BOOK REVIEWS

Homological algebra. By Henri Cartan and Samuel Eilenberg. Princeton, The Princeton University Press, 1956. 15+390 pp. $7.50.

At last this vigorous and influential book is at hand. It took nearly three years from completed manuscript to bound book; Princeton is penalized 15 yards for holding.

Homological algebra deals both with the homology of algebraic systems and with the algebraic aspects of homology theory. The first topic includes the homology and cohomology theories of groups, of associative algebras, and of Lie algebras. The second topic includes the care and feeding of exact sequences and spectral sequences, as well as the manipulation of functors of chain complexes. For example, the Künneth problem reads: Given the homology of complexes K and L, what is the homology of $K\otimes L$? Again, the universal coefficient problem reads: Given a group G and the homology of a complex K, what is the homology of the complexes $K\otimes G$ and Hom (K, G)? These problems and these two functors, tensor product and Hom, are treated not just for groups, but in proper generality for left modules over an arbitrary ring Λ. Explicitly, if A and G are such modules, Hom_Λ (A, G) denotes the group of Λ-module homomorphisms of A into B. When G is a right Λ-module and A a left module—a situation denoted neatly as $(G_\Lambda, {}_\Lambda A)$—the tensor product taken over Λ is written as $G\otimes_\Lambda A$. A Λ-complex K is a graded differential left Λ-module; its homology $H(K)$ is the usual graded module $H(K) = \sum H_n(K)$, it has the usual definition and an unusual definition (Chap. IV), dual to the usual one.

The various aspects of homological algebra all meet in the notion of a projective resolution (Chap. V). A left module P is *projective* (Chap. I) if any homomorphism of P into a quotient module B/C can be "lifted" into a homomorphism of P into B. (Thus a free module is projective, but not necessarily vice versa.) A projective resolution of A is an exact sequence $\cdots \to X_n \to X_{n-1} \to \cdots \to X_0 \to A \to 0$ composed of A and a complex X which consists of projective modules X_n, $n=0, 1, \cdots$. Given two such resolutions X and X' for the same A, the familiar method of climbing up one dimension at a time provides a chain transformation of X into X' and proves X and X' chain equivalent. Given any module G_Λ the homology groups $H(G\otimes_\Lambda X)$ are therefore independent of the choice of the resolution of A and depend only on G and A; they are called the torsion products and are denoted (Chap. VI)

$$\mathrm{Tor}_n^{\Lambda}(G, A) = H_n(G \otimes_{\Lambda} X), \qquad n = 0, 1, \cdots;$$

in particular, $\mathrm{Tor}_0(G, A) = G \otimes_{\Lambda} A$. Similarly, given any ${}_{\Lambda}G$ the cohomology groups $H(\mathrm{Hom}_{\Lambda}(X, G))$ depend only on A and G and are written

$$\mathrm{Ext}_{\Lambda}^{n}(A, G) = H^{n}(\mathrm{Hom}_{\Lambda}(X, G)), \qquad n = 0, 1, \cdots;$$

in particular $\mathrm{Ext}^0(A, G)$ is $\mathrm{Hom}_{\Lambda}(A, G)$. Any module homomorphism $\gamma: A \to B$ extends to a map of a resolution of A to a resolution of B; hence each Tor_n is a covariant functor of its arguments G and A, while Ext^n is contravariant in A and covariant in G.

These two functors now apply to the typical problems of homological algebra. Let Λ be a hereditary ring (i.e., every left or right ideal is projective as a module). In the Künneth problem, if K and L are complexes of projective modules, there is an exact sequence

$$(1)\quad 0 \to H(K) \otimes_{\Lambda} H(L) \xrightarrow{\alpha} H(K \otimes_{\Lambda} L) \xrightarrow{\beta} \mathrm{Tor}_1^{\Lambda}(H(K), H(L)) \to 0,$$

the sequence splits, and the natural homomorphisms α and β have degrees 0 and 1, respectively. The same sequence, with L a module (differentiation zero), solves the universal coefficient problem for homology, under somewhat weaker conditions, while for the coefficient problem in cohomology one has the exact sequence

$$0 \to \mathrm{Ext}_{\Lambda}^{1}(H(K), G) \xrightarrow{\alpha'} H(\mathrm{Hom}_{\Lambda}(K, G)) \xrightarrow{\beta'} \mathrm{Hom}_{\Lambda}(H(K), G) \to 0,$$

valid under similar conditions, with maps α' and β' of degrees 1 and 0, respectively. This sequence is a case of a "Cokünneth" Theorem (Theorem 3.1.a of Chap. VI) for $\mathrm{Hom}_{\Lambda}(K, L)$; this theorem, insufficiently emphasized by the authors, contains in particular a homotopy classification of maps of K onto L.

