incorrect, for the integral and Z_p cohomology rings are in fact independent of the Z_p action. The homotopy types can be distinguished only by getting chosen generators from the reduction of integral to mod p cohomology, or from the Hurewicz homomorphism. The most crucial mistake is in 14.8 on p. 311, where the author says that $\tilde{E}^0(B)=0$ if $\boldsymbol{E}$ is a connected spectrum when in fact one needs $\pi_q(E)=0$ for $q>0$. This error is compounded in the following discussions of orientation. In particular, 14.9 is false except for ordinary cohomology and the proof of 14.18 is only valid then. The exercise on p. 312 suffers badly from misprints but seems to involve the same error, and if I correctly interpret it, it is false.

R. STONG

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Almost periodic differential equations, by A. M. Fink, Lecture Notes in Mathematics, No. 377. Springer-Verlag, Berlin, Heidelberg, New York, 1974, vii+336 pp.

Nonlinear differential equations of higher order, by R. Reissig, G. Sansone and R. Conti, Noordhoff, Leyden, 1974, xiii+669 pp.

Functional differential equations, by J. K. Hale, Applied Mathematical Sciences, No. 3. Springer-Verlag, New York, Heidelberg, Berlin, 1971, viii+238 pp.

In this review we trace some of the major developments in the study of the qualitative behavior of solutions of ordinary differential equations and show how these books fit into this general theory.

I. **Origins of the qualitative theory.** The qualitative theory of ordinary differential equations began nearly a century ago with the work of H. Poincaré in France and A. Lyapunov in Russia. Prior to this time the major emphasis in differential equations had been on the methods of "solving" various equations either in closed form by an explicit formulation, or in terms of series, cf. Ince [**17,** pp. 529–539] for example. This interest in solving equations was undoubtedly influenced by the strong interconnection between the study of differential equations and the problems of physics. To put it in modern language, the existence of a solution is clearly the first logical step in establishing the validity of a given mathematical model for a physical phenomenon. Naturally, the first attempts at finding solutions were in terms of explicit formulae. This line of research reached its dénouement during the period from 1875 to 1900 when the work of Lipshitz, Picard, Peano, and others established the so-called fundamental theory, i.e., the theory of the existence, uniqueness and continuity of solutions. While investigations into the fundamental theory continue even today, one finds that the major emphasis in the study of ordinary differential equations now seems to be in the qualitative behavior of solutions.

It is interesting to note that the origins of the qualitative theory, in the works of Poincaré and Lyapunov, are found at the same time (1875–1900) as the fundamental theory was reaching its zenith. Poincaré started a trend towards the use of topological techniques (the analysis situs) in the study of differential equations. The Poincaré-Bendixson theory of solutions of differential equations in the plane is a good illustration of the influence of Poincaré. Lyapunov introduced a new analytic technique, the theory of Lyapunov functions, into the study of stability. The qualitative theory begins, both logically and historically, where the fundamental theory ends. One assumes the existence and continuity of solutions, and one then uses the topological structure of the phase space or the analytic structure of the vector field to derive qualitative information (such as stability, periodicity, recurrence, etc.) about the behavior of solutions.

Research into the theory of differential equations was not only stimulated by physics, but was also influenced by developments in other areas of mathematics. One finds a rather interesting trend beginning around the turn of the century, viz. an introduction of the techniques and theories of both topology and functional analysis into the study of differential equations. The influence of these disciplines has been a factor of continuing growing importance in the qualitative theory. Of special significance is the work of G. D. Birkhoff during the period 1912–1931. Birkhoff, whose research was strongly influenced by Poincaré, used topological techniques in his study of limit sets, recurrence and the structure of minimal sets, cf. [**3**]. On the other hand, Birkhoff's work on invariant measures and the ergodic theorem [**4**] illustrates the influence of Lebesgue, Hilbert, von Neumann, and other researchers in functional analysis, in the study of the qualitative behavior of solutions of differential equations.

One of the consequences of the growing use of topological and functional analytic techniques in mathematics, in general, has been a tendency towards abstraction. In the qualitative theory this tendency has resulted in the notion of a flow or dynamical system. Birkhoff, in his writings, made numerous attempts at defining a flow, and in [**5**] one finds a distinction between a continuous flow (solutions of a differential equation) and a discrete flow (homeomorphisms on a manifold). However, this concept, as we understand it today, was formulated later and appears for the first time[1] about thirty years ago, cf. Nemyckiĭ and Stepanov [**24**].

