

point set whose elements need not all be distinct points and talking about points of accumulation of "sets" of this kind (he regards a sequence $\{P_n\}$ as such a set). Now, a "point set" in this latter sense is in reality a function (a sequence being a function defined on the positive integers). There is no need to introduce the concept "point of accumulation" except in the sense that is customary in topology. Strict adherence to this point of view makes for clarity and for less difficulty on the part of students. All that the author wishes to accomplish can be done by suitable discussion of points of accumulation of a set and their relation to convergent sequences chosen from the set. A second criticism relates to the author's definition of a closed set as one which contains all its points of accumulation *and is bounded*. This departure from the usual definition spoils the duality between open and closed sets, complicates the statements of many theorems, and has no advantages apparent to the reviewer. The topological definition of connectedness is not given; the definition which is given on p. 39 (connectedness by polygonal paths) is unsatisfactory, except for open sets, and the remarks about connected regions on p. 42 puzzled the reviewer. The intermediate value theorem for continuous functions is not presented as a connectedness theorem.

ANGUS E. TAYLOR

Conformal mapping. By Z. Nehari. New York, McGraw-Hill, 1952. 8+396 pp. \$7.50.

This is a textbook that will fill two needs. The author has designed the first four chapters to serve as the basis for a one term introductory course in complex variables, while the remainder of the book can be used in a graduate course in conformal mapping. It is claimed in the preface that only a knowledge of advanced calculus is necessary to read this book. (Perhaps a slightly better knowledge of the properties of real numbers is actually assumed than is given in most courses in advanced calculus.)

In Chapter I, the properties of harmonic functions in the plane are developed. The author discusses the solutions of the boundary value problems of the first and second kinds, introducing the Green's and Neumann's functions, and the harmonic measure. The first chapter closes with a derivation of the Hadamard variation formula giving the dependence of the Green's function on the domain.

The complex number system is explained in the first part of Chapter II, culminating in a discussion of sequences and series of complex numbers. After an analytic function is defined to be one which has a derivative, the connection between analyticity, the

Cauchy-Riemann equations, and the Taylor series is demonstrated. The chapter concludes with a study of the elementary functions. Chapter III is devoted to the complex integral calculus, collecting together those topics which relate to the Cauchy integral theorem and formula. To lay the groundwork for the numerous extremal problems to be encountered, Chapter IV takes up normal families of analytic functions.

Chapter V is devoted to the conformal mapping of simply-connected domains, beginning with an extensive study of the linear fractional transformations. Then Schwarz's lemma leads the way to the study of bounded analytic functions. The Riemann mapping theorem is proved together with the continuity of the mapping function on the boundary of a smooth Jordan region. On a more practical level, the functions mapping polygons and domains bounded by circular arcs onto a half-plane are found. Several extremal problems in the class of functions univalent in $|z| < 1$ are solved, including the distortion theorems and certain coefficient problems. The principle of subordination is introduced to solve extremal problems in larger classes of functions, dropping the univalence requirement. The chapter continues with a discussion of the Bergman kernel function for simply-connected domains, and closes with the mapping of nearly circular domains.

In Chapter VI, the mappings produced by certain special functions are studied. Included are the rational functions of second degree, the exponential and trigonometric functions, and the elliptic functions. The last chapter deals with the mapping of multiply-connected domains onto canonical domains. The existence proofs here are based upon extremal problems analogous to the one used in proving the Riemann mapping theorem, lending unity to the book. The Dirichlet problem for multiply-connected domains is then discussed. The book ends with a treatment of several special extremal problems: maximizing outer area or minimizing inner area, the Bergman kernel function, and bounded functions in multiply-connected domains.

The author has ended each section with a long list of carefully selected problems, many of which will present a challenge to the student. The book, written in an informal style, is very readable. In some places, however, this informality leads to carelessness in stating hypotheses. For example, in Chapter I, the author states, "the boundary value problem of the second kind can be reduced to a boundary value problem of the first kind." He then gives a proof of this statement ignoring the fact that the domain must be simply-connected. In fact, in the following section, he notes that connectivity

was not referred to in the preceding section; but "these results are true regardless of whether the domains in question are simply-connected or multiply-connected." He does at least point out here that one does have to worry about single-valued conjugate harmonic functions in multiply-connected domains. Another example appears when the Cauchy integral theorem is proved for functions with a derivative on the boundary and later, in proving the symmetry principle, is used for functions only continuous on the boundary. Also in Chapter VII, the author claims "to prove the existence of the solution of the Dirichlet problem in the case of a general domain of finite connectivity," not stating how *general*. In reality, he proves it for domains bounded by smooth Jordan curves. The reviewer regrets that the lucid style of the author was not utilized to give a textbook presentation of the theory of prime ends, which would have been appropriate in a book of this type and is lacking in the literature. It is also rather unfortunate that the author followed the only too prevalent custom of claiming that this work, being a textbook, need not be documented. No references to original sources or to other works are given. This is true even when, in general discussions, the author makes statements he does not prove in the book. The value of this excellent book to the graduate student would have been enhanced considerably if it had also furnished a key to further study.

G. SPRINGER

The theory of functions of a real variable. By R. L. Jeffery. (Mathematical Expositions, No. 6.) University of Toronto Press, 1951. 13+232 pp. \$6.00.

This book consists of two distinct parts. The first part (Chapters I-V) gives a general introduction to functions of a real variable, measure, and integration, while the second part (Chapters VI and VII, with Chapter VIII as a kind of appendix) treats the problem of inverting the derivative of continuous functions, leading to the Denjoy integrals, and studies the derivatives and approximate derivatives of functions of a real variable on arbitrary linear sets. The author himself, who in previous papers has made some valuable contributions to these topics, considers the presentation of this second part as the main purpose of his book. In both parts only functions of one real variable are discussed.

After an introduction concerning the real number system, Chapter I deals with sets, sequences, and functions and Chapter II with metric properties of sets. Here the author considers only the outer measure of a set A , called by him the metric of A and designated by $|A|^0$. Two