Exact sequences can be grown. If $0 \to A \to B \to C \to 0$ is a given exact sequence of left modules, then for each right module G the derived sequence

$$0 \to G \otimes_{\Lambda} A \to G \otimes_{\Lambda} B \to G \otimes_{\Lambda} C \to 0$$

in general fails to be exact at $G \otimes_{\Lambda} A$. Exactness returns when the left-hand zero is replaced by $\mathrm{Tor}_1(G, C)$, and the sequence is continued to the left as

$$(2)\quad \cdots \to \mathrm{Tor}_{n+1}^{\Lambda}(G, C) \xrightarrow{\theta} \mathrm{Tor}_n^{\Lambda}(G, A) \to \mathrm{Tor}_n^{\Lambda}(G, B) \to \mathrm{Tor}_n^{\Lambda}(G, C) \to$$

where θ is a suitable "connecting homomorphism." Similarly, given ${}_{\Lambda}G$, the derived sequence

$$0 \to \mathrm{Hom}_\Lambda (C, G) \to \mathrm{Hom}_\Lambda (B, G) \to \mathrm{Hom}_\Lambda (A, G) \to 0$$

becomes exact when the right-hand zero is replaced by $\mathrm{Ext}^1_\Lambda (C, G)$, and the sequence is continued to the right with the higher Ext^n functors.

The homology of various algebraic systems can be written with torsion products. To get the homology of a group Π (Chap. X), take Λ to be the integral group ring $Z(\Pi)$ and regard the ring Z of integers as the left $Z(\Pi)$-module for which $xm = m$ for each integer m and each $x \in \Pi$. For modules $G_{Z(\Pi)}$ and ${}_{Z(\Pi)}A$ the homology and cohomology groups of Π are then defined by

$$(3) \qquad H_n(\Pi, G) = \mathrm{Tor}_n^{Z(\Pi)}(G, Z), \qquad H^n(\Pi, A) = \mathrm{Ext}^n_{Z(\Pi)}(Z, A).$$

The latter are the cohomology groups of Π originally defined by Eilenberg-Mac Lane in terms of a standard complex. This complex can be viewed as a particular projective resolution of the $Z(\Pi)$-module Z; for special Π the use of other resolutions expedites the calculation of H^n (e.g., for Π cyclic see Chap. XII, §7).

To get the homology of an associative algebra Γ over a commutative ring K (Chap. IX), the authors first turn bimodules into left modules. More exactly, let Γ^* be the algebra anti-isomorphic to Γ and set $\Gamma^e = \Gamma \otimes_K \Gamma^*$; any two-sided Γ-module A can then be interpreted as a left Γ^e-module. The homology and cohomology of Γ is now defined as

$$(4) \qquad H_n(\Gamma, A) = \mathrm{Tor}_n^{\Gamma^e}(A, \Gamma), \qquad H^n(\Gamma, A) = \mathrm{Ext}^n_{\Gamma^e}(\Gamma, A).$$

These are the groups defined originally by Hochschild, who used a standard complex which may be regarded as a projective resolution of the Γ^e-module Γ.

To get the homology of a Lie algebra L over a commutative ring K (Chap. XIII), first take the enveloping associative algebra L^e, defined as the tensor algebra T of the K-module L modulo the ideal spanned by all $a \otimes b - b \otimes a - [a, b]$ for $a, b \in L$ and $[a, b]$ the bracket product in L. The augmentation $\epsilon: T \to K$ of the tensor algebra (identity on scalars, zero on elements of L) induces an algebra homomorphism $\epsilon: L^e \to K$, and K becomes a left L^e-module with operators $xk = \epsilon(x)k$ for $x \in L^e$ and $k \in K$. The rôle of L^e is that each left representation module of the Lie algebra L is a left L^e-module, and vice versa. For modules in the situation $(A_{L^e}, {}_{L^e}C)$ one then defines the homology and cohomology groups of L as

$$(5) \qquad H_n(L, A) = \mathrm{Tor}_n^{L^e}(A, K), \qquad H^n(L, C) = \mathrm{Ext}^n_{L^e}(K, C).$$

In case L is free as a K-module, the Birkhoff-Witt theorem shows that these groups agree with those originally defined by Chevalley-Eilenberg, using a more complicated complex constructed directly from L.