II. **Almost periodicity.** The techniques of topology and functional analysis have, of course, been used on other areas of mathematics besides differential equations. One such area, the theory of almost periodic functions, is of especial interest here because of the important role this theory has played in

[1] It is very difficult to be precise about the historical questions concerning the evolution of the concept of a flow. The notion of a flow has its origins in the theory of transformation groups, which goes back to Lie, but it is the connection with the qualitative theory of differential equations which interests us here. This connection, which evolved through the research of Poincaré and Birkhoff, came of age about a generation ago.

the study of the qualitative behavior of solutions. The theory of almost periodicity began with the pioneering papers of Bohr [**9**] in 1924–1926, and many significant contributions to the subject were made in the decade immediately following Bohr's work, cf. Bochner [**7**], von Neumann [**25**], and van Kampen [**19**], for example.

During the 1920's research into almost periodicity was directed towards the development of a theory of Fourier series for almost periodic functions. The papers of Bohr and Bochner cited above were fundamental contributions to this theory. Then starting[2] in the 1930's, the theory of almost periodic functions on the real line R was extended to a theory of almost periodic functions on a group G. This later trend grew out of an interest in the mathematical community to study groups of linear operators, in particular, and topological groups, in general. .One of the interesting consequences of this newer trend was a tendency to divide the study of almost periodic functions into two almost nonoverlapping disciplines. The original emphasis on the Fourier series of almost periodic functions has grown into an important area of study in "hard" analysis. Most of the research into ordinary differential equations with almost periodic coefficients would fit into this discipline. The second area, which started with the study of functions on groups, has grown into the theory of abstract harmonic analysis, cf. Hewitt and Ross [**16**] for example.

The theory of almost periodic functions is, of course, an extension of the theory of periodic functions, and certainly one of the important successes of this newer theory was the development of a rather complete theory of Fourier series for almost periodic functions. One of the reasons researchers in differential equations have been interested in almost periodic solutions is because of this theory of Fourier series. Another reason for this interest is related to the successful investigations of Floquet in 1883 into the theory of linear differential equations with periodic coefficients. The general desire to "extend" the Floquet theory to equations with almost periodic coefficients has motivated countless papers during the last fifty years. A third reason, and perhaps the most important reason, for the interest in almost periodic phenomena is that some differential equations, especially equations arising from physics, simply have almost periodic solutions.

During the past fifty years many books have been written which treat the theory of almost periodic phenomena in ordinary differential equations. Out of all of these books it seems that two of them are particularly noteworthy. One is the book by Favard [**13**], which appeared in 1933 and is the first book written on this subject. The other is *Almost periodic differential equations*, by A. M. Fink which was published in 1974.

In Favard's book one finds a treatment of the general theory of almost periodic functions, with applications of this theory to the study of linear

[2] It is a bit difficult to pinpoint a *unique* paper which inaugurated this trend. Basic papers of Haar and von Neumann appeared in 1933; however, one could argue that the Peter and Weyl paper in 1927 anticipated this trend. For a detailed historical discussion see Maak [**21**, pp. 222–235].

differential equations $x'=A(t)x+f(t)$, where the coefficients $A(t)$ and $f(t)$ are almost periodic. The main problem studied in this book may be described as the problem of determining sufficient conditions in order that the above equation admits an almost periodic solution.

During the four decades between the appearance of these two books, many important advances have been made in the study of almost periodic phenomena for differential equations. First, by using various notions of separatedness and stability many authors have been able to extend Favard's theory from linear to nonlinear equations. Secondly, the method of averaging due to Krylov and Bogoliubov has been shown to be an important tool in the study of various perturbation phenomena arising in the theory of almost periodic differential equations. Thirdly, a very important development occurred in 1962 when Bochner [**8**] showed that the concept of almost periodicity could be reformulated in terms of an iterated limit. (There have been, of course, other major developments in the study of almost periodic phenomena for differential equations, and we will have more to say about this shortly.)

The book by Fink is up-to-date treatment of the theory of almost periodic functions and differential equations which treats the Bohr–Bochner–Favard theory together with the advances noted above. This book has many features which will be of value to both experts and students. First, it fully incorporates the recent Bochner characterization into the text. As a result, one finds in this book a more unified treatment of the subject of almost periodic functions as well as a very useful technique for applications to differential equations, especially differential equations with almost periodic coefficients. Secondly, most of the techniques and results in the theory of almost periodic phenomena for both linear and nonlinear differential equations have now been collected together in one source. This book is perhaps the most complete reference available on almost periodic phenomena in the study of differential equations. Thirdly, the text includes a good selection of historical notes, which is supplemented with an excellent bibliography of 548 items.

