Periodic Solutions and Global Attraction for N-Dimension Discrete-Time Neural Networks with Time-Varying Delays

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Abstract

In this paper, easily verifiable conditions for the existence and global attraction of periodic solutions of n-dimension nonautonomous discrete-time neural networks with time-varying delays are established by using the well–known Gaines–Mawhin theorem and some analysis techniques. These results obtained are more generalizable than the previously known results. Moreover, an example is provided to illustrate the validity of our results.

1 Introduction and Preliminaries

It is well known that artificial neural networks have been widely used to various fields. Such as information processing, continuous and discrete models. Artificial neural networks have been demonstrated by authors to have complex dynamics including asymptotically stable fixed points, periodic orbits, bifurcation and chaotic attractors [2,5,6–10].

Taboas [8] considered the following system of delay differential equations:

$$\begin{cases} x'(t) = -x(t) + \alpha f_1(x(t-\tau), y(t-\tau)), \\ y'(t) = -y(t) + \alpha f_2(x(t-\tau), y(t-\tau)). \end{cases} \tag{1.1}$$

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It arises as a model for a network of two saturating amplifiers (or neurons) with delayed outputs, where $\alpha > 0$ is a constant, $f_1,\ f_2$, are bounded C^3-functions on R^2 satisfying:

$$\frac{\partial f_1}{\partial y}(0,0) \neq 0 \text{ and } \frac{\partial f_2}{\partial x}(0,0) \neq 0,$$

and the negative feedback conditions $yf_1(x,y) > 0,\ y \neq 0;\ xf_2(x,y) < 0,\ x \neq 0$. Taboas showed that there is an $\alpha_0 > 0$, such that for $\alpha > \alpha_0$, there exists a non-constant periodic solution with period greater than 4. Taboas's investigation received many authors's attention [1,3,5,7]. Baptisini and Taboas [1], Godoy and dos Reis [5] studied the global existence of periodic solutions of system (1.1). Ruan and Wei [7], Chen and Wu [3] investigated the non-constant periodic solutions and slowly oscillating periodic solutions respectively for planar systems with two delays.

Recently, Huang et.al. [6] investigated the following planar systems with time-varying delays:

$$\begin{cases} x'(t) = -a_1(t) + b_1(t)f_1(x(t-\tau_1(t)),\ y(t-\tau_2(t))) + I_1(t), \\ y'(t) = -a_2(t) + b_2(t)f_2(x(t-\tau_3(t)),\ y(t-\tau_4(t))) + I_2(t), \end{cases}$$

where $a_i \in C(R,\ [0,\ \infty)),\ b_i,\ I_i \in C(R,\ R),\ f_i \in C\left(R^2,\ R\right),\ i = 1,\ 2$ are periodic of a common period $\omega\ (> 0)$, $\tau_i \in C(R,\ [0,\ \infty)),\ i = 1,\ 2,\ 3,\ 4$ being $\omega-$periodic. They proved the existence and exponential stability of periodic solutions for the above system.

The above works considered the continuous planar system. However, owing to the shortcomings of equipment and technology and the needs of people, the connection weight from one neuron to another, input bias of a neuron and self-recurrent bias of a neuron are changing with time and usually are not continuous. Delays in artificial neural networks usually time-varying, and sometimes they vary violently with time due to the finite switching speed of amplifiers and faults in the electrical circuit. They slow down the transmission rate and tend to introduce some degree of instability in circuits. Therefore, a fast response must be required in practical artificial neural-network designs. The technique to achieve fast response troubles many circuit designers [10].

