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ON A PROBLEM OF ROTA

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Let $S(n, k)$ denote the Stirling numbers of the second kind, and let K_n be such that $S(n, K_n) \geqslant S(n, k)$ for all k. Rota's problem [3] is to prove or disprove the following:

For all n, the largest possible incomparable collection of partitions of an n-set contains $S(n, K_n)$ partitions.

An "incomparable collection" of partitions is one in which no partition in the collection is a refinement of some other partition in the collection.

DEFINITION. Let $\mathcal{S}(n, k)$ denote the collection of all partitions of an n-set into k nonempty blocks. If $\mathcal{C} \subseteq \mathcal{S}(n, k)$, define Span($\mathcal{C}$) by

$$\text{Span}(\mathcal{C}) = \{\pi \in \mathcal{S}(n, k+1): \pi \text{ is a refinement of some } \pi' \in \mathcal{C}\}.$$

THEOREM. *For all sufficiently large n, there is a collection $\mathcal{C} \subseteq \mathcal{S}(n, j)$ such that*

(i) $j + 1 = K_n$,

(ii) $|\text{Span}(\mathcal{C})| < |\mathcal{C}|$, *where* $|\ |$ *denotes cardinality.*

Consequently, $(\mathcal{S}(n, j+1) - \text{Span}(\mathcal{C})) \cup \mathcal{C}$ *is an incomparable collection with more than $S(n, K_n)$ partitions.*

REMARKS. $\mathcal{C}$ *consists of all* $\pi \in \mathcal{S}(n, j)$ having exactly l blocks of size $\leqslant M$ and exactly $j - l$ blocks of size $> M$ and $\leqslant 2M$, where l and M are appropriately defined.

The proof of the Theorem requires [2] to estimate $|\mathcal{C}|$ and $|\text{Span}(\mathcal{C})|$; and also requires [1] to know the approximate value of K_n.

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