11. H. Matsumura, *Commutative algebra*, Benjamin, New York, 1970. MR 42 #1813.

12. S. McAdam, *Going down and open extensions*, Canad. J. Math. (to appear).

13. I. J. Papick, Ph. D. Dissertation, Rutgers University, New Brunswick, N. J., 1975.

14. M. Raynaud, *Anneaux locaux henséliens*, Lecture Notes in Math., vol. 169, Springer-Verlag, Berlin and New York, 1970. MR 43 #3252.

15. F. Richman, *Generalized quotient rings*, Proc. Amer. Math. Soc. 16 (1965), 794–799. MR 31 #5880.

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G-TRANSVERSALITY

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Let G be a compact Lie group and N, M and $Y \subset M$ be smooth G manifolds. Suppose $f: N \longrightarrow M$ is a proper G map. We give an obstruction theory (Theorem 1) for a proper G homotopy between f and a map g transverse to Y written $f \pitchfork Y$. In this generality we cannot say more; however, when $f: N \longrightarrow M$ is a quasi-equivalence of G vector bundles over Y, this can be considerably improved (Theorem 2) by removing the dependence of the map f. By definition f is a quasi-equivalence if N and M are G vector bundles over Y and f is proper, fiber preserving and degree 1 on fibers. *To be concise we suppose G is abelian* and omit applications and insights, referring to [1] and [2] for further information.

Let K be a subgroup of G and $\hat{K}$ the set of real irreducible K modules. If Γ and Ω are real K modules, let $V_{\Gamma,\Omega}$ denote the space of surjective real K homomorphisms of Γ to Ω. By Schur's lemma $V_{\Gamma,\Omega} = \Pi_{\psi \in \hat{K}} V^{\psi}_{\Gamma,\Omega}$ where $V^{\psi}_{\Gamma,\Omega}$ has the homotopy type of the Stiefel manifold of b_ψ frames in the D_ψ vector space of dimension a_ψ. Here D_ψ is the division algebra of real K endomorphisms of ψ and $\Gamma = \Sigma_{\psi \in \hat{K}} a_\psi \psi$, $\Omega = \Sigma_{\psi \in \hat{K}} b_\psi \psi$.

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Let L denote the set of isotropy groups of the action of G on N partially ordered by inclusion. If L has T elements, choose a 1-1 function α from L to the integers 1 through T with the property that $\alpha(K) < \alpha(H)$ if $K > H$. Suppose f is transverse to Y on $\bigcup_{\alpha(H)<k} N^H = Z_{k-1}$ and $\alpha(K) = k$. Without loss of generality, we may suppose $f^K \pitchfork Y^K$ and define $X^K = (f^K)^{-1} Y^K$ where Y^K is the fixed set of K acting on Y. The G normal bundle of Y in M is denoted by $\nu(Y, M)$. Define $\nu(Y, M)_K$ to be the G complement of $\nu(Y^K, M^K)$ in $\nu(Y, M)|_{Y^K}$. Define a function $V(K)$ from the set of components of X^K to topological spaces whose value at a component C of X^K is $V_{\Gamma,\Omega}$. For p a point in the component C of X^K, $\Gamma = \nu(N^K, N)|_p$ and $\Omega = \nu(Y, M)_{K|p}$. Set $X_K = \bigcup_{H>K;H\in L} X^H$.

THEOREM 1. *There is a sequence of obstructions*

$$O_*(K) \in H^*(X^K/G, X_K/G, \pi_{*-1}(V(K)))$$

(in the sense that $O_j(K)$ is defined if $O_i(K) = 0$ for $i < j$) whose vanishing implies f is properly G homotopic rel Z_k *to a function transverse to Y on Z_k.*

THEOREM 2. *Let $f'\colon N \longrightarrow M$ be a quasi-equivalence of G vector bundles over Y. Suppose f is propertly G-homotopic to f' and is transverse to Y on Z_{k-1} and $f^k \pitchfork Y^K$. There are obstructions*

$$O'_*(K) \in H^*(Y^K/G, Y_K/G, \pi_{*-1}(V'(K)))$$

whose vanishing implies f is properly G homotopic rel Z_k *to a function transverse to Y on Z_k. (Here $V'(K)$ is a function of the components of Y^K.)*

REFERENCES

1. T. Petrie, *Obstructions to transversality for compact Lie groups*, Bull. Amer. Math. Soc. **80** (1974), 1133–1136.

2. ———, *Equivariant quasi-equivalence, transversality and normal cobordism*, Proc. Internat. Congress Math. (Vancouver, 1974), Canadian Mathematical Congress (to appear).

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