In this paper, we will consider the following n-dimension nonautonomous discrete-time neural networks with time-varying delays:

$$\Delta y(k) = Ay(k) + B(k)B_\tau(k) + I(k), \tag{1.2}$$

where

$$A = \begin{pmatrix} a_1 & 0 & ... & 0 \\ 0 & a_2 & ... & 0 \\ \vdots & & & \\ 0 & 0 & ... & a_n \end{pmatrix},\quad B(k) = \begin{pmatrix} b_1(k) & 0 & ... & 0 \\ 0 & b_2(k) & ... & 0 \\ \vdots & & & \\ 0 & 0 & ... & b_n(k) \end{pmatrix}$$

and

$$\begin{aligned}
y(k) &= \left(y_1(k),\ y_2(k), \ldots,\ y_n(k)\right)^T, \\
I(k) &= \left(I_1(k), I_2(k),\ ...,\ I_n(k)\right)^T, \\
\Delta y(k) &= (\Delta y_1(k), \Delta y_2(k), ..., \Delta y_n(k),)^T, \\
\Delta y_i(k) &= y_i(k+1) - y_i(k), \\
B_\tau(k) &= \left(F_1(k),\ F_2(k),\ ...,\ F_n(k)\right)^T, \\
F_i(k) &= \sum_{j=1}^{n} f_{ij}\left(y_1\left(k-\tau_{i1}(k)\right), \ldots,\ y_n\left(k-\tau_{in}(k)\right)\right),
\end{aligned}$$

for $i = 1, 2, \ldots, n$ and $k \in Z$. By using the well–known Gaines–Mawhin theorem and some analysis technique, we will give some easily verifiable conditions for the existence and global attraction of the periodic solutions of system (1.2). The results of this paper are new and they generalize the previously known results. Moreover, an example is given to illustrate the results obtained.

To facilitate the discussion below, we define

$$I_\omega := \{0,\ 1,\ ...\ \omega - 1\},$$

and denote

$$\overline{f(k)} = \frac{1}{\omega}\sum_{k=0}^{\omega-1} f(k),\ \ \overline{y(k)} = \left(\overline{y_1(k)},\ \overline{y_1(k)}, ...,\ \overline{y_n(k)}\right)^T,$$

where $f(k)$ and $y_i(k)$ are ω-periodic sequences (i.e., $f(k+\omega) = f(k)$, $y_i(k+\omega) = y_i(k)$), and

$$H(k) = \left(H_1(k), H_2(k), \ldots,\ H_n(k)\right)^T$$

with

$$H_i(k) = -a_i(k)y_i(k) + b_i(k)F_i(k) + I_i(k)$$

for i=1, 2, ..., n.

In order to explore the existence of periodic solutions of (1.2), we first make some preparations.

Let X and Z be Banach spaces. Consider a operator equation:

$$Lx = \lambda Nx, \lambda \in (0,1)$$

where $L : DomL \cap X \to Z$ is a linear operator, $N : X \to Z$ is a continuous operator and λ is a parameter. Let P and Q denote two projectors $P : X \to X$ and $Q : Z \to Z$ such that $ImP = KerL$ and $ImL = KerQ = Im(I - Q)$. It follows that $L|_{DomL \cap KerP} : (I - P)X \to ImL$ is invertible. We denote the inverse of this map by Kp. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega}$ if $QN(\bar{\Omega})$ is bounded and $Kp(I - Q)N : \bar{\Omega} \to X$ is compact. Because ImQ is isomorphic to $KerL$, there exists an isomorphism $J : Im\ Q \to KerL$.

Recall that operator L will be called a Fredholm operator of index zero if $\dim KerL = codim\ ImL < \infty$.

The following lemma is crucial in our discussion, which can be found in [4].

Lemma 2.1 *(Continuation Theorem). Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega}$. Suppose*

(a) *for each* $\lambda \in (0,1)$, *every solution* x *of* $Lx = \lambda Nx$ *is such that* $x \notin \partial\Omega$;

(b) $QNx \neq 0$ *for each* $x \in \partial\Omega \cap KerL$;

(c) $\deg\{JQN, \Omega, 0\} \neq 0$.

Then the operator equation $Lx = Nx$ *has at least one solution in* $DomL \cap \bar{\Omega}$.

2 Existence of periodic solutions

In this section, we are concerned with the existence of periodic solutions of model (1.2). We approach our result by using the well–known Gaines–Mawhin theorem.