The book, however, does have some shortcomings. Major topics in the study of almost periodicity in differential equations have been omitted. Perhaps the most noticeable shortcoming is that there is no treatment of the theory of quasi-periodic solutions of Hamiltonian differential systems, cf. Birkhoff [**6**], Kolmogorov [**18**], Arnol'd [**1**], and Moser [**23**]. Unfortunately, the "small divisor problem", which is a central problem in the theory of quasi-periodic solutions, is not even mentioned.

Another omission is that there is no discussion of the Cartwright Theorem which gives an upper bound for the dimension of the Fourier frequency module in the case of an almost periodic solution of a differential equation, cf. [**10**] and [**11**]. Actually, this is perhaps the one aspect of the study of almost periodic solutions of differential equations where the two branches of almost periodicity (i.e., the hard analysis branch and the topological group branch) could be united for a simple proof of this important theorem. The proof would use the Pontryagin Duality Theorem together with the fact that the dimension of the Fourier frequency module is precisely the rank of the character group. The Cartwright Theorem would then follow as a consequence of the relationship between the topological dimension of a compact connected abelian group and the rank of the associated character group, cf. Pontryagin [**26,** pp. 148–149]. This result from the theory of differential equations should have been included. Furthermore, the primary audience for this book, viz. researchers in differential equations, would have found a proof along the above lines very instructive and valuable.

The bibliography, as we have already noted, is one of the highlights of the books and most readers will find this to be very valuable. However, it is a bit unfortunate that some of the important historical papers, including the original papers of Bohr [**9**], as well as several of the papers cited here, have not been included.

Our final objection to Fink's book concerns the author's decision to omit the aspect of the theory of almost periodicity that is related to dynamical systems or flows. This seems to have been an unfortunate decision for several reasons. Let us look at two.

First, researchers in ordinary differential equations with almost periodic coefficients have known, since the time of Favard, that it is usually not adequate to consider a single differential equation for an analysis of certain qualitative behavior, but rather one must consider an entire class of equations from the "hull". The precise mathematical reason for the preference of the hull over a single equation has been shrouded in mystery for many years. However, recent investigations of almost periodic differential equations in terms of flows have begun to unravel this puzzle.[3] By not incorporating the theory of flows into the book, the author has lost a chance to explain this interesting new development.

Secondly, by using the notion of a flow one can give a characterization of an almost periodic minimal set in terms of equicontinuity (cf. Franklin [**14**]), which in turn is simply a form of Lyapunov stability, cf. Nemyckiĭ and Stepanov [**24**], and Sell [**28**]. The author presents several theorems which establish the existence of an almost periodic solution as a consequence of a stability property. For the most part, these theorems are consequences of the existence of almost periodic minimal sets in a flow. This fact certainly would have introduced greater clarity into the relationship between almost periodic differential equations and various stability properties, cf. Sell [**29**].

In summary, the book by Fink is an excellent, well-written book and, within the limitations chosen by the author, it is complete. Even though it cannot be judged a definitive work, it is, nevertheless, the best book available on the subject. Since it appears now in a Lecture Notes series, we

[3] For a recent account of the preference of the hull over a single equation, including some references which have appeared after the publication of the book by Fink, see Miller and Sell [**22**].

hope that an expanded version, which includes the fundamental topics mentioned above, will be forthcoming.

III. **Lyapunov theory.** As is well known, the influence of Poincaré was felt immediately throughout the mathematical community. In the area of differential equations we have seen this influence in the work of Birkhoff. The contributions of Lyapunov, on the other hand, remained essentially unknown for nearly a half a century. During the period from 1930 to 1950 one can find a gradual reawakening of interest in the theory of stability and Lyapunov functions. The work of Malkin, K. P. Persidskiĭ and Massera was especially important in this period. The renewal of interest in Lyapunov functions spawned a renewal of research in the qualitative theory of differential equations in general. During the last 25 years many of the modern techniques of mathematics have been used, sometimes with great success, to derive new insight into the structure of solutions of differential equations. Fixed point methods, topological degree, semigroup theory, homological theory, algebraic geometrical methods, and dynamical systems are just a few of these techniques.

