Existence of Local Solutions of Nonlinear Wave Equations in n-Dimensional Space *

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Abstract

In this paper, we investigate the local existence of solutions in H^s for n-dimensional nonlinear wave equations with special nonlinear terms, such as

$$u_{tt} - \Delta u = u^k |\nabla u|^l, \ \ x \in R^n, \ \ k \in Z^+, \ \ l \geq 2.$$

where $\nabla u = (\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \cdots, \frac{\partial u}{\partial x_n})$. Meanwhile, we obtain that the regular index s of Sobolev space H^s satisfies $s > \max\{\frac{n+5}{4}; \frac{n}{2} + 1 - \frac{1}{l-1}\}$, $n > 3$.

1 Introduction

In the paper we are concerned with the following nonlinear wave equation

$$u_{tt} - \triangle u = u^k |\nabla u|^l, \ \ (x,t) \in R^n \times R^+ \tag{1}$$

with the initial value conditions

$$u(x,0) = \varphi(x), \ u_t(x,0) = \psi(x), \ x \in R^n, \tag{2}$$

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where $k \in Z^+$, $l \geq 2$, $\nabla u = (\frac{\partial u}{\partial x_1}, \frac{\partial u}{\partial x_2}, \cdots, \frac{\partial u}{\partial x_n})$. We want to study the minimal Sobolev regularity in order to deal with the existence of local solutions to the problem (1)-(2) in Sobolev space H^s. Gustavo Ponce and Thomas C. Sideris [10] show that the lower bound for the Sobolev exponent can be reduced from $\frac{5}{2}$ to $s = s(l) = \max\{2, \frac{5l-7}{2l-2}\}$ in three space dimensions when the nonlinearity in (1) grows no faster than order l. Working in three dimensions, Klainerman and Machedon [5] show that if the nonlinearity with $l = 2$ satisfies an additional null condition, the Sobolev exponent $s = 2$ can be achieved.

The case $l = 2$, $k = 0$ and $n \geq 5$ was solved by Tataru [12] using fixed point argument by constructing an appropriate function space. D.Tataru's main result read as follows:

Theorem 1.1 *Assume that $l = 2$, $n \geq 5$. then the problem (1)-(2) as $k = 0$ is locally well-posed in H^s for all $s > \frac{n}{2}$. More precisely, given an initial data $(u_0, u_1) \in H^s \times H^{s-1}$ there exists an unique H^s local solution u within the space F^s defined below. Furthermore, the solution depends analytically on the initial data.*

The most effective way to prove Theorem1.1 is to use the $X^{s,\theta}$ space[6,7,11] associated to the wave equation and the nonlinear term estimates in $X^{s,\theta}$ space. However, the desired estimates are true at fixed frequency; its failure in general is due to the interaction between the high and low frequencies. Tataru's approach is first to find a suitable modification F^s of the $X^{s,\frac{1}{2}}$ space for which the appropriate estimates hold. Next, he prove the local wellposedness using the fixed point argument by establishing the appropriate multiplicative estimates in F^s for the nonlinear term.

The object of this manuscript is concerned with the local regularity of (1)-(2) in R^n for $n > 3$, $l \geq 2$, $k \geq 0$. The Sobolev exponent s satisfying $s > \max\{\frac{n+5}{4}; \frac{n}{2} + 1 - \frac{1}{l-1}\}$ is obtained in the paper. Under the assumptions on initial value $\varphi(x)$ and $\psi(x)$ as in [10], the local wellposedness of the problem (1)-(2) is proved using the contraction mapping principle by establishing estimates of linear and nonlinear wave equations.

