five countries. Plainly Professors Gillman and Jerison have written a standard work, which will undoubtedly be apotheosized with the passage of time to the status of a classic.

EDWIN HEWITT

Theory of Markov processes. By E. B. Dynkin (translated from the Russian by D. E. Brown, edited by T. Kovary). Prentice-Hall Inc., Englewood Cliffs, N. J. and Pergamon Press, Oxford-London-Paris, 1961. 9+210 pp. $11.95.

Die Grundlagen der Theorie der Markoffschen Prozesse. By E. B. Dynkin. (Die Grundlehren der Mathematischen Wissenschaften, Band 108), translated from the Russian by Joseph Wloka. Springer Verlag, Berlin-Göttingen-Heidelberg, 1961. 12+174 pp. D.M. 29.80.

These are translations of the first of two books by the author on Markoff processes. The second, concerning relationships between these processes and semi-groups of linear operators, will appear in Russian shortly.

There has been much work in recent years on continuous time-parameter Markoff processes in an arbitrary state space and on their relationships with certain objects in analysis, but, until now, no one has put forward a single unified framework within which all this research could be carried on. Such a structure is provided here, and its description is carried out in a systematic and skillful manner.

Although the book is formally self-contained, its real prerequisites are a knowledge of general measure theory together with the amount of the theory of Markoff processes to be found in the books of Feller and Gnedenko. Even a reader with this background may find the reading tedious, for while he will be able to follow the formal development, he will get no glimpse of the interesting research to which this extensive measure-theoretic apparatus is appropriate. Thus, the book tends to look like one hundred and seventy pages of preliminaries. A few comments indicating the applications and related research would have added little to the length of the volume and much to the enjoyment of the reader. Viewed, however, as a rigorous survey of the foundations, this book is a complete success.

Chapter One contains a survey of the necessary measure-theoretic facts, including conditional expectation.

In Chapter Two the general concept of a Markoff process is introduced. A transition function is assumed so that rather than having one measure on the sample space we have a family of them, one for each starting point (and for each time in the non-time-homogeneous case). The distinction is made quite explicitly between this approach and the customary one in which the basic measure is regarded as fixed and the transition function is regarded as an auxiliary datum. Workers in the field have always mentally used the two approaches interchangeably. In addition the processes may have a random length of life, that is a typical path of the process is defined on the interval $[0, \zeta)$ where ζ is a positive (possibly infinite) random variable having certain extra measurability properties and called the terminal time. Other useful generalizations are given as well. Processes with the same transition function are called equivalent. The author considers transformations of the sample space which change a process into an equivalent one, and offers as a substitute (in this case) for Doob's separability, a theorem saying when a process can be replaced by an equivalent one whose sample functions have prescribed regularity properties. The definition and properties of time-homogeneous processes are obtained by specialization from the general case with a resulting loss in ease of reference for people only interested in the time-homogeneous case. Only a few extra pages would have been required for a separate treatment of the latter.

Chapter Three presents the author's research on multiplicative functionals. Starting with a process, a new Markoff process, called a subprocess, is obtained by suitably decreasing the original terminal time. Previously, this was done constructively by extinguishing a path of the original process when it first hit some fixed set or by imposing a death rate on it. Dynkin extends these operations by requiring merely that for the subprocess proceeding from time s, the conditional probability that the terminal time exceeds $t>s$, given the entire development of the original process from time s on, be given by a random variable α_t^s which depends only on the behavior of the original process in the time interval $[s, t]$. From this requirement, the relationship $\alpha_t^s\alpha_u^t=\alpha_u^s$ $(s<t<u)$ follows, and therefore α is called a multiplicative functional. The regularity properties of multiplicative functionals and their relation to the subprocesses are investigated in detail. Since the appearance of the Russian edition of this book, some interesting new research has appeared on the structure and use of these functionals.

Chapter Four contains the standard theorem on the existence of a Markoff process with prescribed transition function.

Chapter Five is concerned with the strong Markoff property. In terms of the time-homogeneous case, a positive random variable T is "independent of the future" if for every constant t the occurrence or not of the event $T<t$ is determined by the behavior of the process only in the time interval $[0, t]$. It has been a widely applied heuristic principle that with regard to Markoffian properties such random variables behave like constants. Processes for which this principle is actually valid are said to have the strong Markoff property or to be strong Markoff processes. Dynkin describes the situation rigorously and derives the standard sufficient conditions for a process to be a strong Markoff process. The development is more than adequate for almost all applications. There has been interesting research on minimal conditions under which the strong Markoff property holds, in which even the formulation of the problem is a delicate matter and the methods come from several parts of analysis. Most of these results appeared after publication of the Russian edition.

Chapter Six treats the problem of replacing a process by an equivalent one whose sample functions have specified boundedness or continuity properties. The author's approach is via the theorem of Chapter Two, so that he obtains the standard theorems in a more systematic manner than have other authors, who used separability.

An Appendix contains a proof of a theorem of Choquet on capacitability with applications, following Hunt, to measurability problems connected with the time of first hitting a set.

There is a useful index of notations, in addition to the usual index.

We note that the terminology and notations of this book have become standard for Russian research in Markoff processes, so that those wishing to follow these developments will find this book a necessary reference.

R. M. BLUMENTHAL