, $p'q'>pq$, such that, on level $N'=p'q'$, (c,h) is the first intersection encountered on $C'_{p,q}$, starting from $c=1$.

Proposition 3. *The first intersections on $C'_{p,q}$ are the intersections $F_{p,q,k}$ of $C'_{p,q}$ and $C_{p',q'}=C_{q+k-1,p+k}$, $k \geqq 1$. $F_{p,q,k}$ is the point $h=h_{p,q}(m)$, $m=p+q+k-1$. Each of these first intersections is, at level $p'q'$, the intersection of exactly two vanishing curves.*

Proof. The proof is straightforward algebra. □

It immediately follows that

Proposition 4. *The discrete list* (5) *consists exactly of the first intersections, on all the vanishing curves $C'_{p,q}$.*

Define $R_{1,1}$ to be the open quadrant $c<1$, $h<0$. Define $R_{p,1}=R_{1,p}$, for $p>1$, to be the open region bounded by $C'_{p,1}$, $C'_{p-1,1}$, and $C'_{1,p}$. For $p,q>1$, define $R_{p,q}$ to be the open region bounded by $C'_{p,q}$, $C'_{p-1,q-1}$, and $C'_{q-1,p}$.

Proposition 5. *No vanishing curves on level $N=pq$ intersect $R_{p,q}$.*

Proof. A vanishing curve which did intersect $R_{p,q}$ would have to intersect its boundary. By Proposition 3, this does not happen. □

Proposition 6. $S_N - S_{N-1} = \bigcup\limits_{pq=N} R_{p,q} \bigcup\limits_{pq=N} C'_{p,q}$.

Proposition 7. *Except possibly for the curves $C'_{p,q}$, $pq \leqq N$, all of S_N is eliminated on levels $\leqq N$.*

Proof. The proof is by induction in N. The proposition is clearly true for $N=1$, because S_1 is the quadrant $c<1$, $h\leqq 0$, and $C'_{1,1}$ is the line $h=0$, $c<1$. Now suppose the proposition is true for $N-1$. We show that it is also true for N. By Proposition 6, we need to show that the $R_{p,q}$, $pq=N$, are eliminated on level N.

We say that two connected regions of the (c,h) plane are *contiguous on level N* if they can be connected by a path which does not intersect any vanishing curves on level N. If two regions are contiguous on level N, then the signature of M_N is the same in both regions, because the signature can only change when a vanishing curve is crossed. For each $C_{p,q}$ on level N, for $pq\leqq N$, choose a neighborhood U of $C_{p,q}$ small enough so that the only other vanishing curves on level N which intersect U also intersect $C_{p,q}$. $U-C_{p,q}$ has two connected components. Define the $c>1$ *side* of $C_{p,q}$ to be the connected component on the right of $C_{p,q}$, moving forward, if $p\geqq q$, and on the left, moving forward, if $p<q$. The other component is called the $c>1$ *side* of $C_{p,q}$. The motivation for this terminology is that the $c>1$ side of $C_{p,q}$, for c near 1, is contiguous on level $N=pq$ with the region $c>1$, $h>0$. This is easily verified by expanding $h_{p,q}(c)$ around $c=1$. It follows that $M_N(c,h)$ is a positive matrix on the $c>1$ side of $C_{p,q}$ for c near 1. $\det M_N$ vanishes to first order on $C_{p,q}$. Therefore $\det M_N(c,h)$ is negative on the $c<1$ side of $C_{p,q}$, for c near but not at 1. The sign of $\det M_N(c,h)$ can only change at a vanishing curve, so $\det M_N(c,h)$ is negative in the entire region of the $c<1$ half-plane which is contiguous on level pq to the $c<1$ side of $C_{p,q}$ for c near but not at 1. By Proposition 5, this region is $R_{p,q}$. So the region $R_{p,q}$ is eliminated. The induction step now follows from Proposition 6. □