In 1954 Cesari [**12**] wrote an excellent summary of the qualitative theory of differential equations. There have been so many developments within the last twenty years that today it would be almost impossible to write a comprehensive treatment of the subject. Since the appearance of Cesari's book one finds, for the most part, special purpose books which deal with selective topics in the qualitative theory. The recent book by R. Reissig, G. Sansone, and R. Conti on *Nonlinear differential equations of higher order* is such a book.

This book presents the general theory of Lyapunov functions (as well as the related subject of comparison theorems) as applied in the study of stability, boundedness, and periodicity. The general discussion of nth order differential systems is a standard treatment which one would expect to see in a book on Lyapunov stability theory. The unique feature of this book is an extensive treatment of third and fourth order differential equations.[4] For example, Chapter 4 is a 100 page treatment of the study of solutions of the third order equation

$$x''' + ax'' + \varphi(x, x') + f(x) = p(t)$$

where $p(t)$ is periodic in t.

Applied mathematicians, whose primary interest is in the analysis of the stability properties of rather specific models, may find this book to be quite valuable. As one surveys the literature one finds that the books on stability theory fall roughly into two broad classes. First, general books on the nth order systems, like Hahn [**15**], which are suitable for a overall introduction to stability. Such a book might be of interest to a beginner. The second class would consist of books like [**27**] and LaSalle and Lefschetz [**20**] which deal

[4] This book, which is an English translation of a German edition which first appeared in 1969, should not be confused with an earlier book [**27**] by the same authors. The emphasis in this earlier book was on second order equations.

extensively with second order equations. These books generally use special topological properties of the plane (such as the Poincaré-Bendixson theory) to derive information above the qualitative behavior of solutions of second order differential equations. Researchers, who are studying equations of higher order ($n \geqq 3$), may find the first type of book to be too general, and the second type of book to be too specific. Differential equations of higher order oftentimes require very special techniques and it is the hope of finding such techniques that would bring a reader to the Reissig–Sansone–Conti book.

At this point, the subject of mathematics tends to be more of an art than a science. While it is true that the material covered is completely rigorous, it is not possible for a reviewer to predict that a given technique will solve a given problem. All we can say is that this book contains an essentially complete discussion of the status of our knowledge of the qualitative behavior of solutions of third and fourth order equations. It is a good account of the subject, but it is intended for specialists only.

The final chapter of the book should be of interest to researchers in control theory. It consists of a rather complete discussion of the Lure problem together with the two techniques for solving this problem, viz. the technique of Lyapunov functions and a technique due to Popov which is based on Laplace transform methods. The book concludes with a brief discussion comparing these two techniques.

IV. **Equations with delays.** The introduction of functional analytic techniques into the study of differential equations has paid other dividends. Because of this, it then became quite natural to study the theory of differential equations in infinite dimensional spaces. In fact, the mathematical theory of quantum mechanics and the theory of semigroups of unitary operators grew out of such investigations. Applications of semigroup theory to partial differential equations and stochastic differential equations are familiar to many persons. We will not go into these applications here. Instead, we shall look at a theory that is more reminiscent of the methods and results of ordinary differential equations in finite dimensional spaces. This is the theory of ordinary differential equations with time delays.

A recent book by J. K. Hale, *Functional differential equations*, is of particular interest here because of the important contributions this book offers in the qualitative study of solutions of functional differential equations. The theory of functional differential equations, or more specifically, the theory of retarded functional differential equations, as presented in Hale's book, can be considered as a theory of a semi-flow on a Banach space $C=C([-r,0],R^n)$ where $[-r,0]$ denotes the basic delay interval. It is not quite accurate to consider this theory only as a theory of ordinary differential equations on the Banach space C. Retarded functional differential equations have more structure, and as a result, this theory is more closely related to the theory of ordinary differential equations in finite-dimensional spaces.

Many of the qualitative properties of ordinary differential equations in finite-dimensional spaces can be extended to the theory of retarded functional differential equations. This extension, which is by no means as simple as it is stated, is one of the recurring themes in Hale's book. Perhaps the most interesting version of this extension is the saddle property. By assuming that the spectrum misses the imaginary axis, the author shows that in the case of linear equations the phase space $C([-r, 0], R^n)$ splits into the direct sum of a stable space S and a finite-dimensional unstable space U. Furthermore, this splitting, as in the ordinary differential equation case, is preserved by small nonlinear perturbations.

