BOOK REVIEWS

The theory of groups. By Marshall Hall, Jr., New York, MacMillan Co., 1959. 13+434 pp. $8.75.

When, in 1911, W. Burnside published the second edition of his *Theory of groups of finite order*, his work contained all the essential group theoretical knowledge of his time with the exception of the theory of continuous groups (as they were called fifty years ago). In spite of the large number of results found and of methods developed since Burnside's time, Hall's *Theory of groups* can claim to be a "Burnside brought up-to-date." Clearly, this cannot mean any more an account of all things known which now would require a book of thousands of pages. But Hall introduces the reader to the most important concepts and methods available in group theory (outside of the theory of Lie groups and topological groups), and he leads him in many cases to the frontier of our knowledge. Almost throughout, complete proofs are given for all results stated as theorems. Notable exceptions are Theorems 17.2.1 (p. 314) on the standard form of an element in a free product with amalgamated subgroups and 18.4.8 (p. 337) on the complete solution of Burnside's problem for exponent 6 by Higman, P. Hall and the author. However, it is not astonishing that these exceptions occur but that they are so rare.

The first four chapters of the book deal with definitions, homomorphisms, direct and Cartesian products, Sylow theorems, elementary results on abelian groups and the basis theorem for finite abelian groups. Although all of this is standard material which has been presented many times in many places, the author approaches it in a new and very efficient manner; at the end, he is able to discuss the groups of order p, p^2, pq, p^3 in three pages. Abelian groups are taken up again and discussed more fully in Chapter 13.

The fifth chapter on permutation groups is a brief but important account of this subject, including a discussion of the alternating group and a proof of a generalization of a theorem of Jordan referring to four-fold transitive groups. The chapter ends with a discussion of the wreath product and the Sylow subgroups of the symmetric groups.

Chapter 6 on automorphisms and Chapter 8 on lattices and composition series contain basic facts; the topic of Chapter 8 is taken up again in Chapter 14 on lattices and subgroups. It ends with the proof of a theorem due to Iwasawa which characterizes as supersolvable the finite groups in which all maximal subgroup chains have the same length.

Chapters 7, 11 and 17 deal respectively with free groups, basic commutators and free and amalgamated products. In a sense, they belong together, although they are separated for valid reasons of expediency. These chapters contain a proof of Schreier's Theorem on the subgroups of free groups, an important reduction process due to J. Nielsen, a presentation of Philip Hall's collecting process for commutators and the author's remarkable proof of the basis theorem on commutators. Chapter 17 contains a proof of Kurosh's theorem about the subgroups of free products, and touches briefly on free products with amalgamated subgroups.

Again, Chapters 9, 10, 12 and 18 belong together, dealing with solvable, supersolvable, nilpotent and p-groups, and with the Burnside Problem. Throughout, full attention is given to the results found and the ideas introduced by Philip Hall, and the proofs include those of numerous rather recent results.

Chapters 14 and 16 deal with monomial representations, applications of the concept of transfer and with the theory of group representations in a field the characteristic of which is coprime with the order of the group. Chapter 16 (on group representation) is a straightforward account which should be particularly useful since it occurs frequently that a concise, self-contained presentation of this theory is needed which does not involve any unnecessary elements.

Chapter 15 on group extensions and cohomology of groups introduces the very first beginnings of cohomology theory but it gives the important application to extension theory.

The final chapter (no. 20) on group theory and projective planes is a surprise even to a reader who, through the previous chapters, has been well acquainted with the author's ability to present many results in a streamlined form. It brings proofs of many results which have been published not long before the author's book came out, and merely going through the definitions and theorems makes fascinating reading.

Hall's work is well equipped with exercises, many of which are far from trivial and require the proof of results which supplement the general theory in an interesting manner. References are given only where the paper in question is actually used in the text or where a source for more advanced information is given. For instance, the monograph by Kaplansky on infinite abelian groups and Kurosh's book on group theory are quoted for the reader who wishes to learn more about abelian groups. The reviewer would like to add two references of this type to those given by the author, namely B. H. Neumann's *Essay on free products of groups with amalgamations*, Philos. Trans. Roy. Soc. London no. 919 vol. 246 (1954) pp. 503–554 and M. Lazard's paper *Sur les groupes nilpotents et les anneaux de Lie*, Ann. Sci. Ecole Norm. Sup. (3) vol. 71 (1954) pp. 101–190. These papers may also be considered as monographs on the subjects described in their titles.

The author states that a knowledge of Birkhoff and MacLane's *Survey of modern algebra* will enable a student to read his book. This is true, with respect to both factual knowledge and training in abstract thinking. However, the style of the book is concise and demanding like that of a research paper,— a well written and lucid research paper, but with few "asides" and with little leisurely exposition. (Incidentally, the same may be said about large portions of Burnside's book.) A student who studies Hall's book should find himself well equipped with both the mathematical background and maturity required for the reading of current literature in group theory.

WILHELM MAGNUS

Approximate methods of higher analysis. By L. V. Kantorovich and V. I. Krylov. Translated from the fourth Russian edition by Curtis D. Benster. New York, Interscience Publishers, 1958. xii+681 pp. $17.00.

The publication of an English translation of the remarkable work of Kantorovitch and Krylov is highly to be commended. The first Russian edition appeared in 1936; several subsequent editions, incorporating only minor revisions, appeared from 1941 on. Thus the material is not new. Yet a great part of it has been relatively inaccessible in Russian journals. The remarkably clear and thorough exposition would justify publication of an English version in any case.

The scope of the book is not broad. Its purpose is to study a few approximate methods for solution of the boundary value problems of mathematical physics, to provide in most cases a complete mathematical treatment of the methods, and to expound in full detail the technique of applying them to specific numerical problems. The authors make only brief allusions to digital computers, but it is evident that their procedures have many applications to solution of problems on the modern high-speed machines.

The principal methods studied are the following: series of orthogonal or nonorthogonal functions, infinite systems of simultaneous equations, solution of integral equations by successive approximations and by replacement by finite systems of algebraic equations,