There is a striking parallel between the formulas (3), (4), and (5); this the authors exploit by a new notion of an augmented ring (of which more later). In the first and third cases the ring $\Lambda = Z(\Pi)$ or $\Lambda = L^e$ is an algebra over Z or K respectively. These two cases are unified in terms of supplemented algebras. A *supplemented algebra* (Chap. X) is an algebra Λ over a commutative ring K together with a K-algebra homomorphism $\epsilon:\Lambda \rightarrow K$. This ϵ induces on K a left Λ-module structure; the homology groups of the supplemented algebra Λ are defined in terms of this structure for each A_Λ as $\mathrm{Tor}_n^\Lambda (A, K)$. In case Λ is K-free, these groups are isomorphic (Theorem 2.1) to the Hochschild homology groups of Λ, with A regarded, through ϵ, as a bimodule. By this token the homology either of groups or of (K-free) Lie algebras can be regarded as special cases of the Hochschild homology.

This brief outline of homological algebra does not adequately represent the generality of the treatment in *Homological algebra*. For example, much of the discussion is carried out for arbitrary covariant functors $T(A)$ which are *additive* (Chap. II) in the sense that $T(\alpha_1+\alpha_2) = T(\alpha_1)+T(\alpha_2)$ for any sum of module homomorphisms α_1 and α_2 (a condition which implies that $T(A+B) = T(A)+T(B)$ for a direct sum of modules A and B). Any such functor has a left and a right satellite functor (Chap. III). For example, the left satellite $S_1T(A)$ is found from any exact sequence $0 \rightarrow M \rightarrow P \rightarrow A \rightarrow 0$, P projective, as the kernel of $T(M) \rightarrow T(P)$, and is independent of the choice of that sequence, while the right satellite is defined dually (reverse all arrows). For any exact sequence $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ there is a connecting homomorphism $S_1T(C) \rightarrow T(A)$. The functor T is called half exact if the related sequence $T(A) \rightarrow T(B) \rightarrow T(C)$ is always exact at $T(B)$; in this case the connecting homomorphisms yield an infinite exact sequence of iterated satellites in the form

$$\cdots \rightarrow S_2T(C) \rightarrow S_1T(A) \rightarrow S_1T(B) \rightarrow S_1T(C) \rightarrow T(A) \rightarrow \cdots.$$

The functors Tor_n are the iterated left satellites of $\otimes_\Lambda$, while Ext^n are the right satellites of Hom_Λ. There is an axiomatic description of satellites (Theorem 5.1; see also the elegant characterization of Tor given in the introduction).

Homological methods applied to arbitrary functors are more powerful. For a contravariant additive functor $T(A)$ the nth right derived functor is defined as the nth homology group of the complex $T(X)$, where X is a projective resolution of A, and similarly for covariant functors and functors of several arguments. Chapter V discusses these notions, relates the derived functors to satellites and describes the "balanced" covariant functors $T(A, G)$ for which resolution of either factor yields the same derived functor—the one relevant example being the functor $G \otimes_\Lambda A$. One also employs injective resolutions (dual to projective resolutions). For example (p. 107) the nth right derived functor of $\operatorname{Hom}_\Lambda^n (A, C)$ is $\operatorname{Ext}_\Lambda^n (A, C)$; it may be obtained as the nth homology group of any one of $\operatorname{Hom}_\Lambda (X, C)$, $\operatorname{Hom}_\Lambda (A, Y)$ or $\operatorname{Hom}_\Lambda (X, Y)$, where X is a projective resolution of A and Y an injective one of C.

Chapter VI introduces dimension concepts. The *projective dimension* of a Λ-module A is the least non-negative inreger n such that there is a projective resolution X of A with $X_k = 0$ for all $k > n$. The *left global dimension* of a ring Λ is the least non-negative integer m such that all left Λ-modules A have projective dimension $\leqq m$, or, equivalently, such that $\operatorname{Ext}_\Lambda^{m+1} = 0$. In the exercises (of which most chapters have many, giving additional results) one finds also a weak dimension. Another interesting result is Rose's theorem (Chapter IX, Proposition 7.4) on the dimension of the tensor product of algebras. Hilbert's Theorem on chains of syzygies is comprised in the beautiful Theorem 6.5 of Chapter VIII: if Λ is the polynomial ring in n indeterminates over a field K, then the projective dimension of any Λ-module is at most n and that of any ideal of Λ is at most $n-1$. The demonstration uses a projective resolution due to Koszul; there is a similar theorem (Theorem 6.5′) for formal or convergent power series rings.

