BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 1, January 1975

# A STRONG NONIMMERSION THEOREM FOR $RP^{8l+7}$

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Communicated by Edgar Brown, Jr., August 13, 1974 [2]

In this paper we shall sketch the proof of a nonimmersion theorem for real projective spaces of dimension $8l + 7$ which is conjectured to be best possible. Details will appear elsewhere.

THEOREM. *Let* $\alpha(n)$ *denote the number of* 1*'s in the binary expansion of* $n$. *Let*

$$\beta(n) = \begin{cases} 2\alpha(n) & \text{if } \alpha(n) \equiv 1, 2(4), \\ 2\alpha(n) + 1 & \text{if } \alpha(n) \equiv 0(4), \\ 2\alpha(n) + 2 & \text{if } \alpha(n) \equiv 3(4). \end{cases}$$

*If* $n \equiv 7(8)$, *then* $\mathbf{R}P^n \not\subseteq \mathbf{R}^{2n-\beta(n)}$.

This result was announced in [4] but difficulties [2] were found in the argument sketched there. It was conjectured in [4] that if $n \equiv 7(8)$, then $\mathbf{R}P^n \subseteq \mathbf{R}^{2n-\beta(n)+1}$. Using techniques of [1] we have proved these immersions when $\alpha(n) = 5, 6, 8$, or 9 (unpublished), thus establishing the precise immersion dimension in these cases.

It is well known that the theorem is equivalent to showing that the map $\mathbf{R}P^n \xrightarrow{f} BSp$ which classifies $(2^L - n - 1)\xi_n$ does not lift to $\widetilde{BSp}_{n-\beta(n)}$ [1] (where $L$ is any sufficiently large integer). We prove the nonexistence of this lifting by showing that a $bo$-secondary obstruction is nonzero with zero indeterminacy.

As in [5] we define $bo_i^4$ to be connective $\Omega$-spectrum whose $(8k + 4 - i)$th-space is $BO(8k + 4, \infty)$ localized at 2. Let $\theta: bo \longrightarrow bo_4^4$ be the map inducing the Adams operation $\psi^3 - 1$, and let $bJ$ denote the

*AMS (MOS) subject classifications* (1970). Primary 55G45, 57A35; Secondary 55B20, 55D15, 55F05.

*Key words and phrases.* Immersions of projective spaces, obstruction theory, modified Postnikov towers,

[1] Both authors supported in part by NSF Grant GP25335.

[2] Originally received July 2, 1974.

fibre of $\theta$. Let $B_N^0 = \widetilde{BSp}_N \wedge_{BSp} bo$ denote the space which was called $E_N^0$ in [1], and similarly define $B_N^J = \widetilde{BSp}_N \wedge_{BSp} bJ$.

In stable dimensions ($\leqslant 2N$), using techniques of [1], [5] and [6], we can form the first two stages of a $bo$-resolution of $V_N \longrightarrow \widetilde{BSp}_N \longrightarrow BSp$

$$\begin{array}{ccccc} & & B_N^J & & \\ & & \downarrow & & \\ V_N \wedge bo & \xrightarrow{i} & B_N^0 & \xrightarrow{c_1} & V_N \wedge bo_4^4 \\ & & \downarrow & & \\ \mathbf{R}P^n & \xrightarrow{f} & BSp & \xrightarrow{c_0} & \Sigma V_N \wedge bo, \end{array}$$

where $c_1 \circ i = 1 \wedge \theta$.

Let $N = n - \beta(n)$. The theorem is proved by showing that there is a lifting of $f$ to $B_N^0$ which does not lift to $B_N^J$, and that the indeterminacy $(1 \wedge \theta)_*$: $[\mathbf{R}P^n, V_N \wedge bo] \longrightarrow [\mathbf{R}P^n, V_N \wedge bo_4^4]$ is zero. To prove the non-lifting to $B_N^J$ we construct an $(n-1)$-modified Postnikov tower [3], $B_N^J \longrightarrow E_r \longrightarrow \cdots \circ \longrightarrow E_1 \longrightarrow BSp$ and show using [1, Theorem 1.8] that $\mathbf{R}P^{n-1}$ lifts to $E_{\alpha(n)-3}$. Using the Serre spectral sequence we show that the map of 7-connected coverings $B_N^J(8, \infty) \longrightarrow E_{\alpha(n)-3}(8, \infty)$ is induced through dimension $n-1$ by a map $E_{\alpha(n)-3}(8, \infty)^{(n-1)} \xrightarrow{\bar{c}} Y$, and if $\tilde{f}: \mathbf{R}P^{n-1} \longrightarrow E_{\alpha(n)-3}(8, \infty)$ is a lifting, then we show that $\bar{c}\tilde{f}$ is nontrivial, so $\tilde{f}$ does not lift to $B_N^J(8, \infty)$. This is then used to show that a lifting to $B_N^0$ does not lift to $B_N^J$.

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