THE APRIL MEETING IN NEW YORK

The four hundred seventy-ninth meeting of the American Mathematical Society was held at Columbia University, New York City, on Friday and Saturday, April 25–26, 1952. The meeting was attended by about 225 persons, including the following 207 members of the Society:

C. R. Adams, M. I. Aissen, A. A. Albert, A. C. Allen, Warren Ambrose, R. D. Anderson, R. L. Anderson, Joseph Andrushkiw, R. G. Archibald, Sholom Arzt, Joshua Barlaz, J. H. Barrett, R. G. Bartle, W. R. Baum, Enrique Báyo, R. J. Beeber, E. G. Begle, Stefan Bergman, S. D. Bernardi, Lipman Bers, D. W. Blackett, Jerome Blackman, A. L. Blakers, J. H. Blau, I. E. Block, H. W. Bode, Samuel Borofsky, E. H. Boyle, J. L. Bricker, H. W. Brinkmann, F. E. Browder, A. B. Brown, Bailey Brown, R. G. Brown, R. H. Brown, D. W. Bushaw, Jewell H. Bushey, Eugenio Calabi, P. W. Carruth, Y. W. Chen, F. E. Clark, E. A. Coddington, L. W. Cohen, R. M. Cohn, Philip Cooperman, T. F. Cope, Richard Courant, A. B. Cunningham, M. L. Curtis, R. B. Davis, C. R. DePrima, Avron Douglis, D. M. Dribin, Nelson Dunford, Samuel Eilenberg, C. C. Elgot, M. P. Epstein, M. E. Estill, R. M. Exner, A. L. Fass, J. M. Feld, William Feller, R. M. Foster, L. G. Fourès, J. B. Freier, Gerald Freilich, Bernard Friedman, Orrin Frink, I. S. Gál, A. S. Galbraith, David Gale, G. N. Garrison, B. H. Gere, Abolghassem Ghaffari, H. A. Giddings, B. P. Gill, Leonard Gillman, Samuel Goldberg, J. K. Goldhaber, J. W. Green, L. W. Green, Emil Grosswald, Felix Haas, Marshall Hall, Jr., Carl Hammer, Harish-Chandra, F. S. Hawthorne, E. V. Haynsworth, K. E. Hazard, C. M. Hebbert, G. A. Hedlund, M. H. Heins, Einar Hille, Banesh Hoffmann, L. A. Hostinsky, J. L. Howell, C. C. Hsiung, L. C. Hutchinson, Eugene Isaacson, W. S. Jardetzky, Fritz John, M. L. Juncosa, Shizuo Kakutani, Aida Kalish, Hyman Kamel, J. F. Kiefer, H. S. Kieval, J. W. Kitchens, George Klein, E. R. Kolchin, B. O. Koopman, Jack Kotik, P. G. Kvick, R. H. Kyle, A. W. Landers, C. W. Langley, Solomon Leader, Joseph Lehner, Benjamin Lepson, M. E. Levenson, Howard Levi, D. C. Lewis, E. R. Lorch, Eugene Lukacs, N. H. McCoy, Brockway McMillan, L. A. MacColl, G. W. Mackey, H. M. MacNeille, Wilhelm Magnus, Irwin Mann, A. J. Maria, M. H. Maria, L. F. Markus, W. T. Martin, A. N. Milgram, K. S. Miller, Don Mittleman, E. E. Moise, E. F. Moore, Marston Morse, W. J. Nemerever, D. J. Newman, I. L. Novak, F. G. O'Brien, A. M. Ostrowski, J. C. Oxtoby, A. J. Penico, I. D. Peters, R. M. Peters, E. L. Post, Walter Prenowitz, W. W. Proctor, M. H. Protter, H. W. Raudenbush, G. E. Raynor, Helene Reschovsky, Moses Richardson, J. H. Roberts, M. S. Robertson, S. L. Robinson, David Rosen, Maxwell Rosenlicht, H. D. Ruderman, Walter Rudin, J. P. Russell, Jacob Samoloff, Arthur Sard, A. T. Schafer, R. D. Schafer, J. A. Schatz, Abraham Schwartz, I. E. Segal, D. B. Shaffer, Daniel Shanks, H. N. Shapiro, I. M. Sheffer, K. M. Siegel, Annette Sinclair, I. M. Singer, James Singer, J. J. Sopka, G. Y. Sosnow, S. K. B. Stein, C. F. Stephens, R. L. Sternberg, F. M. Stewart, Walter Strodt, R. L. Swain, R. L. Taylor, J. M. Thomas, M. L. Tomber, A. W. Tucker, Annita Tuller, A. H. Van Tuyl, B. W. Volkmann, H. V. Waldinger, G. C. Webber, J. V. Wehausen, H. F. Weinberger, Alexander Weinstein, David Wellinger, Franc Wertheimer, Albert Wilansky, Arthur Wouk, J. A. Zilber, Leo Zippin.