Chapter XI proliferates products. Given left modules A and C over the K-algebra Γ and similar modules over Γ' there is an obvious natural homomorphism

$$\operatorname{Hom}_\Gamma (A, C) \otimes \operatorname{Hom}_{\Gamma'} (A', C') \to \operatorname{Hom}_{\Gamma \otimes \Gamma'} (A \otimes A', C \otimes C'),$$

where all tensor products are taken over K. Upon replacing the arguments A and A' by resolutions one obtains an "external" product

$$\vee : \operatorname{Ext}_\Gamma^p (A, C) \otimes \operatorname{Ext}_{\Gamma'}^q (A', C') \to \operatorname{Ext}_{\Gamma \otimes \Gamma'}^{p+q} (A \otimes A', C \otimes C');$$

it is a homomorphism which reduces to the previous one when $p = q = 0$. Given a "diagonal map" $D : \Gamma \to \Gamma \otimes \Gamma$ and $\Gamma' = \Gamma$ one induces a corresponding internal product

$$\cup : \operatorname{Ext}_\Gamma^p (A, C) \otimes \operatorname{Ext}_\Gamma^q (A', C') \to \operatorname{Ext}_\Gamma^{p+q} (A \otimes A', C \otimes C').$$

This yields the usual cup products in the cohomology of algebras or of groups, and can there be expressed by the usual Čech-Alexander formulas in the standard complexes. There are three more external products, each with an internal version, which spring as above from the three natural homomorphisms

$$(C \otimes A) \otimes (C' \otimes A') \to (C \otimes C') \otimes (A \otimes A'),$$

$$\operatorname{Hom}(A \otimes A', \operatorname{Hom}(C, C')) \to \operatorname{Hom}(C \otimes A, \operatorname{Hom}(A', C')),$$

$$\operatorname{Hom}(C, C') \otimes (A \otimes A') \to \operatorname{Hom}(\operatorname{Hom}(A, C), C' \otimes A').$$

The third one gives "cap" products. After handling these products expeditiously, with effective use of the unifying concept of a supplemented algebra, the chapter closes with a generalization of the Eilenberg-Mac Lane cup-product reduction theorem.

Chapter XII, one of the most interesting of the book, presents hitherto unpublished results of Tate and Artin-Tate on the homology and cohomology of finite groups. For a left module A over the group ring of a finite group Π one has the norm homomorphism $a \to \sum xa$, the sum being taken over all $x \in \Pi$. This induces a norm homomorphism $H_0(\Pi, A) \to H^0(\Pi, A)$ and hence makes it possible to combine the homology and cohomology groups of Π with coefficients in A into a single (doubly infinite) sequence. To calculate these groups one can use a suitably designed "complete" resolution; there is a (cup) product which works for the whole sequence. For certain finite groups Π there is known to be a period q; that is, an integer such that there is an isomorphism $H^n(\Pi, A) \cong H^{n+q}(\Pi, A)$ given for all n by a cup product. The chapter ends with the beautiful theorem that Π has such a period if and only if every abelian subgroup of Π is cyclic.

Homological algebra arose from extension problems: $\operatorname{Ext}_1^Z(A, C)$ is the group of abelian group extensions of C by A; $H^2(\Pi, G)$ is the group of extensions of the Π-group G by the group Π, and similarly for algebras and Lie algebras. These matters are treated systematically, elegantly, and belatedly in Chapter XIV. The most interesting known results in these directions (three-dimensional cohomology classes as obstructions to extension problems) are omitted.

The next chapter sets up the formalism of spectral sequences. The exposition is clear; there is a good explanation (§7) of how spectral sequences arise from relative homology; there is (finally!) a decent notation for a filtration. Unhappily the authors continue the conspiracy of silence according to which the rectangular diagrams, used by all the experts, never appear in print. As the authors note, spectral sequences were discovered by Leray (1945); as they do not note, they were independently discovered by Lyndon (Harvard thesis, May, 1946; cf. also Duke Math. J. vol. 15 (1948) pp. 271–292), who was the first to consider the subject matter of these chapters—spectral sequences applied to algebraic problems. The authors improperly credit Lyndon's spectral sequences to a much later paper by Hochschild-Serre.

