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# TWO KOROVKIN-TYPE THEOREMS IN MULTIVARIATE APPROXIMATION

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ABSTRACT. Let $\Omega$ be a compact convex subset of $\mathbb{R}^d$ and let $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive linear operators that map $C(\Omega)$ into itself. We establish two Korovkin-type theorems in which the limit of the sequence of operators is not necessarily the identity.

## 1. INTRODUCTION AND NOTATION

Let $C[a,b]$ be the linear space of all real-valued continuous functions on $[a,b]$ and let $T$ be a linear operator which maps $C[a,b]$ into itself. We say that $T$ is *positive* if for every non-negative $f \in C[a,b]$, we have $T[f](x) \geq 0$ for all $x \in [a,b]$.

In its simplest form, the theorem of Korovkin may be stated as follows.

**Theorem A.** *Let $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive linear operators that map $C[0,1]$ into itself. Suppose that the sequence $(L_n[f])_{n\in\mathbb{N}}$ converges to $f$ uniformly on $[0,1]$ for the three special functions $e_i : x \mapsto x^i$, where $i = 0,1,2$. Then this sequence converges to $f$ uniformly on $[0,1]$ for every $f \in C[0,1]$.*

This theorem became known mainly by the book of Korovkin [7]. Preliminary forms with special classes of positive linear operators are due to Bohman [2] and Korovkin [6].

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As soon as Theorem A is available, a proof of Weierstrass' approximation theorem becomes very simple; see, e.g., Cheney [3] or Meinardus [8]. In addition, Theorem A has various applications to approximation problems. For example, it yields a simplified proof of Fejér's result concerning approximation by means of Hermite interpolation with derivatives controlled at the Chebyshev nodes; see Cheney [3], where further applications along these lines will be found.

Because of its powerful applications, Korovkin's result has been extended in many directions. There is an extensive literature on Korovkin-type theorems, which may have had a summit already about twentyfive years ago. In particular, there exist abstract results that cover many naturally arising concrete cases. The contributions up to about 1994 are excellently documented in the book of Altomare and Campiti [1].

The following more recent result by Wang Heping [9] seems to be not covered by the results in [1].

**Theorem B.** *Let $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive linear operators that map $C[0,1]$ into itself and satisfy the following conditions:*

(i) *The sequence $(L_n[e_2])_{n\in\mathbb{N}}$ converges uniformly on $[0,1]$.*

(ii) *For every convex function $f \in C[0,1]$, and any $x \in [0,1]$, the sequence $(L_n[f](x))_{n\in\mathbb{N}}$ is non-increasing.*

*Then there exists a linear operator $L_\infty$ on $C[0,1]$ such that $L_n[f]$ converges uniformly to $L_\infty[f]$ on $[0,1]$ for every $f \in C[0,1]$.*

In this paper, we want to establish two multivariate Korovkin-type theorems, one of them being an analogue of Theorem B.

We now make some agreements for the multivariate case. Throughout this paper, let $\Omega$ be a convex compact subset of $\mathbb{R}^d$. By $C(\Omega)$, we denote the class of all real-valued continuous functions on $\Omega$ and by $C^k(\Omega)$ the subclass of all functions that are $k$ times differeniable in the following sense. For each $\boldsymbol{x} \in \Omega$ and any $\boldsymbol{y} \in \mathbb{R}^d$ such that $\boldsymbol{x}+\boldsymbol{y} \in \Omega$, the directional derivatives

$$D^j_{\boldsymbol{y}} f(\boldsymbol{x}) := \frac{\mathrm{d}^j}{\mathrm{d}t^j} f(\boldsymbol{x}+t\boldsymbol{y})\bigg|_{t=0} \qquad (j=0,\dots,k)$$

exist and depend continuously on $\boldsymbol{x}$. When the directional derivative exists for $\boldsymbol{y}$, it can be extended to multiples by defining

$$D^j_{\lambda\boldsymbol{y}} f(\boldsymbol{x}) := \lambda^j D^j_{\boldsymbol{y}} f(\boldsymbol{x}) \qquad (\lambda \in \mathbb{R}).$$

It is convenient to agree that $C^0(\Omega) := C(\Omega)$.