At a general session on Friday Professor Marston Morse of the Institute for Advanced Study delivered an invited address on *The* *generalized Fréchet variation and theorems of Riesz-Young-Hausdorff type.* Professor Nelson Dunford presided. An invited address on *Galois theory of differential fields* was delivered at a general session on Saturday by Professor E. R. Kolchin of Columbia University. Professor Orrin Frink presided.

The sessions for contributed papers on Friday were presided over by Professors Lipman Bers and Marshall Hall, Jr. Professor Einar Hille, D. C. Lewis, N. H. McCoy, and Arthur Sard presided over the sessions on Saturday.

The Trustees met at the Faculty Club at 6:00 P.M. on April 25.

The Council met at the Faculty Club at 8:00 P.M. on April 25.

The Secretary announced the election of the following one-hundred eleven persons to ordinary membership in the society:

Dr. Allan George Anderson, Instructor, Oberlin College;
Sister Madeleine Rose Ashton, President, College of the Holy Names, Oakland, California;
Mr. Stanley Edward Asplund, Teaching Fellow, Cornell University;
Miss Phyllis Hunt Belisle, Instructor, Bennett Junior College, Millbrook, New York;
Mr. Charles Bernard Bell, Jr., Research Laboratory Analyst, Douglas Aircraft Company, Inc., Santa Monica, California;
Mr. James Clyde Bradford, North Texas State College, Denton, Texas;
Mr. Wallace Franklin Branham, Mathematician, Long Range Proving Ground, Cocoa, Florida;
Mr. Frederick Arnold Brown, Chief, Computers Branch, T. T. D., Headquarters Air Force Missile Test Center, Melbourne, Florida;
Assistant Professor John Lawrence Brown, Jr., Pennsylvania State College;
Mr. Robert Eugene Bryan, Assistant in Instruction, Yale University;
Mr. Roger Thomas Businger, Chemical Engineer, Pittsburgh Plate Glass Company, Pittsburgh, Pennsylvania;
Mr. Paul Leo Butzer, Research Assistant, University of Toronto;
Mr. Jean-Francois Canu, Mathematician, U. S. Naval Proving Ground, Dahlgren, Virginia;
Mr. John W. Caulder, Assistant Director, Mech. Test Laboratory, Wright Field, Dayton, Ohio;
Mr. Wayne Hwa-wei Chen, Pre-doctoral Associate, University of Washington;
Mr. Heron Sherwood Collins, Teaching Fellow, Tulane University;
Mr. Virgil Lee Collins, State Teachers College, Troy, Alabama;
Mr. James Vernon Davis, Research Engineer, Fuller Brush Company, and Instructor, Hillyer College, Hartford, Connecticut;
Mr. Julian Leonard Davis, Associate Staff Member, Institute for Cooperative Research, Johns Hopkins University;
Mr. Eugene DeArmand, Instructor, East Central Junior College, Decatur, Mississippi;
Mr. Richard Clyde DePrima, Assistant Research Mathematician, Carnegie Institute of Technology;
Dr. George Francis Denton Duff, C. L. E. Moore Instructor, Massachusetts Institute of Technology;
Mr. Donald W. Dubois, Research Fellow, University of Oklahoma;