The following function spaces are used throughout this paper: $L^p = L^p(R^n)$ denotes the Lebesgue space on R^n with the norm $\|\cdot\|_p$, $1 \leq p \leq \infty$. For $s \in R$ and $1 < p < \infty$, Let $H^{s,p} = H^{s,p}(R^n) = (1-\triangle)^{-\frac{s}{2}} L^p(R^n)$, the inhomogeneous Sobolev space in terms of Bessel potentials with norm $\|\cdot\|_{H^{s,p}} = \|\mathcal{F}^{-1}(1+|\xi|^2)^{-\frac{s}{2}}\mathcal{F}\cdot\|_p = \|(1-\Delta)^{-\frac{s}{2}}\cdot\|_p$; let $\dot{H}^{s,p} = \dot{H}^{s,p}(R^n) = (-\triangle)^{-\frac{s}{2}} L^p(R^n)$, the homogeneous Sobolev space in terms of Riesz potentials with norm $\|\cdot\|_{\dot{H}^{s,p}} = \|\mathcal{F}^{-1}|\xi|^s\mathcal{F}\cdot\|_p = \|(-\Delta)^{-\frac{s}{2}}\cdot\|_p$; and write $H^s = H^s(R^n) = H^{s,2}(R^n)$ and $\dot{H}^s = \dot{H}^s(R^n) = \dot{H}^{s,2}(R^n)$. For any Banach space X, we denote by $L^r(R^+; X)$ the space of strongly measurable functions from R^+ to X with $\|u(\cdot)\|_X \in L^r(R^+)$. For any $r \in [1, +\infty)$, r' is the dual number of r, i.e. $\frac{1}{r} + \frac{1}{r'} = 1$. Moreover, C denotes a constant which can be changed from line to line. R^+ is a positive real number set, Z^+ is a positive integer set; $\mathcal{F}$ and $\mathcal{F}^{-1}$ always denote the spatial Fourier transform and its inverse.

2 Some Lemmas

We list up some lemmas here for the following discussion.

Lemma 2.1 ([1]) (Sobolev inequality) *For* $\frac{n}{p} - s = \frac{n}{q}$ *with* $1 < p \leq q < +\infty$, *we have* $\|f\|_{L^q} \leq C\|f\|_{\dot{H}^{s,p}}$. *In contrast, for* $sp > n$ *and* $p \geq 1$; $H^{s,p} \hookrightarrow L^\infty$.

This estimate combined with usual interpolation yields counterpart estimate for $H^{s,p}$, i.e. $H^{s,p} \hookrightarrow L^q$ with $\frac{n}{p} - s \leq \frac{n}{q}$, $q \geq p$ and $s > 0$.

Lemma 2.2 ([2,4,8]) *Let* $u_0(x,t)$ *is the solution of the following linear homogeneous wave equation*

$$u_{tt} - \Delta u = 0, \ \ (t,x) \in R^+ \times R^n$$

with initial value

$$u(x,0) = \varphi(x), \ \ u_t(x,0) = \psi(x), \ \ x \in R^n,$$

namely

$$u_0(t,x) = \cos(-\Delta)^{\frac{1}{2}} t\varphi + (-\Delta)^{-\frac{1}{2}} \sin(-\Delta)^{\frac{1}{2}} t\psi,$$

then

$$\|(-\Delta)^\rho u_0(t,x)\|_{L^r(R^+;L^q(R^n))} \leq C(\|(-\Delta)^{\frac{1}{2}}\varphi\|_{L^2} + \|\psi\|_{L^2}).$$

where $\frac{2}{r} = (n-1)(\frac{1}{2} - \frac{1}{q}), \ \ q > 2; \ \rho = \frac{1}{2}(1 - \beta(q)), \ \beta(q) = \frac{n+1}{2}(\frac{1}{2} - \frac{1}{q})$.

Lemma 2.3 ([3]) *If* $f(x), \ g(x) \in S(R^n)$ *and* $\frac{1}{p_i} + \frac{1}{q_i} = \frac{1}{2}$, $i = 1, 2$, *with* $2 < p_i \leq \infty$, *then for* $s > 0$, *we have the following inequality*

$$\|fg\|_{H^s} \leq C(\|f\|_{L^{p_1}}\|g\|_{H^{s,q_1}} + \|g\|_{L^{p_2}}\|f\|_{H^{s,q_2}}).$$