For $f, g \in C(\Omega)$, we write $f \le g$ if $f(\boldsymbol{x}) \le g(\boldsymbol{x})$ for all $\boldsymbol{x} \in \Omega$. By $\|\cdot\|_\Omega$, we denote the supremum norm on $\Omega$, that is,

$$\|f\|_\Omega := \sup_{\boldsymbol{x}\in\Omega} |f(\boldsymbol{x})| \ .$$

When $(g_n)_{n\in\mathbb{N}}$ is a sequence of functions in $C(\Omega)$ and $\lim_{n\to\infty} \|g-g_n\|_\Omega = 0$, or equivalently, $\lim_{n\to\infty} g_n(\boldsymbol{x}) = g(\boldsymbol{x})$ uniformly for $\boldsymbol{x} \in \Omega$, we simply write $\lim_{n\to\infty} g_n = g$.

By $e_i$, we denote the projection

$$e_i \,:\, \boldsymbol{x} = (x_1,\dots,x_d) \longmapsto x_i$$

and write $\boldsymbol{e} := (e_1, \ldots, e_d)$ for the identity on $\mathbb{R}^d$, that is,

$$\boldsymbol{e}(x_1, \ldots, x_d) = (x_1, \ldots, x_d).$$

The class of all linear operators that map $C(\Omega)$ into itself shall be denoted by $\mathcal{L}(\Omega)$. When $L \in \mathcal{L}(\Omega)$ and

$$\boldsymbol{f} = (f_1, \ldots, f_d) \in C(\Omega)^d,$$

where $C(\Omega)^d$ is the Cartesian product of $d$ copies of $C(\Omega)$, we define

$$L[\boldsymbol{f}] := \big(L[f_1], \ldots, L[f_d]\big).$$

This way, $L$ is extended to an operator

$$L : C(\Omega)^d \longrightarrow C(\Omega)^d.$$

Furthermore, we write $\|\boldsymbol{f}\|^2$ for the mapping $\boldsymbol{x} \mapsto \|\boldsymbol{f}(\boldsymbol{x})\|^2$, where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^d$.

After these preparations, a multivariate Korovkin-type theorem which is closest to Theorem A can be stated as follows.

**Theorem C.** *Let $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive operators from $\mathcal{L}(\Omega)$. Suppose that*

(i) $\lim_{n\to\infty} L_n[1] = 1,$

(ii) $\lim_{n\to\infty} L_n[\boldsymbol{e}] = \boldsymbol{e},$

(iii) $\lim_{n\to\infty} L_n\left[\|\boldsymbol{e}\|^2\right] = \|\boldsymbol{e}\|^2.$

*Then $\lim_{n\to\infty} L_n[f] = f$ for every $f \in C(\Omega)$.*

Theorem C is a special case of known results that are themselves consequences of more abstract Korovkin-type theorems. For example, with $\Omega$ not necessarily convex, Theorem C can be found in [4, p. 363, Corollary 3], and with any strictly convex function $u \in C(\Omega)$ taking the role of $\|\boldsymbol{e}\|^2$ in (iii), the result is in the book of Altomare and Campiti [1, p. 234, Corollary 4.3.9]. While in these sources of Theorem C, the limit of $L_n$ in the hypotheses (i) and (ii) and in the conclusion is always the identity, we want to establish an extension of Theorem C where this need not be the case.

## 2. Statement of the Results

As a generalization of Theorem C, where the limit of the sequence $(L_n)_{n\in\mathbb{N}}$ is not necessarily the identity, we have the following result.

