(3) No mention is made of Chevalley's version and application of the Tannaka theorem, nor of the work of Krein in related connections. (4) It is mentioned that the unimodular group in $n(>1)$ dimensions is minimally almost periodic; notably more generally, the same is true for any noncompact simple Lie group.

There is some occasional vagueness that may bother the non-expert reader. Thus, in the discussion of Hilbert's fifth problem, "finite-dimensional" is used as if it meant "finite-dimensional and locally euclidean," or at least "finite-dimensional and locally connected." And in quoting van der Waerden's result on the continuity of representations of semi-simple groups, the author neglects to make explicit the qualification that the representation be finite-dimensional.

This booklet conveys the scope of the theory of almost periodic functions on groups more rapidly and pleasantly than any other treatment with which the reviewer is familiar, and as such performs a valuable and important service.

I. E. SEGAL

Undecidable theories. By A. Tarski in collaboration with A. Mostowski and R. M. Robinson. Amsterdam, North Holland Publishing Company, 1953. 9+98 pp. $2.50.

The material of this book is based on results which were obtained by the authors, in particular by Tarski, between 1938–1952, and were originally intended to be published as a series of papers in some mathematical journal. Fortunately they are combined here in one volume.

A given axiomatic theory T is called decidable or undecidable according as we can or cannot find a mechanical procedure which permits us to decide (in a finite number of steps), for each particular sentence formulated in the symbolism of T, whether this sentence is provable by means of the devices available in T. (The number of steps necessary will, however, in general depend on the structure of the sentence under test, just as in applications of the Euclidean algorithm the number of divisions necessary depends on the two numbers whose g.c.d. we wish to find.) The decision problem of a theory T is the problem of determining whether T is decidable or undecidable. The problem of deciding which axiomatic theories have decision procedures and which do not is one of the central problems of modern symbolic logic, and should be of interest to all mathematicians, not just logicians. Here the authors describe all the principal methods and results in this field. The exposition throughout is excellent, so that the book is valuable not only to the expert, but also to students.

The first of the three papers is by Tarski and it outlines the general methods in proofs of undecidability and gives the necessary definitions and background. It forms both an introduction and outline of the whole work. Two general methods have been developed to show that a theory T (formalized in the first order predicate calculus) is undecidable. The first, direct, method can be used only in case T allows us to build up a sufficiently large part of number theory so that we can carry through arguments based on Gödel numbering and use diagonal procedures. The second method is indirect: The decision problem for a theory T_1 is reduced to that for some theory T_2 which has already been shown to be undecidable. In many applications, T_2 is taken to be arithmetic based on Peano's axioms, for Rosser has shown that every consistent extension of this is undecidable. This can be expressed by saying that Peano's arithmetic is essentially undecidable. If now T_1 is obtained from an undecidable theory T_2 by deleting a finite number of axioms (without, however, removing any constant from the symbolism of T_2), then T_1 is undecidable. The same holds if some essentially undecidable theory T_2 is interpretable in T_1. Moreover, T_1 is undecidable if some finitely axiomatizable and essentially undecidable theory T_2 is interpretable in some consistent extension of T_1. Some fragments of arithmetic are both finitely axiomatizable and essentially undecidable and applications of the last result showed the undecidability of many formal theories, such as the elementary theory of groups, rings, fields and lattices.

Paper II is by A. Mostowski, R. M. Robinson, and A. Tarski. Here the first, direct, method is used to give a very elegant proof of the essential undecidability of a subtheory of the arithmetic of natural numbers which is based on only 7 axioms. The first such theory was constructed by Mostowski and Tarski in 1939 and the present simplified version is due to Robinson. It is also shown that every subtheory of arithmetic (having the same constants) is also undecidable. The results are extended to systems of arithmetic with different sets of constants; and, using the indirect method, the elementary theories of various kinds of rings can be shown to be undecidable, and in some cases essentially undecidable.

The third and last paper is by A. Tarski and shows that the elementary theory of groups is undecidable. More precisely, if we consider the class of all sentences which can be deduced from the axioms $x\circ(y\circ z)=(x\circ y)\circ z$, $\bigvee z(x=y\circ z)$, $\bigvee y(x=y\circ z)$ by the first order functional calculus, then there is no general algorithm for deciding whether a given sentence is in this class or not. [This is not yet the same as the word-problem for groups which deals with sentences of a particular form. The word-problem for groups is still unsolved.] The proof is by the indirect method by interpreting a translation of the finitely axiomatized essentially undecidable arithmetic theory in an extension of group theory.

Much credit is also due to the editors of the series on "Studies in logic and the foundations of mathematics" for their part in the publication of this and many other excellent volumes.

ILSE NOVAK GÁL

Pages choisies d'analyse générale. By M. Fréchet. Paris, Gauthier-Villars; Louvain, Nauwelaerts, 1953. 216 pp. 2.000 fr. ($5.94).

Jubilees or seventieth anniversary celebrations of savants usually carry with them the publication of an issue of a journal dedicated to the celebrant, or a special volume consisting of papers in his main field of interest, contributed by his friends, pupils and admirers. The present volume, which is not in either category, was not planned for in connection with the Jubilee of M. Fréchet. It is, however, an outgrowth of this celebration in that friends and pupils on that occasion urged on him the desirability of issuing a volume which might contain some of the material planned for in a second volume of his *Espaces abstraits*, had not a change of positions and consequent change of fields of interest and research intervened in 1928. The volume under discussion might fall into the category of "excerpts from collected works" which would be insufficiently inclusive in that it is limited to re-publication of some papers connected with the topic of general spaces and not intended to give a systematic insight into the development of these researches. We have then before us a group of papers published in various periodicals, a half dozen of which may be somewhat inaccessible at present. The selection was made by the author and can be considered an indication of what phases of his researches in this field he considers worthy of reproduction and important.

The papers are roughly grouped into chapters headed as follows: 1. Survey of sets (1 paper); 2. Functional spaces (13 papers); 3. Functional analysis (13 papers); 4. Abstract spaces (4 papers); 5. General analysis (6 papers). However, there is no sharp distinction between the topics included in the various chapters. For instance, papers pertaining to the differential are found in the third and fifth chapters, matters pertaining to the generalization of the Weierstrass theorem of approximation in the third and fifth chapters, considerations per-