In order to obtain the estimate of the solution of the inhomogeneous equation

$$u_{tt} - \triangle u = u^k |\nabla u|^l, \ \ (x,t) \in R^+ \times R^n$$

with the initial value conditions

$$u(x,0) = 0, \ u_t(x,0) = 0, \ x \in R^n,$$

we are going to use Theorem 1.3 in [9], now it reads as follows

Theorem 2.1 ([9]) *The solution of the problem*

$$u_{tt} - \Delta u = h, \ \ u(0) = u_t(0) = 0$$

fulfills

$$\sup_{t\in[0,T]} \|\nabla u(t)\|_{H^s} + \sup_{t\in[0,T]} \|u_t(t)\|_{H^s} \leq c\int_0^T \|h(t)\|_{H^s} dt.$$

Here c is independent of T. We have $\nabla u, \ u_t \in C^0([0,T], H^s)$, *if* $h \in L^1((0,T), H^s)$.

3 Estimates of Linear and Nonlinear Wave Equations

Theorem 3.1 *Let $\varphi \in H^s$, $\psi \in H^{s-1}$; then the solution $u_0(t,x)$ of the Cauchy problem for homogeneous linear wave equation*

$$u_{tt} - \Delta u = 0,\ \ u(0) = \varphi(x),\ u_t(0) = \psi(x)$$

fulfills

$$\sup_{t\in[0,T]} \|u_0(t)\|_{H^s} + \sup_{t\in[0,T]} \|u_{0t}(t)\|_{H^{s-1}} \le 2\big[\|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}}\big].$$

Proof It is well known that

$$\begin{aligned}\|u_0(t)\|_{L^2} &= \Big\|\cos[(-\Delta)^{\frac12}t]\varphi + (-\Delta)^{-\frac12}\sin[(-\Delta)^{\frac12}t]\psi\Big\|_{L^2}\\ &\le \|\varphi\|_{L^2} + T\|\psi\|_{L^2},\end{aligned} \tag{3}$$

$$\begin{aligned}\|u_0(t)\|_{\dot H^s} &= \|(-\Delta)^{\frac{s}{2}}u_0(t)\|_{L^2}\\ &= \Big\|(-\Delta)^{\frac{s}{2}}\cos[(-\Delta)^{\frac12}t]\varphi + (-\Delta)^{\frac{s}{2}-\frac12}\sin[(-\Delta)^{\frac12}t]\psi\Big\|_{L^2}\\ &\le \|\varphi\|_{\dot H^s} + \|\psi\|_{\dot H^{s-1}}.\end{aligned} \tag{4}$$

The fact that $H^s = \dot H^s \cap L^2$ if $s > 0$ (see [1], Theorem 6.3.2) together with (3) and (4) implies

$$\|u_0(t)\|_{H^s} \le \|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}}.$$

Similarly, we have

$$\|u_{0t}(t)\|_{H^{s-1}} \le \|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}}.$$

Consequently

$$\sup_{t\in[0,T]} \|u_0(t)\|_{H^s} + \sup_{t\in[0,T]} \|u_{0t}(t)\|_{H^{s-1}} \le 2\big[\|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}}\big].$$

Theorem 3.2 *Let $n > 3$, $s > \max\{\frac{n+5}{4}, \frac{n}{2} + 1 - \frac{1}{l-1}\}$; and assume that $u(t,x)$ is a solution of the inhomogeneous equation*

$$u_{tt} - \triangle u = u^k|\nabla u|^l,\ \ (x,t) \in R^+ \times R^n$$

with the initial value conditions

$$u(x,0) = 0,\ u_t(x,0) = 0,\ x \in R^n,$$

namely

$$u(t,x) = \int_0^t K(t-\tau)(u^k|\nabla u|^l)(\tau)d\tau,$$

then $u(t,x)$ satisfies the following estimate

$$\begin{aligned}\sup_{t\in[0,T]} \|u(t)\|_{H^s} &+ \sup_{t\in[0,T]} \|u_t(t)\|_{H^{s-1}} \le C\{T \sup_{t\in[0,T]} \|u(t)\|^{k+l}_{H^s}\\ &+T^{\frac{r+1-l}{r}}\|(-\Delta)^{\sigma-\frac12}u(t)\|^{l-1}_{L^r(0,T;L^q)} \sup_{t\in[0,T]} \|u(t)\|^{k+1}_{H^s}\},\end{aligned}$$