**Theorem 2.1.** *Let $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive operators from $\mathcal{L}(\Omega)$. Suppose that*

(i) $\lim_{n\to\infty} L_n[1] = 1,$

(ii) $\lim_{n\to\infty} L_n[\boldsymbol{e}] = \boldsymbol{\varphi} \in C(\Omega)^d,$

(iii) $\lim_{n\to\infty} L_n\left[\|\boldsymbol{e}\|^2\right] = \|\boldsymbol{\varphi}\|^2.$

*Then* $\lim_{n\to\infty} L_n[f] = f \circ \boldsymbol{\varphi}$ *for every* $f \in C(\Omega)$.

This theorem may be interpreted as follows. If $\lim_{n\to\infty} L_n[f]$ exists for all affine functions and for the Euclidean norm $\|\boldsymbol{e}\|^2$ and if it reproduces these functions under a co-ordinate transformation $\boldsymbol{x} \mapsto \boldsymbol{\varphi}(\boldsymbol{x})$, then the same holds for every $f \in C(\Omega)$.

Now we turn to the announced multivariate analogue of Theorem B.

**Theorem 2.2.** *Let* $(L_n)_{n\in\mathbb{N}}$ *be a sequence of positive operators from* $\mathcal{L}(\Omega)$ *satisfying the following conditions:*

(i) *The sequence* $(L_n[\|\boldsymbol{e}\|^2])_{n\in\mathbb{N}}$ *converges uniformly on* $\Omega$.

(ii) *For every convex function* $f \in C(\Omega)$ *and any* $\boldsymbol{x} \in \Omega$, *the sequence* $(L_n[f](\boldsymbol{x}))_{n\in\mathbb{N}}$ *is non-increasing.*

*Then there exists an operator* $L_\infty \in \mathcal{L}(\Omega)$ *such that* $\lim_{n\to\infty} L_n[f] = L_\infty[f]$ *for every* $f \in C(\Omega)$.

## 3. Auxiliary Results

We shall need generalizations of the following results.

**Proposition 3.1.** *Let* $L \in \mathcal{L}(\Omega)$ *be a positive operator that reproduces affine functions. Then* $f \le L[f]$ *for every convex function* $f \in C(\Omega)$.

For the second result, we introduce a notation which will be used from now on. If $f \in C^2(\Omega)$, we define

$$\left| D^2 f \right|_\Omega := \sup_{\boldsymbol{x}\in\Omega} \sup \left\{ \left| D^2_{\boldsymbol{y}} f(\boldsymbol{x}) \right| \;:\; \boldsymbol{y} \in \mathbb{R}^d,\ \|\boldsymbol{y}\| = 1 \right\}.$$

**Proposition 3.2.** *Let* $L \in \mathcal{L}(\Omega)$ *be a positive operator that reproduces affine functions. Then*

$$|L[f] - f| \;\le\; \frac{\left| D^2 f \right|_\Omega}{2} \left( L\left[\|\boldsymbol{e}\|^2\right] - \|\boldsymbol{e}\|^2 \right)$$

*for every* $f \in C^2(\Omega)$. *Equality is attained for* $f = c\|\boldsymbol{e}\|^2 + P$, *where* $c \in \mathbb{R}$ *and* $P$ *is any affine function.*

These propositions were derived and employed in a recent manuscript [5]. Here we want to relax the hypothesis that $L$ reproduces affine functions. The following lemma contains Proposition 3.1 as a special case.

**Lemma 3.3.** *Let* $L \in \mathcal{L}(\Omega)$ *be a positive operator such that* $L[1] = 1$. *Then* $L[\boldsymbol{e}]$ *maps* $\Omega$ *into itself and*

$$f \circ L[\boldsymbol{e}] \;\le\; L[f] \tag{3.1}$$

*for every convex function* $f \in C(\Omega)$.