Another major topic studied in Hale's book is the question of periodic solutions of some of the classical delay equations, including

$$x'(t) = -\alpha x(t-1)[1+x(t)]$$

as well as the delayed Lienard equation

$$x''(t)+f(x(t))x'(t)+g(x(t-r))=0.$$

This investigation is based on an abstract theorem describing eigenvectors for mappings on a cone. The reader will find this theory to be a beautiful application of functional analytic methods. Hale's book will make an excellent companion volume to the earlier book by Bellman and Cooke [**2**].

Unlike the theory of almost periodicity and the theory of stability, which were the topics of the other two books reviewed here, the theory of functional differential equations is relatively new. One finds it now in a rapidly changing state. The subject is changing faster than one can write. It may take another five or ten years before one can hope to see a definitive book on the subject. In the meantime, the book by Hale, together with Bellman and Cook [**2**], will serve as a very good introduction to the study of functional differential equations.

V. **Epilogue.** This rather brief account of the status of the qualitative theory of differential equations is by no means complete. Some selectivity was necessary because of the need to discuss the three books reviewed here. Many major topics in the qualitative theory have not even been mentioned, and others have been stressed beyond their relative importance. This undoubtedly reflects the personal biases of the reviewer, and for this I apologize.

The three books we have reviewed here have been, within the limits described above, attempts to present a reasonably up-to-date account of the state of the art in each of these areas. They have, in my opinion, been successful in this endeavor and represent important contributions to the literature.

There are other areas in the qualitative theory of differential equations which have gone through an important evolution during the past two decades. Perhaps it is time now to look for definitive books in the areas of Hamiltonian differential systems, differentiable dynamics, and the use of algebraical-topological techniques in the study of differential equations. I, for one, hope we see them soon.

REFERENCES

1. V. I. Arnol'd, *Proof of a theorem of A. N. Kolmogorov on the preservation of quasi-periodic motions under a small perturbation of the Hamiltonian*, Uspehi Mat. Nauk **18** (1963), no. 5 (113), 13–40 = Russian Math. Surveys **18** (1963), no. 5, 9–36. MR **29** #328.

2. R. E. Bellman and K. L. Cooke, *Differential-difference equations*, Academic Press, New York, 1963. MR **26** #5259; erratum, **27**, p. 1399.

3. G. D. Birkhoff, *Collected mathematical papers*. Vol. 1, Amer. Math. Soc., Providence, R. I., 1950; reprint, Dover, New York, 1968, pp. 654–751; Vol. 2, pp. 111–229, 283–332, 530–661. MR **38** #4271a,b.

4. ———, *Collected mathematical papers*. Vol. 2, Amer. Math. Soc., Providence, R.I., 1950; reprint, Dover, New York, 1968, pp. 295–332, 360–394, 398–408, 462–465, 713–717. MR **38** #4271b.

5. ———, *Collected mathematical papers*. Vol. 2, Amer. Math. Soc., Providence, R.I., 1950; reprint, Dover, New York, 1968, pp. 710–712. MR **38** #4271b.

6. ———, *Collected mathematical papers*. Vol. 2, Amer. Math. Soc., Providence, R.I., 1950; reprint, Dover, New York, 1968, pp. 295–332. MR **38** #4271b.

7. S. Bochner, *Beiträge zur Theorie der fastperiodischen Funktionen*. I, Math. Ann. **96** (1927), 119–147.

8. ———, *A new approach to almost periodicity*, Proc. Nat. Acad. Sci. U.S.A. **48** (1962), 2039–2043. MR **26** #2816.

9. H. Bohr, *Zur Theorie der fastperiodischen Funktionen*. I, II, III, Acta Math. **45** (1924), 29–127; **46** (1925), 101–214; **47** (1926), 237–281.

10. M. L. Cartwright, *Almost periodic flows and solutions of differential equations*, Proc. London Math. Soc. (3) **17** (1967), 355–380; corrigenda, ibid. (3) **17** (1967), 768. MR **36** #6697.

11. ———, *Almost periodic differential equations and almost periodic flows*, J. Differential Equations **5** (1969), 167–181. MR **39** #548.

12. L. Cesari, *Asymptotic behavior and stability problems in ordinary differential equations*, Ergebnisse der Mathematik und ihrer Grenzgebiete, N.F., Heft 16, Springer-Verlag, Berlin, 1959. MR **22** #9673.

13. J. Favard, *Leçons sur les fonctions presque-périodiques*, Gauthier-Villars, Paris, 1933.

14. P. Franklin, *Almost periodic recurrent motions*, Math. Z. **30** (1929), 325–331.

15. W. Hahn, *Stability of motion*, Die Grundlehren der math. Wissenschaften, Band 138, Springer-Verlag, New York, 1967. MR **36** #6716.