Given Propositions 2 and 7, we are left with the task of eliminating the intervals on the curves $C'_{p,q}$ in between the points in the discrete list (5). Let $I_{p,q,k}$, $k\geqq 2$, be the open interval on $C'_{p,q}$ between $F_{p,q,k-1}$ and $F_{p,q,k}$. Let $I_{p,q,1}$ be the open subset of $C'_{p,q}$ beyond $F_{p,q,1}$. That is, $I_{p,q,1}$ is the open subset of $C'_{p,q}$ with $m<p+q$. Clearly,

Proposition 8.

$$C'_{p,q}=\bigcup_{k\geqq 0} I_{p,q,k}\bigcup_{k\geqq 1} F_{p,q,k}\,. \tag{8}$$

The goal is to eliminate the open intervals $I_{p,q,k}$, $k\geqq 1$. Recall that, when $C_{p',q'}=C_{q+k-1,p+k}$ first appears on level $N'=p'q'$, there is a negative metric state on its $c<1$ side, near $c=1$. We will show that this negative metric state continues to exist on the $c<1$ side of $C_{p',q'}$ moving away from $c=1$, and in particular exists on $C'_{p,q}$ on the $c<1$ side of $C_{p',q'}$. That part of $C'_{p,q}$ is a subset of $I_{p,q,k}$, and, by the definition of first intersections, there are no intersections on $I_{p,q,k}$ at level N'. It will then follow that there is a negative metric state on all of $I_{p,q,k}$, and we will be done.

Proposition 9. *On level $N'=p'q'$, the first k successive intersections on $C_{p',q'}$, are with $C'_{p+k-j,q+k-j}$, $1\leqq j\leqq k$. These are the first intersections $F_{p+k-j,q+k-j,j}$ on $C'_{p+k-j,q+k-j}$, occurring at $m=p+q+2k-j-1$.*

Proof. The proof is straightforward algebra. □

Proposition 10. *Suppose (c', h') is on some $C_{p,q}$, $pq=N$, but is not on an intersection of vanishing curves at level N. Then the null space on level N is one dimensional at (c', h').*

Proof. $\det M_N(c,h)$ vanishes to first order at $C_{p,q}$ near (c', h'). □

Proposition 11. *At $F_{p,q,k}$, the intersection of $C'_{p,q}$ and $C_{p',q'}=C_{q-1+k,p+k}$, $k\geqq 1$, occurring at $c=c(m)$, $h=h_{p,q}(c)$, $m=p+q+k-1$,*

$$\det M_{p'q'-pq}(c, h+pq)\neq 0. \tag{9}$$

Proof. If this determinant were zero, then $(c, h+pq)$ would be on a vanishing curve $C_{r,s}$ on level $rs=p'q'-pq$. Direct calculation of $p'q'-pq$ gives

$$rs=m(m+1)-(m+1)p-mq. \tag{10}$$

The condition that $(c, h+pq)$ lie on $C_{r,s}$ is

$$(m+1)p+mq=\pm((m+1)r-ms). \tag{11}$$

It follows from Eqs. (10, 11) that $r=m$ or $s=m+1$. But this gives a contradiction if we take Eq. (10) mod m or mod $m+1$, since $1\leqq p<m$ and $1\leqq q<m+1$. □

Proposition 12. *For $j=1,2,\dots,k$ there exists an open neighborhood $U_{p',q',j}$ of*

$$F_{p+k-j,q+k-j,j}=F_{q'-j,p'+1-j,j},$$

and a nowhere zero analytic function $v_j(c,h)$, defined on $U_{p',q',j}$ with values in level $N'=p'q'$ of $V(c,h)$, such that $v_j(c,h)$ is in the null space of level N' if and only if (c,h) is on $C_{p',q'}$.