*Proof.* Assume that there exists a $\boldsymbol{y} \in \Omega$ such that $L[\boldsymbol{e}](\boldsymbol{y}) \notin \Omega$. Since $\Omega$ is a closed convex set, there exists, by a familiar separation theorem (see, e.g., [10, p. 65, Theorem 2.4.1]), a point $\boldsymbol{x}^* \in \Omega$ such that

$$\left\langle L[\boldsymbol{e}](\boldsymbol{y}) - \boldsymbol{x}^*,\, \boldsymbol{x} - \boldsymbol{x}^* \right\rangle \;\le\; 0 \qquad (\boldsymbol{x} \in \Omega),$$

where we have used the standard inner product in Euclidean spaces on the left-hand side. Hence the function

$$\phi \, : \, \boldsymbol{x} \longmapsto \big\langle L[\boldsymbol{e}](\boldsymbol{y}) - \boldsymbol{x}^*, \, \boldsymbol{x} - \boldsymbol{x}^* \big\rangle$$

is non-positive on $\Omega$. Now we apply $L$ to $\phi$ and evaluate the result at $\boldsymbol{y}$. Since $L$ is a positive operator that reproduces constants, we find that

$$0 \, \geq \, L[\phi](\boldsymbol{y}) \, = \, \big\langle L[\boldsymbol{e}](\boldsymbol{y}) - \boldsymbol{x}^*, \, L[\boldsymbol{e}](\boldsymbol{y}) - L[\boldsymbol{x}^*](\boldsymbol{y}) \big\rangle \, = \, \big\| L[\boldsymbol{e}](\boldsymbol{y}) - \boldsymbol{x}^* \big\|^2 \, .$$

Hence $L[\boldsymbol{e}](\boldsymbol{y}) = \boldsymbol{x}^*$, which is a contradiction. This proves the first assertion.

Next we want to prove (3.1). Assume that there exists a convex function $f \in C(\Omega)$ and a point $\boldsymbol{y} \in \Omega$ such that

$$f\big(L[\boldsymbol{e}](\boldsymbol{y})\big) \, > L[f](\boldsymbol{y}) \, . \tag{3.2}$$

Now, consider the epigraph

$$E \, := \, \big\{ (\boldsymbol{x}, u) \in \mathbb{R}^{d+1} \, : \, \boldsymbol{x} \in \Omega, \; u \in \mathbb{R}, \; u \geq f(\boldsymbol{x}) \big\}$$

of $f$, which is a closed convex set in $\mathbb{R}^{d+1}$. Because of (3.2), the point

$$\big( L[\boldsymbol{e}](\boldsymbol{y}), L[f](\boldsymbol{y}) \big) \in \mathbb{R}^{d+1}$$

does not belong to $E$. By the aforementioned separation theorem, now used in $\mathbb{R}^{d+1}$, there exists a point $(\boldsymbol{x}^*, u^*) \in E$ such that

$$H(\boldsymbol{x}, u) \, := \, \Big\langle \, \big( L[\boldsymbol{e}](\boldsymbol{y}), L[f](\boldsymbol{y}) \big) - (\boldsymbol{x}^*, u^*) \, , \, (\boldsymbol{x}, u) - (\boldsymbol{x}^*, u^*) \Big\rangle \, \leq \, 0$$

for all $(\boldsymbol{x}, u) \in E$. Since $(\boldsymbol{x}, f(\boldsymbol{x})) \in E$ for $\boldsymbol{x} \in \Omega$, the function $\Phi \, : \, \Omega \to \mathbb{R}$, defined by $\Phi(\boldsymbol{x}) := H(\boldsymbol{x}, f(\boldsymbol{x}))$, is non-positive on $\Omega$. Applying $L$ to $\Phi$ and evaluating the result at $\boldsymbol{y}$, we find that

$$0 \, \geq \, L[\Phi](\boldsymbol{y}) \, = \, \big\| \, \big( L[\boldsymbol{e}](\boldsymbol{y}), L[f](\boldsymbol{y}) \big) \, - \, (\boldsymbol{x}^*, u^*) \big\|^2 \, ,$$

which is a contradiction. This completes the proof. □

The next lemma is a generalization of Proposition 3.2.