Theorem 2.1. *Assume that the following statement are valid for positive* ω

(i) $f_{ij} \in C(R^n,\ R)$, *are bounded continuous functions, with* $y_i(0) \geq 0$.

(ii) $a_i(k),\ b_i(k),\ I_i(k),\ \tau_{ij}(k)$ *are* $\omega - periodic$ *with* $a_i(k) > 0$.

Then model (1.2) has at least one $\omega - periodic$ *solutions for* $i, j = 1, 2, ..., n$.

Proof. In order to apply Lemma 2.1 to system (1.2), we take

$$X = Z = \{y = \{y(k)\} : y(k) \in R^n,\ y(k+\omega) = y(k),\ k \in Z\},$$

with $\|y\| = \sum_{i=1}^{n} \max\{|y_i(0)|,\ |y_i(1)|, \ldots,\ |y_i(\omega-1)|\}$. Then X and Z are Banach spaces. Define $L:\ DomL \subset X \to Z$ by

$$Ly(k) = \Delta y(k).$$

Let $N : \Omega \in X \to Z$ is given by

$$Ny(k) = H(k).$$

We then define two projectors P and Q by

$$Py(k) = Qy(k) = \overline{y(k)},\ y \in X.$$

It is easily verified that $KerL = \{y = \{y(k)\} \in X : y(k) = h \in R^n\}$,

$$ImL = \left\{ y \in X : \sum_{k=0}^{\omega-1} y_i(k) = 0,\ i = 1,\ 2, \ldots,\ n \right\}$$

is closed in X and $dimKerL = codimImL = n$. Therefore, L is a Fredholm mapping of index zero. Furthermore, the generalized inverse $K_p : ImL \to DomL \cap KerP$ has the form

$$K_p y(k) = \Theta,$$

where $\Theta = (E_1, ..., E_n)^T$ with

$$E_i = \sum_{l=0}^{k-1} y_i(l) - \frac{1}{\omega} \sum_{s=0}^{\omega-1} \sum_{l=0}^{s} y_i(l)$$

for $i = 1, 2, \ldots, n$. Thus $QN: \ \Omega \to Z$ has the form

$$QNy(k) = \overline{H(k)}$$

and $K_p(I-Q)N: \ \Omega \to X$ has the form

$$K_p(I-Q)Ny(k) = \Xi,$$

where $\Xi = (U_1, ..., U_n)^T$ with

$$U_i = \sum_{k_1=0}^{k-1} H_i(k_1) + \overline{kH_i(k)} + \left(\frac{1-\omega}{2} - k\right) \overline{H_i(k)}.$$

It is easy to see that QN and $K_p(I-Q)N$ are continuous, and $QN\,(\bar{\Omega})$ is bounded. Since X is of finite dimension, $K_p(I-Q)N\,(\bar{\Omega})$ is relatively compact for any open bounded set $\Omega \subset X$ by Arzela-Ascoli theorem. Therefore, N is L-compact on $\bar{\Omega}$.

We now search for an appropriate open bounded subset Ω for the application of Lemma 2.1. Corresponding to the operator equation $Ly = \lambda Ny$, $\lambda \in (0,1)$, we have

$$\Delta y(k) = \lambda H(k). \tag{2.1}$$

Suppose that $y \in X$ is a solution of (2.1) for $\lambda \in (0,1)$. From (1.2) we have

$$|\Delta y_i(k)| < |a_i(k) y_i(k)| + |b_i(k) F_i(k) + I_i(k)| \,. \tag{2.2}$$

By virtue of (2.2) and the discrete Gronwall inequality we obtain that for $k \leq \omega$

$$\begin{aligned} |y_i(k)| \ &\leq \ \left(y_i(0) + \sum_{k=0}^{\omega-1} a_i(k)\right) \exp\left\{\sum_{k=0}^{\omega-1} |b_i(k)F_i(k) + I_i(k)|\right\} \\ &\leq \ (y_i(0) + \omega A_i) \exp\{\omega G_i\} := R_i, \end{aligned} \tag{2.3}$$

where $A_i = \max\{a_i(k) : \ k \in I_\omega\}$ and $G_i = \max\left\{\sum_{k=0}^{\omega-1} |b_i(k)F_i(k) + I_i(k)| : \ k \in I_\omega\right\}$.

