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Lectures on the asymptotic theory of ideals, by David Rees. Cambridge University Press, Cambridge (London Math Society, Lecture Notes #113), 1988, 200 pp., $24.95. ISBN 0-521-31127-6

Commutative ring theory was born in the early part of this century, a child of Emmy Noether and Wolfgang Krull (David Hilbert filling in as grandpa). It grew up on a tough block, living between algebraic number theory and algebraic geometry. Those two have always been bigger and brasher, and maybe tried to bully it a bit.

Yet our hero held its ground, gracefully accepting the fact that it would probably never win the accolades afforded its more notorious companions, and secure in the knowledge that its elegance often encompassed surprising depth. The child quickly learned to be idealistic, although due to its mother's influence, most of its ideals are finitely generated. When it reached young adulthood, it accepted the guidance of some loving uncles, among them Samuel and Nagata. Yet it is another uncle who most concerns us here, a man who might have been chosen for the part by central casting, he so well fits the role. Over the last thirty-five years, few people could hope to rival the influence which David Rees has had in commutative ring theory.

In a Noetherian ring, large powers of a given ideal tend to be well behaved. The first major evidence for this was the Hilbert polynomial. If J is an ideal in such a ring, and if $v(J)$ denotes the minimal number of generators of J, then there is a polynomial $g(X)$ such that for large n, $v(J^n) = g(n)$. This thread was then taken up by Samuel, who in 1952 considered a function f on a Noetherian ring defined by $f(x) = \sup\{n | x \in J^n\}$. He showed that $\mathbf{f}(x) = \lim_{n\to\infty} f(x^n)/n$ exists. Shortly thereafter, Nagata showed that this limit, if not infinite, must be rational. Rees went further. His valuation theorem showed there are finitely many integer valued valuations $v_1, \dots, v_k$ and positive rational numbers $e_1, \dots, e_k$, such that $\mathbf{f}(x) = \min\{v_j(x)/e_j | j = 1, \dots, k\}$, for all x. Since the middle of this century, Rees has dominated that part of commutative ring theory which deals with asymptotic properties of ideals, the Rees ring and the Artin–Rees lemma bearing testimony to this claim.

It is appropriate that this book appears at the time of Rees's retirement, for it is, to a large extent, a book of memories. However, while many of the basic ideas are old, they are generalized from dealing with powers of an ideal, to dealing with Noetherian filtrations. Also, some of the proofs are new. Furthermore the final chapters on a generalized degree formula for mixed multiplicities contain some new material. Reflecting its birth as a series of lectures (presented at Nagoya University during the winter of 1982–1983), the text has a chatty style, similar to Northcott's *Lessons on Rings, Modules, and Multiplicities.* However, in that famous work, the reader always knows the goal of each paragraph in advance, while occasionally in the present text, one does not know where one is going until one gets there. Yet, in all, it is pleasant reading. A commendable feature of the work, making it particularly suitable for a graduate seminar, is that it is essentially self-contained. Since the Mori–Nagata theorem is used, it is proved via Matijevic's theorem. As completions appear, their needed properties are reviewed. Since the multiplicity of a filtration is defined via Koszul complexes, an introduction to those complexes is presented. (Bless you, Sir.) The most serious shortcoming of the work, to this reader's mind, is that not enough is said about the history of the subject. A book of memories can, and even should, wax a bit nostalgic. Perhaps modesty is the reason why so little is said about how these ideas developed.

The first three chapters of the book are essentially preparatory, discussing graded rings, filtrations, and the Mori–Nagata theorem. The first real goal is reached in Chapter 4, which gives a new proof, for Noetherian filtrations, of the valuation theorem. Chapter 5 then proves the strong valuation theorem, again for filtrations. Chapters 6 and 7 characterize those proper valuations on a Noetherian ring which are associated to some Noetherian filtration on the ring via the valuation theorem. Those valuations are then used to give alternate proofs of some known results concerning saturated chains of prime ideals. Chapter 8 develops the concept of the multiplicity function of an M-primary filtration on a finite module over a local ring (A, M). In Chapter 9, the multiplicity function is used to define the degree function of a Noetherian filtration, and that degree function is then characterized in terms of the valuations discussed in Chapters 6 and 7. The final three chapters are essentially independent of the earlier chapters. In 1973, Teissier introduced mixed multiplicities. Rees then used the theory of complete and joint reductions of a set of ideals to generalize the degree formula to deal with mixed multiplicities. However, in Chapters 10, 11, and 12, he uses a different approach, general elements, to give a new version of this generalized degree formula.

Those who already know something of Rees's work will be happy to see familiar ideas unified and extended, and will likely get new insights from the text. They might also be envious of new students, able to reap such a rich harvest from one acre.

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