**Lemma 3.4.** *Let $L \in \mathcal{L}(\Omega)$ be a positive operator such that $L[1] = 1$. If $f \in C^2(\Omega)$, then*

$$\big| L[f] - f \circ L[\boldsymbol{e}] \big| \leq \frac{\big| D^2 f \big|_\Omega}{2} \left( L\left[ \|\boldsymbol{e}\|^2 \right] - \big\| L[\boldsymbol{e}] \big\|^2 \right) \, . \tag{3.3}$$

*Equality is attained for $f = c\|\boldsymbol{e}\|^2 + P$, where $c \in \mathbb{R}$ and $P$ is any affine function.*

*Proof.* The functions

$$g_\pm \, := \, \frac{\big| D^2 f \big|_\Omega}{2} \|\boldsymbol{e}\|^2 \, \pm \, f$$

are convex for both choices of the sign. Applying Lemma 3.3, we find that

$$\frac{\big| D^2 f \big|_\Omega}{2} \, \big\| L[\boldsymbol{e}] \big\|^2 \, \pm \, f \circ L[\boldsymbol{e}] \, \leq \, \frac{\big| D^2 f \big|_\Omega}{2} \, L\left[ \|\boldsymbol{e}\|^2 \right] \, \pm \, L[f],$$

or equivalently,

$$\mp \big( L[f] - f \circ L[\boldsymbol{e}] \big) \, \leq \, \frac{\big| D^2 f \big|_\Omega}{2} \left( L\left[ \|\boldsymbol{e}\|^2 \right] \, - \, \big\| L[\boldsymbol{e}] \big\|^2 \right),$$

which shows that (3.3) holds.

The statement on the occurrence of equality is easily verified. □

## 4. Proofs of the Theorems

*Proof of Theorem 2.1.* Without loss of generality, we may assume that $L_n[1] = 1$ for all $n \in \mathbb{N}$. Indeed, the operators

$$\widetilde{L}_n \,:\, f \longmapsto \frac{L_n[f]}{L_n[1]}$$

have this property and, because of (i),

$$\lim_{n\to\infty} \widetilde{L}_n[f] \,=\, \lim_{n\to\infty} L_n[f]$$

as soon as one of these limits exists.

Now, let $f$ be any function from $C(\Omega)$. Under our assumptions on $L_n$, it follows that

$$|L_n[f]| \,\le\, L_n[|f|] \,\le\, L_n\left[\|f\|_\Omega\right] \,=\, \|f\|_\Omega\, L_n[1] \,=\, \|f\|_\Omega\,.$$

Obviously, when $f = 1$, equality is obtained throughout. Hence

$$\sup_{f\neq 0} \frac{\|L_n[f]\|_\Omega}{\|f\|_\Omega} \,=\, 1\,,$$

which means that the sequence $(L_n)_{n\in\mathbb{N}}$ is uniformly bounded by 1. Taking also into account that $C^2(\Omega)$ is dense in $C(\Omega)$, we see by employing the familiar theorem of Banach–Steinhaus that it suffices to prove the conclusion of the theorem for $f \in C^2(\Omega)$ only.