Obviously, R_i are independent of λ such that $|y_i(k)| \leq R_i$. Take $R > 0$ is so large that

$$R > \max_{1 \leq i \leq n} \left\{ \left| \frac{\mu \overline{b_i(k) F_i(k)} + \overline{I_i\,(k)}}{\overline{a_i(k)}} \right| : \ \mu \in [0,\ 1],\ k \in I_\omega \right\}.$$

Let $R_1 + R_2 + \cdots + R_n + R = M$ and $\Omega = \{y \in X : \|y\| < M\}$. Then $Ly \neq \lambda Ny$ for y$\in \partial\Omega \cap DomL$ and $\lambda \in (0,1)$, which show that the condition (a) in Lemma 2.1 is satisfied.

If $y \in \partial\Omega \cap KerL$, $y \in R^n$ then $\|y\| = M$. The above argument show that $QNy(k) = \overline{H(k)} \neq 0$. This shows that condition (b) in Lemma 2.1 is satisfied.

Finally we will show that condition (c) in Lemma 2.1 is also satisfied. To this end, we define $\Phi: \ DomL \times [0,1] \to Z$ by

$$\Phi(y,\ \mu) = \Psi,$$

where $\Psi = (J_1, ...J_n)^T$ with $J_i = -\overline{a_i(k)y_i(k)} + \overline{I_i(k)} + \mu\overline{b_i(k)F_i(k)}$ for $\mu \in [0,1]$. If $y \in \partial\Omega \cap KerL \subset R^n$, y is a constant vector with $\|y\| = M$. It is easy to show that

$$\Phi(y,\ \mu) \neq 0 \text{ for } \mu \in [0,\ 1].$$

By the property of invariance under a homotopy, we let $J = I : ImQ \to KerL$, i.e., $y \to y$. Then we get

$$\begin{aligned} & \deg\left(JQNy,\ \Omega \cap KerL,\ \theta\right) \\ = & \deg\left(\Phi\left(y,\ 1\right),\ \Omega \cap KerL,\ \theta\right) \\ = & \deg\left(\Phi\left(y,\ 0\right),\ \Omega \cap KerL,\ \theta\right), \end{aligned}$$

where $\theta = (0, 0, ..., 0)^T$. By condition (ii) of Theorem 2.1, one can easily show that

$$\deg\left\{JQNy(k),\ \Omega \cap KerL,\ \theta\right\} = sgn(-1)^n \prod_{i=1}^{n} \overline{a_i(k)} \neq 0.$$

Hence the condition (c) in Lemma 2.1 is also satisfied.

We have shown that Ω verifies all requirements of Lemma 2.1, it follows that model (1.2) has at least one ω-periodic solution in $DomL \cap \overline{\Omega}$. The proof is complete.

3 Global attraction of periodic solutions

In the previous section, we have shown that model (1.2) has at least one ω-periodic solution $\overline{y}(k) = (\overline{y}_1(k),\ \ldots, \overline{y}_n(k))^T$ under some easily verified sufficient conditions. We now shall further strengthen the sufficient conditions of Theorem 2.1 to ensure $\overline{y}(k)$ is the global attractor of all other solutions of model (1.2).

Theorem 3.1. *In addition to the assumption $(i) - (ii)$ made in Theorem 2.1, assume further that*

(iii) $\left|f_{ij}\left(y_1,\ \ldots,\ y_n\right) - f_{ij}\left(\overline{y}_1,\ \ldots, \overline{y}_n\right)\right| \leq m_{ij1}\left|y_1 - \overline{y}_1\right| + \cdots + m_{ijn}\left|y_n - \overline{y}_n\right|$.

