act effectively on a manifold. The last topic is the authors' theorem that a compact effective group on 3-space E^3 is linear (orthogonal) in suitable coordinates, with an interpretation of this result for the axiomatic foundation of Euclidean geometry of E^3.

The first four chapters are essentially self-contained, up to general mathematical education and some references to special topics. But it is true of course, no surprise with a subject as complex as the one under consideration, that actually a good deal of sophistication and preparation (or perseverance) will be required for appreciation of the material.

Usually things are spelled out in detail, in an almost conversational style. The authors have not aimed at maximum elegance or brevity in their presentation; e.g., some theorems carry unnecessarily strong hypotheses. Occasionally, particularly in the last part, the reader is asked to supply a good deal of the argument.

In both of its main parts the book leads close to present day work; it constitutes a rich source of facts, techniques of all kinds, and references, for anybody who is actively interested in the subject; it will be of great value to beginning and mature mathematicians alike, and is therefore a very welcome addition to the mathematical literature.

HANS SAMELSON

Théorie globale des connexions et des groupes d'holonomie. By A. Lichnerowicz. Edizioni Cremonese, Roma, 1955. 15+282 pp. 4000 lire.

This is a very timely book on the modern theory of connections. The classical theory was mainly initiated by Levi-Civita and Schouten and received, partly because of its applications to the general theory of relativity an extensive development. Elie Cartan observed, from his effective applications of the method of moving frames to various geometrical problems, that the group concept is the basic underlying idea. He knew many examples and had on the basis of this knowledge all the important notions of a general theory, but did not have the tools and terminology to express them. In fact, his "tangent space" is a fiber in the modern terminology, and his space of moving frames is what is now called a principal fiber bundle, etc. This is not to minimize the contributions of modern geometers (Ehresmann, Weil, H. Cartan, Chern, Ambrose, Singer, etc.), whose efforts have made what was once a difficult subject into a beautiful theory. It is now the considered opinion that in differential geometry a connection is a concept pertaining to a principal fiber bundle.

The book is divided into five chapters. Chapters I and II give a

lucid account of the basic notions of manifolds and connections. The treatment seems to take the middle road between modern abstraction and classical nonintrinsic formulation. Whenever necessary the author never hesitates to use local coordinates and to make a choice of coordinate frames. In view of applications this attitude can be fully justified. Chapter III studies holonomy groups, to whose foundations the author himself has made important contributions. The holonomy group is a very natural notion in the theory of connections. However, recent investigations by M. Berger and I. M. Singer (Berger, Bull. Soc. Math. France vol. 83 (1955) pp. 279–330; Singer, to be published) have shown that its possibilities are rather limited. Except for homogeneous spaces it is perhaps not a strong invariant. The chapter ends with a theorem of de Rham which states that a complete simply-connected reducible Riemannian manifold is a topological product.

Chapter IV gives an introduction to the theory of harmonic forms of Hodge. The emphasis is on the formal and algebraic aspects of the theory and does not include a proof of the fundamental theorem. The author is mainly interested in the applications of the fundamental theorem to differential geometry. As a generalization of Kähler geometry the author studies the case of a compact orientable Riemannian manifold on which there exists an exterior differential form with covariant derivative equal to zero. If k is the degree of this differential form F, there is defined a sequence of operators K_h, $0 \leq h \leq k$, such that K_0 is the exterior product and K_k the interior product by F. The aim of the computations is to prove that they commute with the Laplacian operator. From this, results are derived on properties of the Betti numbers of a compact Riemannian manifold on which such a form F exists.

As Hodge's pioneering work has shown, a finer analysis of the global properties of a compact manifold is possible, if the manifold has a Kähler metric. A Kähler metric is usually defined to be a positive definite Hermitian metric $ds^2 = \sum_{k,l} h_{kl} dz^k d\bar{z}^l$ with the property that the associated exterior two-form $i \sum_{k,l} h_{kl} dz^k \wedge d\bar{z}^l$ is closed. What accounts for Hodge's theorems on compact Kähler mainfolds can, in the reviewer's opinion, be briefly described by the commutativity of the following diagram

$$\text{Hermitian metric} \longrightarrow \text{Riemannian metric}$$
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$
$$\text{Associated connection} \rightarrow \text{Levi-Civita connection}$$

In fact, one defines from an Hermitian metric a connection with the

unitary group as the structure group; the condition that it reduces to the Levi-Civita connection of the Riemannian metric arising from the given Hermitian metric is equivalent to the Kähler condition. That such commutativity should be basic follows from the fact that the harmonic forms are defined relative to the Riemannian metric. Chapter V starts with an introduction to almost complex and complex manifolds and has as main purpose the derivation of Hodge's theorems. They are formulated in terms of pseudo-Kählerian manifolds; the proofs are identically the same as in the Kählerian case. Recently, L. Nirenberg and A. Newlander succeeded in proving that local complex coordinates can be introduced in a differentiable integrable almost complex structure. As a side result it seems that the adjective "pseudo" can now be dropped in the terminology of these classes of manifolds.

The book should be considered as an account of the author's own researches with sufficient introductory material and does not aim at completeness. For instance, two important topics, characteristic classes and locally flat spaces, are hardly touched. With this in mind the book is highly recommendable as an introduction to modern differential geometry.

<div align="right">S. S. Chern</div>

RESEARCH PROBLEMS

1. Richard Bellman: *Differential equations.*

Consider the Sturm-Liouville problem:

$$u'' + \lambda(f(x) + \epsilon g(x))u = 0, \qquad 0 < x < 1,$$
$$u(0) = u(1) = 0,$$

where $f(x)$ and $g(x)$ are continuous functions over $[0, 1]$ with positive minima.

Let us regard λ_1, the smallest characteristic value, as a function of ϵ. Is it true that λ_1 is an analytic function of ϵ for $R(\epsilon) \geq 0$? In general, where is the singularity nearest the origin ($\epsilon = 0$)? (Received September 24, 1956.)