where $K(t) = (-\Delta)^{-\frac{1}{2}}\sin(-\Delta)^{\frac{1}{2}}t$, $\sigma = \frac{1}{2}(s-\beta(q)+1)$, $\frac{2}{r} = (n-1)(\frac{1}{2}-\frac{1}{q})$, $q > \frac{2(n-1)}{4s-n-5}$.

Proof We get that by Theorem 2.1

$$\sup_{t\in[0,T]}\|u(t)\|_{H^s} + \sup_{t\in[0,T]}\|u_t(t)\|_{H^{s-1}} \le C\int_0^T \|(u^k|\nabla u|^l)(\tau)\|_{H^{s-1}}d\tau. \tag{5}$$

From the fact $H^{s-1} = \dot{H}^{s-1}\cap L^2$ if $s>1$, it is easily to see

$$\begin{aligned}&\int_0^T \|(u^k|\nabla u|^l)(\tau)\|_{H^{s-1}}d\tau\\ &\le \int_0^T \|u^k|\nabla u|^l\|_{L^2}d\tau + \int_0^T \|(-\Delta)^{\frac{s-1}{2}}u^k|\nabla u|^l\|_{L^2}d\tau\\ &\le CT\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+l} + \int_0^T \|(-\Delta)^{\frac{s-1}{2}}u^k|\nabla u|^l\|_{L^2}d\tau.\end{aligned} \tag{6}$$

We may choose $p,\ \tilde{p} > 2$ such that $H^s \hookrightarrow H^{s-1,p}$, $H^{s-1}\hookrightarrow L^{\tilde{p}}$ and $\frac{1}{p}+\frac{1}{\tilde{p}} = \frac{1}{2}$. From Lemma 2.1 and Lemma 2.3 we derive

$$\begin{aligned}\|(-\Delta)^{\frac{s-1}{2}}u^k|\nabla u|^l\|_{L^2} &\le C\|u^k|\nabla u|^l\|_{H^{s-1}}\\ &\le C(\|u^k\|_{L^\infty}\||\nabla u|^l\|_{H^{s-1}} + \|u^k\|_{H^{s-1,p}}\||\nabla u|^l\|_{L^{\tilde{p}}})\\ &\le C\|u^k\|_{H^s}\|\nabla u\|_{L^\infty}^{l-1}\|\nabla u\|_{H^{s-1}} \le C\|u\|_{H^s}^{k+1}\|\nabla u\|_{L^\infty}^{l-1}.\end{aligned} \tag{7}$$

We obtain $\frac{n}{2q}+\frac{1}{2} < \frac{1}{2}(s-\beta(q)) = \sigma - \frac{1}{2}$ from $q > \frac{2(n-1)}{4s-n-5}$. Using Sobolev inequality, we find

$$\begin{aligned}\|\nabla u\|_{L^\infty} &\le C\left\|(1-\Delta)^{(\frac{n}{2q})^+}\nabla u\right\|_{L^q}\\ &\le C\|u\|_{L^q} + C\|(-\Delta)^{(\frac{n}{2q}+\frac{1}{2})^+}u\|_{L^q}\\ &\le C\|u\|_{H^s} + C\|(-\Delta)^{\sigma-\frac{1}{2}}u\|_{L^q}.\end{aligned} \tag{8}$$

Substituting the inequality (8) into the estimate (7) and observation (6) yields

$$\begin{aligned}&\int_0^T \|(u^k|\nabla u|^l)(\tau)\|_{H^{s-1}}d\tau\\ &\le CT\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+l} + C\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+1}\int_0^T \|(-\Delta)^{\sigma-\frac{1}{2}}u\|_{L^q}^{l-1}dt.\end{aligned} \tag{9}$$