16. E. Hewitt and K. A. Ross, *Abstract harmonic analysis*. Vols. I, II, Die Grundlehren der math. Wissenschaften, Bände 115, 152, Academic Press, New York; Springer-Verlag, Berlin, 1963, 1970. MR **28** #158; **41** #7378; erratum, **42**, p. 1825.

17. E. L. Ince, *Ordinary differential equations*, Dover, New York, 1956. MR **6**, 65.

18. A. N. Kolmogorov, *On conservation of quasi-periodic motions for a small change in Hamilton's function*, Dokl. Akad. Nauk SSSR **98** (1954), 527–530. (Russian) MR **16**, 924.

19. E. R. van Kampen, *Almost periodic functions and compact groups*, Ann. of Math. **37** (1936), 78–91.

20. J. P. LaSalle and S. Lefschetz, *Stability by Liapunov's direct method, with applications*, Math. in Sci. and Engineering, vol. 4, Academic Press, New York, 1961. MR **24** #A2712.

21. W. Maak, *Fastperiodische Funktionen*, 2nd ed., Die Grundlehren der math. Wissenschaften, Band 61, Springer-Verlag, Berlin and New York, 1967. MR **35** #5860.

22. R. K. Miller and G. R. Sell, *Topological dynamics and its relation to integral equations and nonautonomous systems*, Proc. Conf. on Dynamical Systems, Providence, August, 1974 (to appear).

23. J. K. Moser, *Convergent series expansions for quasi-periodic motions*, Math. Ann. **169** (1967), 136–176. MR **34** #7888.

24. V. V. Nemyckiĭ and V. V. Stepanov, *Qualitative theory of differential equations*, OGIZ, Moscow, 1947; English transl., Princeton Math. Ser., no. 22, Princeton Univ. Press, Princeton, N.J., 1960. MR **10**, 612; **22** #12258.

25. J. von Neumann, *Almost periodic functions in a group*. I, Trans. Amer. Math. Soc. **36** (1934), 445–492.

26. L. S. Pontrjagin, *Continuous groups*, GITTL, Moscow, 1938; English transl., *Topological groups*, Princeton Math. Ser., no. 2, Princeton Univ. Press, Princeton, N.J., 1939. MR **1**, 44.

27. R. Reissig, G. Sansone and R. Conti, *Qualitative Theorie nichtlinearer Differentialgleichungen*, Edizioni Cremonese, Rome, 1963. MR **28** #1347.

28. G. R. Sell, *Topological dynamics and ordinary differential equations*, Van Nostrand Reinhold, London, 1971.

29. ———, *Topological dynamical techniques for differential and integral equations*, Ordinary Differental Equations, Academic Press, New York, 1972, pp. 287–304.

GEORGE R. SELL

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Fourier analysis on local fields, by M. H. Taibleson, Mathematical Notes, Princeton University Press, Princeton, New Jersey, 1975, xii+294 pp., \$7.00.

This book contains the lecture notes of a course given by the author at Washington University, Saint Louis during the Fall and Spring semester 1972–1973. Many results have appeared earlier in a series of papers, some of them written in collaboration with P. Sally, R. A. Hunt and K. Phillips. We find in this book well-known concepts from classical analysis: the Fourier transform, the Hankel transform, Gamma, Beta and Bessel functions, the Poisson summation formula, Fourier series, Césaro sums, fractional integration and many others. But from the title of the book it is clear that these subjects are treated here for a situation different from the classical one. In the classical case these subjects are discussed in the context of analysis on a euclidean space. In this book, however, the theory is developed for local fields, with emphasis, almost exclusively, on totally disconnected fields and on the analogy between this case and the euclidean case.

It is striking to observe the enormous evolution of the subject in two centuries, especially the revolution in the last fifty years. The great lines of this development are most interesting and they are an excellent illustration of the influence of algebra and topology on the form and contents of contemporary analysis.

Fourier series were studied by D. Bernoulli, D'Alembert, Lagrange and Euler from about 1740 onwards. They were led by problems in mathematical physics to study the possibility of representing a more or less arbitrary function f with period 2π as the sum of a trigonometric series of the form

$$\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx+b_n\sin nx).$$

Dirichlet (1829) and later Riemann (1854) started the study of these series in a more rigorous way. This was continued by Cantor—who showed that a