

terms. For all the functions thus developed can contain only those constituents which have the coefficient 1 or the coefficient 0 (and in the latter case, they do not contain them). Hence they are additive combinations of these constituents; and, since the number of the constituents is 2^n , the number of possible functions is 2^{2^n} . From this must be deducted the function in which all constituents are absent, which is identically 0, leaving $2^{2^n}-1$ possible equations (255 when $n=3$). But these equations, in their turn, may be combined by logical addition, *i. e.*, by alternation; hence the number of their combinations is $2^{2^{2^n}-1}-1$, excepting always the null combination. This is the number of possible assertions affecting n terms. When $n=2$, this number is as high as 32767.¹ We must observe that only universal premises are admitted in this calculus, as will be explained in the following section.

53. Particular Propositions.—Hitherto we have only considered propositions with an *affirmative* copula (*i. e.*, inclusions or equalities) corresponding to the *universal* propositions of classical logic.² It remains for us to study propositions with a *negative* copula (non inclusions or inequalities), which translate *particular* propositions³; but the calculus of

¹ G. PEANO, *Calcolo geometrico* (1888) p. x; SCHRÖDER, *Algebra der Logik*, Vol. II, p. 144—148.

² The *universal affirmative*, "All *a*'s are *b*'s", may be expressed by the formulas

$$(a < b) = (a = ab) = (ab' = 0) = (a' + b = 1),$$

and the *universal negative*, "No *a*'s are *b*'s", by the formulas

$$(a < \bar{b}) = (a = a\bar{b}) = (ab = 0) = (a' + b' = 1).$$

³ For the *particular affirmative*, "Some *a*'s are *b*'s", being the negation of the universal negative, is expressed by the formulas

$$(a \not< \bar{b}) = (a \neq a\bar{b}) = (ab \neq 0) = (a' + b' \neq 1),$$

and the *particular negative*, "Some *a*'s are not *b*'s", being the negation of the universal affirmative, is expressed by the formulas

$$(a \not< b) = (a \neq ab) = (ab' \neq 0) = (a' + b \neq 1).$$

propositions having a negative copula results from laws already known, especially from the formulas of DE MORGAN and the law of contraposition. We shall enumerate the chief formulas derived from it.

The principle of composition gives rise to the following formulas:

$$(c \nless a b) = (c \nless a) + (c \nless b),$$

$$(a + b \nless c) = (a \nless c) + (b \nless c),$$

whence come the particular instances

$$(a b \nplus 1) = (a \nplus 1) + (b \nplus 1),$$

$$(a + b \nplus 0) = (a \nplus 0) + (c \nplus 0).$$

From these may be deduced the following important implications:

$$(a \nplus 0) < (a + b \nplus 0),$$

$$(a \nplus 1) < (a b \nplus 1).$$

From the principle of the syllogism, we deduce, by the law of transposition,

$$(a < b) (a \nplus 0) < (b \nplus 0),$$

$$(a < b) (b \nplus 1) < (a \nplus 1).$$

The formulas for transforming inclusions and equalities give corresponding formulas for the transformation of non-inclusions and inequalities,

$$(a \nless b) = (a b' \nplus 0) = (a' + b \nplus 1),$$

$$(a \nplus b) = (a b' + a' b \nplus 0) = (a b + a' b' \nplus 1).$$

54. Solution of an Inequation with One Unknown.—

If we consider the conditional inequality (*inequation*) with one unknown

$$a x + b x' \nplus 0,$$

we know that its first member is contained in the sum of its coefficients

$$a x + b x' < a + b.$$