ring, proves the "basis theorem" for finitely generated modules and introduces the elementary divisors. The theory is finally applied to the classical elementary divisor theory and yields the classification of the endomorphisms of a vector space.

In concluding I wish again to emphasize the complete success of the work. The presentation is abstract, mercilessly abstract. But the reader who can overcome the initial difficulties will be richly rewarded for his efforts by deeper insights and fuller understanding.

E. ARTIN

Inhalt und Mass. By K. Mayrhofer. Vienna, Springer, 1952. 8+269 pp. $9.30.

This is a very careful and detailed presentation of the Carathéodory measure theory, with special emphasis on Lebesgue measure in R^n. The first chapter, after a short paragraph on rings and fields of sets, defines and studies abstractly the notions of content and measure, the former being simply additive and defined on a field of sets, the latter countably additive and defined on a σ-field of sets (only positive set functions are considered). The usual notions related to content and measure (measurability, exterior and interior content or measure, measurable hull and measurable kernel of a set) are investigated with perhaps greater detail than in any other treatise on the subject; so are the relationship between content and measure, and the well known process of "completion" by which a completely additive content can be extended to a measure (on a larger field of sets). Also treated in this chapter are the products of two contents or measures, although one misses the corresponding facts for infinite products (this is probably the only important part of abstract measure theory which is not covered by the book).

The second chapter is devoted to Jordan content and quarrable sets ("quarrable" is to content as "measurable" is to measure), the third to Borel and Lebesgue measures in R^n; among the special features that should be mentioned are examples of nonquarrable Jordan curves, the Vitali covering theorem and the density theorem, and a study of nonmeasurable sets (for Lebesgue measure). Chapter IV deals with transformation of content and measure by a linear mapping in R^n; as an application the measures of various "elementary volumes" are computed. The general notion of measurable mapping from R^n into R^m is also considered, but, surprisingly enough, the author's definition is not the usual one: he defines a measurable mapping as one which sends measurable sets into measurable sets, with the consequence that a continuous function is not always measurable! Chapter V takes up Carathéodory's theory of abstract "exterior measures," i.e., set functions containing as particular cases the exterior measures deduced from a measure; these are axiomatically characterized among all "general exterior measures," and a similar characterization is given for interior measures. The chapter also includes the Carathéodory criterion for measurability of closed sets in a metric space; finally the connection is made between this theory and that developed in chapter I. The last chapter treats measure theory in boolean algebras and σ-complete boolean algebras. An appendix gives the "transfinite" generation of Borel sets and their main properties in finite-dimensional spaces.

The author's claim that he has "reached or even gone beyond" the limit of what is known today on the subject can hardly be accepted without restriction; for instance, no mention is made, in the last chapter, of the Stone and Loomis representation theorems, although they make the developments of that chapter practically pointless! No mention is ever made of characteristic functions of sets, which would at times make proofs much easier (for instance, the well known derivation of the "boolean ring" structure of a boolean algebra). Finally, the reviewer wants to take exception to the author's statement that measure theory (as understood in this book) is the foundation of the theory of integration. This *was* undoubtedly true some years ago, but is fortunately no longer so, as more and more mathematicians are shifting to the "functional approach" to integration. It is always rash to make predictions, but the reviewer cannot help thinking that, despite its intrinsic merits, this book, as well as its brethren of the same tendency, will in a few years have joined many an other obsolete theory on the shelves of the Old Curiosity Shop of mathematics.

J. DIEUDONNÉ

Lectures in abstract algebra. Vol. II. *Linear algebra.* By N. Jacobson. New York, Van Nostrand, 1953. 12+280 pp. $5.85.

Linear algebra is now universally recognized as perhaps the most important tool of the modern mathematician; its concepts and methods, moreover, when properly reduced to their essential features, are among the simplest and most straightforward imaginable. Nevertheless, it is still not uncommon to find graduate students who are totally unfamiliar with some of the fundamental notions of linear algebra, such as, for instance, the theory of duality. This may perhaps be attributed to the scarcity of good textbooks on the subject; if so, the present volume will undoubtedly do much to remedy this situation. Although this is the second part of a work which will