Application of Hölder's inequality obtains finally

$$\int_0^T \|(-\Delta)^{\sigma-\frac{1}{2}}u\|_{L^q}^{l-1}dt \le T^{\frac{r+1-l}{r}}\|(-\Delta)^{\sigma-\frac{1}{2}}u(t)\|_{L^r(0,T;L^q)}^{l-1}. \tag{10}$$

Substituting (10) into (9), and we have by (5)

$$\begin{aligned}\sup_{t\in[0,T]}\|u(t)\|_{H^s} &+ \sup_{t\in[0,T]}\|u_t(t)\|_{H^{s-1}} \le C\{T\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+l}\\ &+T^{\frac{r+1-l}{r}}\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+1}\|(-\Delta)^{\sigma-\frac{1}{2}}u(t)\|_{L^r(0,T;L^q)}^{l-1}\}.\end{aligned}$$

4 Main Result and Its Proof

Our main result reads as follows:

Theorem 4.1 *Suppose that* $n > 3$, $l \geq 2$, $k \geq 0$ *and* $(\varphi, \psi) \in H^s \times H^{s-1}$ *with* $s > \max\{\frac{n+5}{4}; \frac{n}{2} + 1 - \frac{1}{l-1}\}$*; then there exists* $T > 0$ *depending on* s, $\|\varphi\|_{H^s}$ *and* $\|\psi\|_{H^{s-1}}$ *such that (1)-(2) has a unique solution* $u(t,x)$ *satisfying*

$$u(t,x) \in C([0,T]; H^s(R^n)) \cap C^1([0,T]; H^{s-1}(R^n))$$

and

$$\|(-\Delta)^{\sigma-\frac{1}{2}} u(t)\|_{L^r(0,T;L^q)} < +\infty,$$

where $\sigma = \frac{1}{2}(s - \beta(q) + 1)$, $\frac{2}{r} = (n-1)(\frac{1}{2} - \frac{1}{q})$, $q > \frac{2(n-1)}{4s-n-5}$.

Proof For $s > \frac{n}{2} + 1$, it is clear that Theorem 4.1 holds by means of the classical energy method. Thus we only consider the case of $s \leq \frac{n}{2} + 1$.

For T, $M > 0$, we define the space

$$X = \{u | \ u \in C([0,T]; H^s) \cap C^1([0,T]; H^{s-1}); \ \|u\|_X \leq M\},$$

where

$$\|u\|_X = \sup_{t\in[0,T]} \|u(t)\|_{H^s} + \sup_{t\in[0,T]} \|u_t(t)\|_{H^{s-1}} + \|(-\Delta)^{\sigma-\frac{1}{2}} u(t,x)\|_{L^r(0,T;L^q)}.$$

To solve our problem, we may rewrite (1)-(2) in the equivalent integral equation of the form

$$u(t,x) = u_0(t,x) + \int_0^t K(t-\tau)(u^k|\nabla u|^l)(\tau)d\tau. \tag{11}$$

where $K(t)$ is defined in Theorem 3.2.

Defining the following map by the integral equation (11)

$$\Phi: \ u \longrightarrow \Phi u = u_0(t,x) + \int_0^t K(t-\tau)(u^k|\nabla u|^l)(\tau)d\tau. \tag{12}$$

We have from Theorem 3.1 and 3.2

$$\begin{aligned} \sup_{t\in[0,T]} \|\Phi u(t)\|_{H^s} & + \sup_{t\in[0,T]} \|\Phi u_t(t)\|_{H^{s-1}} \\ & \leq \|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}} + CT \sup_{t\in[0,T]} \|u(t)\|_{H^s}^{k+l} \\ & + CT^{\frac{r+1-l}{r}} \|(-\Delta)^{\sigma-\frac{1}{2}} u(t)\|_{L^r(0,T;L^q)}^{l-1} \sup_{t\in[0,T]} \|u(t)\|_{H^s}^{k+1}. \end{aligned} \tag{13}$$

