

(0, 1); hence they can be made identical by substituting the word "row" for the word "column" in the foregoing statement. And, indeed, since the rule of the consequences concerns only classes of the same column, we are at liberty so to arrange the classes in each column on the rows that the rule of the causes will be verified by the classes in the same row.

It will be noted, moreover, that, by the method of construction adopted for this table, the classes which are the negatives of each other occupy positions symmetrical with respect to the center of the table. For this result, the subclasses of the class N' (the logical whole of the given equality or the logical zero of the opposite equality) must be placed in the first row in their natural order from 0 to N' ; then, in each division, must be placed the sum of the classes at the head of its row and column.

With this precaution, we may sum up the two rules in the following practical statement:

To obtain every consequence of the given equality (to which the table relates) it is sufficient to equate each class to every class in the same column; and, to obtain every cause, it is sufficient to equate each class to every class in the row occupied by its symmetrical class.

It is clear that the table relating to the equality $N = 0$ can also serve for the opposite equality $N = 1$, on condition that the words "row" and "column" in the foregoing statement be interchanged.

Of course the construction of the table relating to a given equality is useful and profitable only when we wish to enumerate all the consequences or the causes of this equality. If we desire only one particular consequence or cause relating to this or that class of the discourse, we make use of one of the formulas given above.

52. The Number of Possible Assertions.—If we regard logical functions and equations as developed with respect to *all* the letters, we can calculate the number of assertions or different problems that may be formulated about n simple

terms. For all the functions thus developed can contain only those constituents which have the coefficient 1 or the coefficient 0 (and in the latter case, they do not contain them). Hence they are additive combinations of these constituents; and, since the number of the constituents is 2^n , the number of possible functions is 2^{2^n} . From this must be deducted the function in which all constituents are absent, which is identically 0, leaving $2^{2^n}-1$ possible equations (255 when $n=3$). But these equations, in their turn, may be combined by logical addition, *i. e.*, by alternation; hence the number of their combinations is $2 \cdot 2^{2^n}-1-1$, excepting always the null combination. This is the number of possible assertions affecting n terms. When $n=2$, this number is as high as 32767.¹ We must observe that only universal premises are admitted in this calculus, as will be explained in the following section.

53. Particular Propositions.—Hitherto we have only considered propositions with an *affirmative* copula (*i. e.*, inclusions or equalities) corresponding to the *universal* propositions of classical logic.² It remains for us to study propositions with a *negative* copula (non inclusions or inequalities), which translate *particular* propositions³; but the calculus of

¹ G. PEANO, *Calcolo geometrico* (1888) p. x; SCHRÖDER, *Algebra der Logik*, Vol. II, p. 144—148.

² The *universal affirmative*, "All *a*'s are *b*'s", may be expressed by the formulas

$$(a < b) = (a = ab) = (ab' = 0) = (a' + b = 1),$$

and the *universal negative*, "No *a*'s are *b*'s", by the formulas

$$(a < \bar{b}) = (a = a\bar{b}) = (ab = 0) = (a' + b' = 1).$$

³ For the *particular affirmative*, "Some *a*'s are *b*'s", being the negation of the universal negative, is expressed by the formulas

$$(a \nless b) = (a \neq ab') = (ab \neq 0) = (a' + b' \neq 1),$$

and the *particular negative*, "Some *a*'s are not *b*'s", being the negation of the universal affirmative, is expressed by the formulas

$$(a \nless \bar{b}) = (a \neq ab) = (ab' \neq 0) = (a' + b \neq 1).$$