(iv) $|1 - a_i(k)| < 1 - n^2K\,|b_i(k)|$.

Where m_{ijl} are positive constants, and $K = \max\{m_{ijl}\} for\ i, j, l = 1, 2, \ldots, n$. Then system (1.2) has a unique ω-periodic solution which is globally attractive.

Proof. Let $y(k)$ be a solution of system (1.2). It follows that

$$\Delta y_i(k) = -a_i y_i(k) + b_i(k)F_{\tau_{ij}}(k) + I_i(k). \tag{3.1}$$

Note that (iv) implies $a_i(k) > 0$. Hence the assumption (ii) of Theorem 2.1 holds. It follows that system (1.2) has at least one ω-periodic solution. Let $\overline{y}(k)$ is a ω-periodic solution of model (1.2). Then we get

$$\Delta\overline{y}_i(k) = -a_i\overline{y}_i(k) + b_i(k)\overline{F}_i(k) + I_i(k). \tag{3.2}$$

where

$$\overline{F}_i(k) = \sum_{j=1}^{n} f_{ij}\left(\overline{y}_1\left(k-\tau_{i1}(k)\right), \ldots, \overline{y}_n\left(k-\tau_{in}(k)\right)\right).$$

By (3.1) , (3.2) and (iii), a simple calculation gives

$$\begin{aligned}
& |y_i(k+1) - \overline{y}_i(k+1)| \\
\leq\ & |1-a_i(k)|\,|y_i(k)-\overline{y}_i(k)| + |b_i(k)| \sum_{j=1}^{n} \big| f_{ij}\left(y_1\left(k-\tau_{i1}(k)\right), \ldots, y_n\left(k-\tau_{in}(k)\right)\right) \\
& - f_{ij}\left(\overline{y}_1\left(k-\tau_{i1}(k)\right), \ldots, \overline{y}_n\left(k-\tau_{in}(k)\right)\right)\big| \\
\leq\ & |1-a_i(k)|\,|y_i(k)-\overline{y}_i(k)| + |b_i(k)| \sum_{j=1}^{n} \left\{ m_{ij1}\left|y_1\left(k-\tau_{i1}(k)\right) - \overline{y}_1\left(k-\tau_{i1}(k)\right)\right| \right. \\
& \left. + \cdots + m_{ijn}\left|y_n\left(k-\tau_{in}(k)\right) - \overline{y}_n\left(k-\tau_{in}(k)\right)\right| \right\} \\
\leq\ & |1-a_i(k)|\,|y_i(k)-\overline{y}_i(k)| + nK\,|b_i(k)| \times \\
& \left\{ \left|y_1\left(k-\tau_{i1}(k)\right) - \overline{y}_1\left(k-\tau_{i1}(k)\right)\right| + \cdots + \left|y_n\left(k-\tau_{in}(k)\right) - \overline{y}_n\left(k-\tau_{in}(k)\right)\right| \right\}.
\end{aligned}$$

Let

$$Z_i(k) = \max_{1\leq i,j\leq n} \left\{ |y_i(k)-\overline{y}_i(k)|,\ \left|y_i\left(k-\tau_{ij}(k)\right) - \overline{y}_i\left(k-\tau_{ij}(k)\right)\right| \right\}.$$

Then we get

$$Z_i(k+1) \leq \Big(|1-a_i(k)| + n^2K\,|b_i(k)| \Big) Z_i(k).$$

By (iv) of Theorem 3.1, we have

$$\alpha = \max_{1\leq i\leq n} \Big\{ |1-a_i(k)| + n^2K\,|b_i(k)| \Big\} < 1.$$

Thus

$$Z_i(k+1) \leq \alpha Z_i(k),\ \alpha < 1. \tag{3.3}$$

Inequality (3.3) implies that $\lim_{k\to\infty} Z_i(k) = 0$. That is

$$\lim_{k\to\infty} |y_i(k) - \overline{y}_i(k)| = 0,\ for\ i = 1,\ 2,\ \ldots,\ n.$$

Therefore, $\overline{y}(k)$ is the global attractor of all other solutions of model (1.2). The proof of Theorem 3.1 is complete.