Under this additional assumption on $f$, Lemma 3.4 applies and yields

$$\big|L_n[f] - f\circ L_n[\boldsymbol{e}]\big| \le \frac{\left\bracevert D^2 f \right\bracevert_\Omega}{2}\,\Big(L_n\left[\|\boldsymbol{e}\|^2\right] - \big\|L_n[\boldsymbol{e}]\big\|^2\Big)\,. \tag{4.1}$$

Since $\|\boldsymbol{e}\|^2$ is a continuous function, we deduce with the help of (ii) that

$$\lim_{n\to\infty} \big\|L_n[\boldsymbol{e}]\big\|^2 \,=\, \Big\|\lim_{n\to\infty} L_n[\boldsymbol{e}]\Big\|^2 \,=\, \|\boldsymbol{\varphi}\|^2\,.$$

Hence it follows from (4.1) and (iii) that

$$\lim_{n\to\infty}\big(L_n[f] - f\circ L_n[\boldsymbol{e}]\big) \,=\, 0\,.$$

Finally, the continuity of $f$ and (ii) imply

$$\lim_{n\to\infty} L_n[f] \,=\, \lim_{n\to\infty}\big(f\circ L_n[\boldsymbol{e}]\big) \,=\, f\circ\Big(\lim_{n\to\infty} L_n[\boldsymbol{e}]\Big) \,=\, f\circ\boldsymbol{\varphi}\,.$$

This completes the proof. □

*Proof of Theorem 2.2.* Let $P$ be any affine function. Since $P$ and $-P$ are both convex, it follows from the hypothesis (ii) that $L_n[P] = L_{n+1}[P]$ for all $n \in \mathbb{N}$. In particular, $L_n[1] = L_1[1]$ and $L_n[\boldsymbol{e}] = L_1[\boldsymbol{e}]$ for all $n \in \mathbb{N}$. Thus, if $f$ is any function from $C(\Omega)$, then

$$|L_n[f]| \,\le\, L_n[|f|] \,\le\, L_n\left[\|f\|_\Omega\right] \,=\, \|f\|_\Omega\, L_n[1] \,=\, \|f\|_\Omega\, L_1[1]\,.$$

This shows that the sequence $(L_n)_{n\in\mathbb{N}}$ is uniformly bounded by $L_1[1]$.

Now, let $f \in C^2(\Omega)$ and consider the functions

$$g_{\pm} := \frac{\left| D^2 f \right|_{\Omega}}{2} \, \|\boldsymbol{e}\|^2 \pm f \, .$$

They are convex for both choices of the sign. Hence, for any $n, k \in \mathbb{N}$, the hypothesis (ii) implies that

$$L_{n+k}\left[g_{\pm}\right] \leq L_n\left[g_{\pm}\right] .$$

Substituting according to the definition of $g_{\pm}$, using the linearity of the involved operators and regrouping terms, we arrive at

$$\mp \left(L_n[f] - L_{n+k}[f]\right) \leq \frac{\left| D^2 f \right|_{\Omega}}{2} \left(L_n\left[\|\boldsymbol{e}\|^2\right] - L_{n+k}\left[\|\boldsymbol{e}\|^2\right]\right) .$$

This shows that

$$\left\|L_n[f] - L_{n+k}[f]\right\|_{\Omega} \leq \frac{\left| D^2 f \right|_{\Omega}}{2} \left\|L_n\left[\|\boldsymbol{e}\|^2\right] - L_{n+k}\left[\|\boldsymbol{e}\|^2\right]\right\|_{\Omega} .$$

Employing the hypothesis (i), we now conclude that $(L_n[f])_{n\in\mathbb{N}}$ is a Cauchy sequence in $C(\Omega)$. Since $C(\Omega)$ is complete, there exists a function $f_\infty \in C(\Omega)$ such that

$$\lim_{n\to\infty} L_n[f] = f_\infty \, . \tag{4.2}$$

Although (4.2) has been obtained for $f \in C^2(\Omega)$ only, it extends to all $f \in C(\Omega)$ by the Banach–Steinhaus theorem. Hence we have a mapping $L_\infty$, say, such that

$$L_\infty : \begin{cases} C(\Omega) \longrightarrow C(\Omega)\, , \\ \quad f \longmapsto f_\infty = \lim_{n\to\infty} L_n[f] \, . \end{cases}$$

Clearly, this mapping is linear and so it belongs to $\mathcal{L}(\Omega)$. □

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