BOOK REVIEWS

Fonctions analytiques. By G. Valiron. Paris, Presses Universitaires de France, 1954. 4+236 pp. 1.500 fr.

This book is intended for the student who has had an introductory course in functions of a complex variable, and the first chapter gives a brief summary of the basic notions of function theory. The second chapter treats the classical theory of univalent functions using the area principle of Gronwall. The author gives the theorem $|a_2| \leqq 2$, the one-quarter theorem and its consequences, and the Littlewood bound for the coefficients of a univalent function.

Boundary values of bounded analytic functions are discussed in the third chapter, culminating in Fatou's theorem and the Riesz theorem on the determination of a function by the boundary values on a set of positive measure. In the next chapter the author discusses the behavior of conformal mappings on the boundary and gives criteria that they be conformal there.

Chapter five takes up "functions of Fatou," i.e., functions defined in the unit circle which are bounded by 1 and have absolute value 1 on the unit circumference. In the following chapter we are led to the study of functions which map the upper half-plane into itself and the iteration of such functions. Here the author introduces the notion of a normal family.

Chapter seven discusses the multiplicative functions of Poincaré and notions of their order of growth. Chapter eight deals with the question of prolongation of a function. The final chapter gives the methods and results of Wiman and Valiron on the growth of entire functions.

Altogether this is a very useful little book which has collected together a number of diverse topics in function theory in a form which is easy to read and quite suitable for the student who wishes to familiarize himself with some of the classical tools available in function theory.

H. L. ROYDEN

Uniformisierung. By R. Nevanlinna. Berlin, Göttingen, Heidelberg, Springer, 1953. 391 pp.

There has long been a lack of a reference for students wanting to learn about Riemann surfaces, and Nevanlinna's new book, which might equally have been called *An introduction to Riemann surfaces*, goes far toward filling this need. The first eight chapters of this book, which the reviewer found very readable, are a complete and modern approach to the classical theory of Riemann surfaces.

The book begins with a chapter which gives the algebraic origins of the notion of a Riemann surface and discusses the problem of uniformizing an algebraic curve. The second chapter adopts the abstract viewpoint and is concerned with the development of the topological tools necessary for Riemann surface theory. The first section begins with point set theory and continues through the definitions of two-dimensional manifold and Riemann surface. A detailed discussion of the Prüfer example is given, but, rather than follow Radó's direct but tedious proof of the separability of a Riemann surface, Nevanlinna prefers to deduce this at a later stage as a consequence of the general uniformization theorem. The second section of this chapter develops the homology theory of surfaces using the singular theory. Remaining sections cover the fundamental group, covering surfaces, cover transformations, and triangulation.

The third chapter develops the exterior differential calculus for Riemann surfaces, discusses analytic functions and differentials and their properties. A concluding section develops the various integral and residue theorems. The integration theory is all developed using singular differentiable simplexes and chains.

Chapter four deals with the fundamental existence theorems for harmonic and analytic functions on Riemann surfaces. The author uses the "alternating method" of Schwarz, feeling that it is superior because it is constructive.

Closed Riemann surfaces are taken up in the next chapter, and the Abelian differentials of first, second, and third kinds. The interchange of argument and parameter is discussed, and the reviewer regrets that Nevanlinna does not follow this up with a proof of the Riemann-Roch theorem, which is alluded to but not stated.

The Riemann mapping theorem for arbitrary simply-connected Riemann surfaces is proved in the sixth chapter. After a chapter on groups of linear transformations, the general Riemann mapping theorem is used to uniformize a Riemann surface by mapping its universal covering surface onto the plane or unit-circle. Having done this, the author takes up in some detail a variety of topics, including the Poincaré hyperbolic metric, the elliptic modular function, and conformal self-mappings of a Riemann surface.

The ninth chapter derives the various mapping functions for planar (schlicht-artige) surfaces. They are characterized by extremal properties, and their use for uniformization is discussed.

While the preceding nine chapters have given a relatively complete coverage of the classical theory of Riemann surfaces, the last chapter turns to the subject of open Riemann surfaces. Since this is an extremely active subject at the present time, it is impossible to give any sort of definitive survey. However Nevanlinna does manage to introduce the reader to the principal problems in the field and to give an idea of some of the methods available.

This book arose out of the author's lectures at the University of Helsinki and is admirably suited for anyone who wishes to learn about Riemann surfaces. The author always has the present work in open Riemann surfaces in mind, so that the reader will find that he is prepared for the literature in this field.

H. L. ROYDEN

Lezioni sulle equazioni a derivate parziali. By F. G. Tricomi. Editrice Gheroni Torino, 1954. 4+484 pp.

This book furnishes an excellent introduction to the rapidly expanding theory of partial differential equations, written in the author's usual lucid and interesting style.

The work is divided into five parts. The first part, consisting of one hundred and four pages, presents a rapid but thorough summary of classical analytic tools required in the remainder of the book. The theory of integral equations, the gamma function, the hypergeometric function, the Legendre and Bessel functions are all treated. This part is well worth reading on its own.

The second part, consisting of seventy-five pages, is devoted to a discussion of the theory of characteristics for equations of the first and second order. It includes a section devoted to the Hamilton-Jacobi theory and its connection with the calculus of variations.

The third part, one hundred pages, is devoted to equations of hyperbolic type. Various classical approaches, such as those of Laplace and Riemann, are presented, and there is large section on the movement of a compressible fluid.

The fourth part, ninety-five pages, treats the equations of elliptic type. The classical techniques are given, together with a discussion of more modern methods based upon difference equations, and numerical methods such as the "relaxation" method of Southwell. A section on incompressible fluids is included.

The fifth and concluding part is devoted to equations of parabolic type and equations of mixed type. The greater part of this section is concerned with equations of mixed type, a topic investigated in great detail by Tricomi in 1923, and which in recent years has become of