Chapter XVI applies spectral sequences to various problems; associativity formulas for torsion products, the relation between homology groups of an algebra and those of an invariant subalgebra, and topological spaces with operators. The next and last chapter discusses the projective resolution of a complex K by a double complex X (i.e., a bigraded module with two differentiations). Given also a functor T, the double complex $T(X)$ has homology independent of the choice of the resolution X, has two filtrations and hence two spectral sequences. The resulting maze of objects constitute the *hyperhomological* invariants of K and T. The chapter ends triumphantly with a return to first problems; the Künneth sequence (1) for complexes K and L over a hereditary ring, previously known to be exact for K and L projective, is shown by spectral sequences to be exact provided only that $H(\mathrm{Tor}_1^\Lambda (K, L)) = 0$. This resounding success leads the authors to assert that the hyperhomological invariants of $K \otimes L$ may be regarded as a general solution of the Künneth problem. This assertion appears to be a case of trimming the problem to fit the technique, since a regard uncorrupted by spectres would formulate the Künneth problem as that of finding a formula for the homology of $K \otimes L$ in terms of a sufficient set of invariants (hyperhomologies, Bocksteins, or what have you) of K and L.

In spite of the delay in its publication, widespread acquaintance with the manuscript and with the ideas of this book has already played an important role in the development of this lively subject. For example, Eckmann and Hilton (unpublished) have investigated the interesting functors derived from Hom (A, B) by replacing the contravariant argument A by an injective resolution or the covariant B by a projective. Serre and Auslander-Buchsbaum (Proc. Nat. Acad. Sci. U.S.A. vol. 42 (1956) pp. 36–38) have achieved decisive results on the dimensions of local rings. More significant for the general presentation of the subject is the fact that the whole mechanism of projective resolutions works not just in the category of left modules over a ring, but equally well for right modules, for bimodules, or in many other categories. In an appendix to the book Buchsbaum sets forth these ideas, together with the necessary axioms on the *additive categories* (he calls them "exact" categories) in which this theory works. Subsequent unpublished work by Grothendieck indicates that this point of view will be convenient also for the homology of a space with coefficients in a sheaf. Hence it seems likely that a future presentation of homological algebra will operate in a suitable category, provided at least that someone concocts a convenient method of chasing diagrams without chasing elements.

The authors' approach in this book can best be described in philosophical terms and as monistic: everything is unified. Consider for instance the homology of groups; in view of its application to class field theory and to topology this topic is central in homological algebra. In this book the homology of groups appears as a special case of the homology of monoids (monoid = associative multiplicative system with identity), which in turn is a special case of the homology of supplemented algebras, again a case of the homology of augmented algebras, which is an instance of a torsion product, which at your choice is an instance of a derived functor or an iterated satellite functor.

Historically, each monistic doctrine is resolved by a subsequent pluralism. So it was here. When the authors started to write, it *was* true that all known cases of homology of algebraic systems (groups, algebras, and Lie algebras) could be neatly subsumed under the resolution, Tor, and Ext pattern. When they finished writing this was no longer so—and this because of the authors' own separate efforts elsewhere! The Eilenberg-MacLane homology of abelian groups (Trans. Amer. Math. Soc. vol. 71 (1951) pp. 294–330) has not yet been expressed by torsion products. The Eilenberg-Mac Lane bar construction (Annals of Math. vol. 58 (1953) pp. 55–106) is a standard construction more general than those produced by standard resolutions. Cartan's beautiful and powerful theory of constructions (Séminaire École Normale Supérieure, 1954/1955) is an extension of the idea of a projective resolution beyond the terms of this book. Still more recently, the as yet unpublished homology theories of Dixmier for Lie rings and of MacLane for rings are other examples of homology of algebraic systems not (at least as yet) obtainable by resolutions.

Perhaps Mathematics now moves so fast—and in part because of vigorous unifying contributions such as that of this book—that no unification of Mathematics can be up to date. The reviewer might also add his strictly personal opinion that the authors have not kept sufficiently in mind the distinction between a research paper and a book: a good research paper presents a promising new idea when it is hot—and when nobody knows for sure that it will turn out to be really useful; a good research book presents ideas (still warm) after their utility has been established in the hands of several workers.