4 An example

In this section, we give an example to demonstrate the results of Theorem 2.1 and Theorem 3.1.

Example 4.1. Consider the following two-dimension nonautonomous discrete-time neural networks with time-varying delays

$$\Delta y_i(k) = -a_i(k)y_i(k) + b_1(k)\sum_{j=1}^{2} f_{1j}\left(y_1\left(k-\tau_{i1}(k)\right), y_2\left(k-\tau_{i2}(k)\right)\right) + I_i(k), \tag{4.1}$$

where $i = 1,\ 2$; $a_1(k) = \frac{1}{4}\sin\frac{k\pi}{2} + \frac{1}{2}$, $a_2(k) = \frac{1}{3}\cos\frac{k\pi}{2} + \frac{2}{3}$, $b_1(k) = \frac{1}{68}\sin\frac{k\pi}{2}$, $b_2(k) = \frac{1}{68}\cos\frac{k\pi}{2}$, $f_{11}(y_1,\ y_2) = f_{12}(y_1,\ y_2) = f_{21}(y_1, y_2) = f_{22}(y_1,\ y_2) = \cos y_1 + \cos y_2$, $\tau_{11}(k) = \tau_{12}(k) = \sin\frac{k\pi}{2}$, $\tau_{21}(k) = \tau_{22}(k) = \cos\frac{k\pi}{2}$, $I_1(k) = e^{\sin\frac{k\pi}{2}}$, $I_2(k) = e^{\cos\frac{k\pi}{2}}$.

It is straightforward to check that all the conditions needed in Theorem 2.1 and Theorem 3.1 are satisfied. Hence, model (4.1) has exactly one 4-periodic solution, which is globally attractive.

5 Concluding remarks

In this paper, n-dimension nonautonomous discrete-time neural networks with time-varying delays have been studied. A set of sufficient conditions for the existence and global attraction of the periodic solutions have been obtained. When $i = 1, 2; n = 2$ system (1.2) may be viewed as a discrete analogue of the neural network model considered in [5] (see, the example 4.1). Therefore our results are new.

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References

[1] M. Baptisini, P. Taboas, On the existence and global bifurcation of periodic solutions to planar differential delay equations, *J. Differential Equations* 127 (1996) 391-425.

[2] L. Chen, K. Aihara, Chaos and asymptotical stability in discrete-time neural networks, *Physica D* 104 (1997) 284-325.

[3] Y. Chen, J. Wu, Slowly oscillating periodic solutions for a delayed frustrated network of two neurons, *J. Math. Anal. Appl.* 259 (2001) 188-208.

[4] R.E. Gaines, J.L. Mawhin, Coincidence Degree and Nonlinear Differential Equations, Springer, Berlin, 1977.

[5] S.M.S. Godoy, J.G. dos Reis, Stability and existence of periodic solutions of a functional differential equation, *J. Math. Anal. Appl.* 198 (1996) 381-398.

[6] C. Huang, L. Huang, Existence and global exponential stability of periodic solution of two-neuron networks with time-varying delays, *Appl. Math. Lett.* 19 (2006) 126-134.

[7] S. Ruan, J. Wei, Periodic solutions of planar systems with two delays, *Proc. Roy. Soc. Edinburgh. Sect. A* 129 (1999) 1017-1032.

[8] P. Taboas, Periodic solution of a planar delay equation, *Proc. Roy. Soc. Edinburgh. Sect. A* 116 (1990) 85-101.

[9] H. Xiang, K.-M.Yan,and B.-Y. Wang, Existence and global stability of periodic solution for delayed discrete high-order Hopfield-type neural networks, *Discrete Dynamics in Nature and Society,* 3 (2005) 281-297.

[10] L. Zou, Z. Zhou. Periodic solutions for nonautonomous discrete-time neural networks, *Appl. Math. Lett.* 19 (2006) 174-185.

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