Proof. Write $p''=p+k-j$, $q''=q+k-j$, $N''=p''q''<N'$. Let $U=U_{p',q',j}$ be a neighborhood of $F_{p+k-j,q+k-j,j}$ small enough that it intersects no vanishing curves but $C'_{p'',q''}$ and $C_{p',q'}$ on level N'. Choose coordinates (x,y) in U, analytic in (c,h) and real for c,h real, such that $C'_{p'',q''}$ is given by $x=0$ and $C_{p',q'}$ is given by $y=0$. This is possible because the intersection is transversal. At level N'', $x=0$ is the only vanishing curve in U. The one dimensional null spaces of level N'' form a line bundle over the vanishing curve $x=0$ near $y=0$. Let $v''_j(0,y)$ be a nowhere zero analytic section of this line bundle, and let $v''_j(x,y)$ be an analytic function on U with values in level N'', which extends this section. Define the subspace $V''(x,y)$ of level N' to be the span of the vectors

$$L_{-k_1}L_{-k_2}\dots L_{-k_n}v''_j(x,y), \quad 1\leqq k_1\leqq k_2\leqq \dots \leqq k_n, \quad \sum k_i=N'-N''. \tag{12}$$

The dimension of $V''(x,y)$ is $P(N'-N'')$. For $y\neq 0$, the order of vanishing of $\det M_{N'}(x,y)$ at $x=0$ is also $P(N'-N'')$. Therefore, for $y\neq 0$, $V''(0,y)$ is the null subspace of level N'. Let $V'(x,y)$ be a subspace of level N' complementary to $V''(x,y)$, so level N' is $V''\oplus V'$. The matrix of inner products $M_{N'}$ can now be written in block diagonal form:

$$M_{N'}(x,y)=\begin{pmatrix} xQ(x,y) & xR(x,y) \\ xR(x,y)^t & S(x,y)\end{pmatrix}, \tag{13}$$

where Q and S are symmetric matrices. Three blocks of $M_{N'}(x,y)$ are divisible by x, as in Eq. (13), because $V''(0,y)$ is in the null subspace of level N'.

The key point now is that $Q(0,0)$ is non-degenerate. To see this, first note that, for $n>0$, the vector $L_n v_j''(0,y)=0$, since $L_n v_j''(0,y)$ is in the null subspace of level $N'-n$, which is trivial. From this, and from the explicit basis (12) for $V''(x,y)$, we see that

$$Q(x,y)=M_{p'q'-p''q''}(c,h+p''q'')+O(x)\,, \tag{14}$$

where (c,h) corresponds to $(0,y)$ under the change of coordinates. Since $(0,0)$ is the first intersection $F_{p'',q'',j}$, Proposition (11) gives $\det Q(0,0)\neq 0$.

Since $\det Q(0,0)\neq 0$, $Q(x,y)$ is non-degenerate on all of U, if necessary replacing U by a smaller neighborhood of $(0,0)$. Let W be the matrix

$$\begin{pmatrix} 1 & -Q^{-1}R \\ 0 & 1 \end{pmatrix}, \tag{15}$$

and make the change of basis

$$M_{N'}\rightarrow W^t M_{N'} W=\begin{pmatrix} xQ(x,y) & 0 \\ 0 & T(x,y) \end{pmatrix}. \tag{16}$$

Let $V'''(x,y)$ be the new complement to $V''(x,y)$, on which $T(x,y)$ is the inner product. The order of vanishing argument implies that $\det T(x,y)$ is nonzero for $y\neq 0$ and vanishes to first order at $y=0$. The one dimensional null space of $T(x,0)$ is the null space of level N' for $x\neq 0$. At $x=y=0$, the one dimensional null space of $T(0,0)$ spans, with $V''(x,y)$, the $P(N')-P(N'')+1$ dimensional null subspace of level N'. By the same argument which gave $v_j''(x,y)$, we can choose a nowhere zero analytic function $v_j(x,y)$ on U, with values in $V'''(x,y)$, such that $v_j(x,0)$ is in the null space of $T(x,0)$ and therefore in the null space of level N'. Since $T(x,y)$ is non-degenerate for $y\neq 0$, $v_j(x,y)$ is not in the null space of level N' if $y\neq 0$. □

