BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 5, September 1976

C^0-DENSITY OF STRUCTURALLY STABLE VECTOR FIELDS[1]

BY M. M. C. de OLIVEIRA[2]

Communicated by P. T. Church, March 5, 1976

Let M^n be a C^∞-compact, connected, n-manifold without boundary. Let $\mathfrak{X}^r(M)$ be the set of C^r-vector fields on M, $1 \leqslant r \leqslant \infty$. $X \in \mathfrak{X}^r(M)$ is C^r structurally stable if there exists a neighbourhood U_X of X in $\mathfrak{X}^r(M)$ such that given $Y \in U_X$ there exists a homeomorphism $h: M \longrightarrow M$ taking oriented trajectories of X to oriented trajectories of Y. Let $\Sigma^r(M) \subset \mathfrak{X}^r(M)$ be the set of C^r-structurally stable vector fields on M. In this paper we announce the proof that $\Sigma^r(M)$ is always dense in $\mathfrak{X}^r(M)$ with respect to the C^0-topology. This result is the same theorem that Smale and Shub proved for diffeomorphisms in [2] and [1].

The main tools for our proof are the theorems of Smale [2], Shub [1] and Zeeman [3]. Details of the proof will appear elsewhere. The author wishes to thank his supervisor Professor E. C. Zeeman for many helpful conversations, suggestions and encouragment.

Main theorem.

THEOREM 1. *Let $1 \leqslant r \leqslant \infty$. Let $X \in \mathfrak{X}^r(M)$. Then X is C^r*-isotopic to *a $Y \in \Sigma^r(M)$ by an isotopy which is arbitrarily small in the C^0 topology.*

COROLLARY 2. *Let $1 \leqslant r \leqslant \infty$. Then $\Sigma^r(M)$ is dense in $\mathfrak{X}^r(M)$ with respect to the C^0 topology.*

For the next theorem, suppose M admits a nonsingular vector field and let $NS^r(M)$ be the set of nonsingular C^r-vector fields on M.

THEOREM 3. *Any $X \in NS^r(M)$ is C^r-isotopic (through nonsingular vector fields) to a $Y \in \Sigma^r(M) \cap NS^r(M)$ by an isotopy which is arbitrarily small in the C^0-topology.*

REFERENCES

1. M. Shub, *Structurally stable diffeomorphisms are dense*, Bull. Amer. Math. Soc. **78** (1972), 817–818. MR **46** #6398.

2. S. Smale, *Stability and isotopy in discrete dynamical systems*, Dynamical Systems (Proc. Sympos., Univ. of Bahia, Salvador, 1971) (M. Peixoto, Editor), Academic Press, New York, 1973, pp. 527–530. MR **49** #3984.

3. E. C. Zeeman, *C^0-density of stable diffeomorphisms and flows*, Proc. Colloquium on Smooth Dynamical Systems, Southampton, 1972. (Southampton University reprint).

MATHEMATICS INSTITUTE, UNIVERSITY OF WARWICK, COVENTRY, ENGLAND

AMS (MOS) subject classifications (1970). Primary 58F10.

[1] This paper summarizes a portion of the author's Ph. D. thesis at Warwick University.

[2] Work supported by a scholarship from CAPES (Brazil).