This book contains too large a proportion of shiny new ideas which have nothing to recommend them but their heat and promise: satellites (these appear in Chapter III and then gradually disappear in later chapters), derived functors of anything but Hom and $\otimes$ (the reviewer watched in vain for other examples), semi-hereditary rings, functors derived simultaneously in several variables, supplemented algebras, and the homology of monoids. The same remark applies to spectral sequences. These sequences *have* proved their worth in topology but have not yet reached decisive results in the homology of algebraic systems: the result is that the uninitiated reader can hardly hope to understand what spectral sequences are all about by reading the three chapters devoted to them in this book. The reviewer is not claiming that spectral sequences and these other notions will not later have significant algebraic uses: some of them will, but until that time comes their presence clutters up the book.

Another danger of shiny new notions is that sometimes the shine proves illusory. For example, the authors define an *augmented ring* as the triple consisting of a ring Λ, a left Λ-module Q and a left Λ-module epimorphism $\epsilon:\Lambda\rightarrow Q$. Now Q and ϵ are determined up to isomorphism by the kernel of ϵ, which is a left ideal in Λ. Hence "augmented ring" is a new name for "left ideal in a ring." The authors have introduced these augmented rings because the homology of an augmented ring includes the homology of algebras, groups, and Lie algebras. The homology groups of an augmented ring $\epsilon:\Lambda\rightarrow Q$ are defined to be the groups $\mathrm{Tor}_n^\Lambda(A, Q)$. This definition does include the three desired cases as already displayed above, but a moment's reflection reveals that the definition has nothing to do with the epimorphism ϵ, and would work for *any* left Λ-module Q. The ϵ really occurs only in some wholly routine calculations of low dimensional homology groups. The only consequential theorem about the homology of augmented rings is a mapping theorem (VIII, 3.1) which gives the machinery for changing from one ring Λ to another. This theorem has nothing to do with a map ϵ; in fact the theorem is clarified by the recognition that it deals with a homomorphism of the system consisting of a ring Λ and a left Λ-module Q into a ring Λ' and a left Λ'-module Q'. The rest of the authors' discussion of "augmented rings" consists of theorems about left ideals.

The authors' treatment of the literature is off-hand. Künneth formulas appear, but no references to Künneth. Torsion products abound, with no credit to early discoverers (example: Čech, in Fundamenta Mathematica vol. 25 (1935) pp. 33–44) had the torsion product for abelian groups, defined essentially as a satellite). The authors have discovered that the Birkhoff-Witt theorem was known to Poincaré before either Birkhoff or Witt was born; they proceed to call it the Poincaré-Witt theorem. The facts of the matter are these. The theorem asserts that if the Lie algebra L over K is free as a K-module, then the natural map of L into its enveloping associative algebra L^e has kernel zero. Birkhoff and Witt both found proofs of this theorem in 1936; Birkhoff's was received by the editors 29 days earlier than Witt's, but there is every reason to suppose that the two were independent and that the theorem was then "in the air." Birkhoff makes a (partial) reference to Poincaré; Witt does not. Birkhoff defines the enveloping algebra, not as a quotient of the tensor algebra, but by "straightening" elements of that tensor algebra. His proof is carelessly done, but with some little trouble (which the reviewer has taken) his proof can be made complete and correct. The world would be happy to honor Poincaré, who was well ahead of his time on this, but not at the expense of a manifest injustice to Birkhoff. Since a three-handled theorem is clumsy, it will doubtless remain Birkhoff-Witt.

This book is very carefully prepared and well proof-read; the reviewer noted only one troublesome misprint: on page 185 the η in the top row of the square diagram and in the next line of the text should be ρ (notation from p. 168). The letter Λ is overworked; it appears variously as a ring, as an augmented ring, or as an algebra. More application of the usual (unexpressed) conventions about different letters for different notions would have helped the reader. The index might have the following additions: complete resolution 240, derivation 168, direct family of homomorphisms 4, direct product 4, direct sum 4, exterior ring 146, free ring 146, homology of differential module 54, image 3, kernel 3, Lie algebra 266, negative graded module 58, normal map 349, normalized standard complex 176, polynomial ring 146, positive graded module 58, 60, $\varnothing$-projective module 30, standard complex 175, syzygies 157.

SAUNDERS MACLANE

Methods in numerical analysis. By K. L. Nielsen. Macmillan, New York, 1956. 13+382 pp. $6.90.

Introduction to numerical analysis. By F. B. Hildebrand. New York, McGraw-Hill, 1956.+10 511 pp. $8.50.

Numerical analysis, with emphasis on the application of numerical techniques to problems of infinitesimal calculus in single variables. By Z. Kopal. New York, Wiley, 1955. 14+556 pp. $12.00.

The demand for computer personnel has been made abundantly clear—in the advertisement pages of our newspapers, and at formal