Let $J_{p',q',j}$, $1<j\leqq k$, be the open interval on $C_{p',q'}$ between

$$F_{p+k-j,q+k-j,j} \quad \text{and} \quad F_{p+k-j-1,q+k-j-1,j+1}\,.$$

Let $J_{p',q',1}$ be the open interval on $C_{p',q'}$ lying between $c=1$ and $F_{p+k-1,q+k-1,1}$. Let $W_{p',q',j}$, $1\leqq j\leqq k$, be a neighborhood in the plane which intersects no vanishing curves on level N' except $J_{p',q',j}$. For $j>1$, require

$$\begin{gathered} J_{p',q',j}\subset U_{p',q',j-1}\cup W_{p',q',j}\cup U_{p',q',j}\,, \\ W_{p',q',j}\cap U_{p',q',j}\neq\emptyset\,, \qquad W_{p',q',j}\cap U_{p',q',j-1}\neq\emptyset\,. \end{gathered} \tag{17}$$

For $j=1$ require only

$$W_{p',q',1}\cap U_{p',q',1}\neq\emptyset\,. \tag{18}$$

Proposition 13. *For each j, $1\leqq j\leqq k$, there is a nowhere zero analytic function $w_j(c,h)$ on $W_{p',q',j}$ with values in level N' such that $w_j(c,h)$ is in the null space of level N' if and only if (c,h) is on $J_{p',q',j}$. On the intersections of their neighborhoods of definition,*

$w_j = f_j v_j$, where f_j is a nonzero function, and $w_j = g_j v_{j-1}$, where g_j is a nonzero function.

Proof. Again, the null space of level N' is trivial on $W_{p',q',j}$ except on $J_{p',q',j}$, where it is one dimensional. □

Proposition 14. *The level N' metric is negative on the vectors $v_{p',q',j}(c,h)$ and on the vectors $w_{p',q',j}(c,h)$, on the $c<1$ side of $C_{p',q'}$.*

Proof. The matrix $M_{N'}$ is positive in $W_{p',q',1}$ on the $c>1$ side of $C_{p',q'}$, by the contiguity argument, since there are no intersections on $C_{p',q'}$ between $W_{p',q',1}$ and $c=1$. The inner product is thus positive on $w_{p',q',1}$ on the $c>1$ side of $C_{p',q'}$. The inner product vanishes to first order on $w_{p',q',1}$ on $C_{p',q'}$. Therefore the inner product is negative on $w_{p',q',1}$ on the $c<1$ side of $C_{p',q'}$. The proposition now follows by induction on the series $w_1, v_1, w_2, v_2, \ldots$, since neighboring vectors in the series differ by nonzero functions f_j or g_j, and since the $w_j(c,h)$ and $v_j(c,h)$ are in the level N' null space only for (c,h) on $C_{p',q'}$. □

Proposition 15. *$I_{p,q,k}$ is eliminated on level $N'=(q+k-1)(p+k)$.*

Proof. By the previous proposition, the metric is negative on $v_{p',q',k}(c,h)$, on the $c<1$ side of $C_{p',q'}$. But $I_{p,q,k}$ approaches arbitrarily close to $C_{p',q'}$ on the $c<1$ side within $U_{p',q',k}$. Therefore $M_N(c,h)$ has a negative eigenvalue at one end of $I_{p,q,k}$. But the signature of $M_N(c,h)$ cannot change along $I_{p,q,k}$, because there are no intersections at level N' on $I_{p,q,k}$. The proposition follows. □

Propositions 2, 7, 8, and 15 imply Theorem 1.

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Note added in proof. A similar but not identical version of the details of the non-unitarity proof has been given by Langlands [10].

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