Noting

$$\begin{aligned} \|(-\Delta)^{\sigma-\frac{1}{2}} u(t)\|_{L^r(0,T;L^q)} & \leq \|(-\Delta)^{\sigma-\frac{1}{2}} u_0(t,x)\|_{L^r(R^+;L^q)} \\ & + \left(\int_0^T \|(-\Delta)^{\sigma-\frac{1}{2}} \int_0^t K(t-\tau)(u^k|\nabla u|^l)d\tau\|_{L^q}^r dt \right)^{\frac{1}{r}} \\ & = I + II. \end{aligned}$$

From Lemma 2.2

$$I \le C(\|(-\Delta)^{\frac{s}{2}}\varphi\|_{L^2} + \|(-\Delta)^{\frac{s-1}{2}}\psi\|_{L^2}) \le C(\|\varphi\|_{H^s} + \|\psi\|_{H^{s-1}}), \tag{14}$$

Using the Minkowski integral inequality first in space and then in time, we can estimate II by

$$\begin{aligned} II &\le \left(\int_0^T (\int_0^t \|(-\Delta)^{\sigma-\frac{1}{2}}K(t-\tau)(u^k|\nabla u|^l)(\tau)\|_{L^q}d\tau)^r dt\right)^{\frac{1}{r}} \\ &\le \int_0^T \left(\int_0^T \|(-\Delta)^{\sigma-\frac{1}{2}}K(t-\tau)(u^k|\nabla u|^l)(\tau)\|_{L^q}^r dt\right)^{\frac{1}{r}} d\tau. \end{aligned}$$

Applying the identity $K(t-\tau) = K(t)K'(\tau) - K'(t)K(\tau)$ and the fact that $K'(\tau)$ and $(-\Delta)^{\frac{1}{2}}K(\tau)$ are bounded in L^2 to get

$$II \le C\int_0^T \|(-\Delta)^{\frac{s-1}{2}}(u^k|\nabla u|^l)\|_{L^2}dt,$$

we also used Lemma 2.2 in the above estimation.

It follows that from (9) and (10)

$$II \le C(T\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+l} + T^{\frac{r+1-l}{r}}\sup_{t\in[0,T]}\|u(t)\|_{H^s}^{k+1}\|(-\Delta)^{\sigma-\frac{1}{2}}u(t,x)\|_{L^r(0,T;L^q)}^{l-1}). \tag{15}$$

Combining (13) and (14) with (15), we conclude that

$$\begin{aligned} \|\Phi u\|_X &\le C\left\{\|\varphi\|_{H^s} + (1+T)\|\psi\|_{H^{s-1}} + (T+T^{\frac{r+1-l}{r}})\|u\|_X^{k+l}\right\} \\ &\le C\left\{(1+T)(\|\varphi\|_{H^s} + \|\psi\|_{H^{s-1}}) + (T+T^{\frac{r+1-l}{r}})\|u\|_X^{k+l}\right\}. \end{aligned} \tag{16}$$

Setting $M = 2C(1+T)(\|\varphi\|_{H^s} + \|\psi\|_{H^{s-1}})$ and choosing sufficiently small $T > 0$ such that $C(T+T^{\frac{r+1-l}{r}})M^{k+l-1} \le \frac{1}{2}$; then we obtain from (16)

$$\|\Phi u\|_X \le M.$$

That is, Φ maps X into itself.

For any $u, v \in X$; we get similarly as the above estimation

$$\|\Phi u - \Phi v\|_X \le C(T+T^{\frac{r+1-l}{r}})M^{k+l-1}\|u-v\|_X,$$

Under the same restrictions on T and M; we have

$$\|\Phi u - \Phi v\|_X \le \frac{1}{2}\|u-v\|_X.$$

Consequently, Φ is a contraction map from X to X.

By Banach fixed point theorem, there exists a unique fixed point $u \in X$ of Φ such that $\Phi u = u$; which implies that u is a solution of the integral equation (11) corresponding to (1)-(2) and fulfills

$$u(t,x) \in C([0,T]; H^s(R^n)) \cap C([0,T]; H^{s-1}(R^n)).$$

The proof of Theorem 4.1 is finished.

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