Mr. Robert L. Duncan, Research Assistant, Ordnance Research Laboratory, State College, Pennsylvania;
Mr. John Phillip Dunnett, U. S. Army in Korea, APO 248, c/o Postmaster, San Francisco, California;
Mr. Jack Gresham Elliott, Assistant, Michigan State College;
Mr. Eugene Rhodes Epperson, Instructor, Miami University;
Mr. John B. Escarda, Instructor, Silliman University, Philippines;
Mr. Michael Angelo Famiglietti, Mathematician, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland;
Mr. George Franklin Feeman, Graduate Assistant, Lehigh University;
Mr. William Robert Ferrante, Brown University;
Assistant Professor Raymond Ira Fields, Speed Scientific School, University of Louisville;
Mr. Abraham Bernard Finkelstein, Instructor, Long Island University;
Assistant Professor Patrick Lang Ford, McNeese State College, Lake Charles, Louisiana;
Mr. James Joseph Gehrig, Mathematician, Ballistic Research Laboratories, Aberdeen Proving Ground, Maryland;
Miss Bernice Goldberg, Mathematician, Geophysics Research Division, Air Force, Cambridge Research Center, Cambridge, Massachusetts;
Mr. Samuel Irving Goldberg, Research Scientist, Defense Research Board of C.A.R.D.E., Valcartier, Quebec;
Mr. Robert Melvin Gordon, Assistant in Instruction, Yale University;
Mr. Victor William Graham, Lecturer, Trinity College, Dublin, Ireland;
Professor Herrick Ernest Herbert Greenleaf, DePauw University;
Mr. Lowell Dean Gregory, Mathematician, Chance Vought Aircraft, Dallas, Texas;
Associate Professor Frederick Griffin, Philander Smith College, Little Rock, Arkansas;
Mr. Douglas Laird Guy, Graduate Assistant, Washington University;
Mr. Robert Ritzinger Hare, Jr., Air Force Missile Test Center, Patrick Air Force Base, Cocoa, Florida;
Miss Carolyn Harriet Hines, Instructor, Southern University, Baton Rouge, Louisiana;
Mr. Dan Barton Hoagland, Instructor, University of Missouri;
Mr. Stewart Baldwin Hobbs, University of New Hampshire;
Assistant Professor Williston C. Hobbs, Morris Brown College, Atlanta, Georgia;
Mr. Robert Earl Hux, University of New Hampshire;
Mr. Ding Hwang, Teaching Assistant, University of California, Berkeley, California;
Rev. Joseph Andrew Janiga, Instructor, St. Mary's College, Orchard Lake, Michigan;
Professor Frederick Klein, St. Louis University;
Assistant Professor George Klein, Mount Holyoke College;
Mr. Jose Quebingco Koppin, Physics Department, Silliman University, Philippines;
Mr. Eric Korngold, Melrose Bag and Burlap Company, New York, New York;
Mr. Harvey Newton Lance, Instructor, Mars Hill College, Mars Hill, North Carolina;
Professor William Isaac Layton, Stephen F. Austin State College, Nacogdoches, Texas;
Associate Professor Arthur Clifford Lindberg, Dana College, Blair, Nebraska;
Assistant Professor Julius O. Luck, Fairleigh Dickinson College, Rutherford, New Jersey;
Assistant Professor Everett William McClane, DePaul University;
Mr. Frank Joseph Malak, Instructor, Youngstown College, Youngstown, Ohio;

Mr. Clifford Wallace Marshall, Instructor, Long Island University;
Mr. Louis Clinton Marshall, Instructor, Southern University, Baton Rouge, Louisiana;
Mrs. Dorothy Cobb Martin, Instructor, Wood Junior College, Mathiston, Mississippi;
Mr. John Stanley Maybee, Physicist, David Taylor Model Basin, U. S. Navy, Carderock, Maryland;
Associate Professor Jean Sylve Mendousse, Catholic University;
Mr. Walter Eugene Mientka, Instructor, University of Massachusetts;
Mr. Raphael Miller, Yale University;
Lt. Duncan Edward Morrill, Commander, Reproduction Platoon, Aerial Photo Reproduction Company, Corps of Engineers, U. S. Army;
Assistant Professor Ray Bradford Murphy, Carnegie Institute of Technology;
Assistant Professor Stanley William Nash, University of British Columbia;
Mr. Ryre A. Newton, University of Georgia;
Mr. Wesley Lathrop Nicholson, Research Assistant, University of Oregon;
Assistant Professor Francis George O'Brien, Fordham University;
Mr. Morris A. Oliver, Head of Mathematics Department, Bennington College, Bennington, Vermont;
Assistant Professor Ingram Olkin, Michigan State College;
Mr. Eric M. Olson, Columbia University;
Mr. Raymond Edward Ozimkowski, Instructor, Fordham University;
Mr. Flemming Per Pedersen, Lecturer, University of Southern California;
Mr. Tullio Joseph Pignani, Instructor, University of North Carolina;
Mr. Morris Plotkin, Electronic Scientist, Naval Air Development Center, Johnsville, Pennsylvania;
Mr. Valdemars Punga, Associate in Mathematics, George Washington University;
Mr. James Milton Reynolds, Chairman, Alabama State College;
Mr. Louis Robinson, Fellow, Syracuse University;
Mr. James Purcell Rodman, Instructor, Mount Union College, Alliance, Ohio;
Mr. Lawrence Rosenfeld, Mathematician, Cambridge Research Center, Cambridge, Massachusetts;
Mr. J. Paul Roth, Research Assistant, Engineering Research Institute, Mathematics Group, University of Michigan;
Mr. Paul Theodore Schaefer, Graduate Assistant, University of Rochester;
Associate Professor Dan R. Scholz, Southwest Louisiana Institute, Lafayette, Louisiana;
Mr. Jerome Saul Shipman, Mathematician, Laboratory for Electronics, Inc., Boston, Massachusetts;
Mrs. Sedalia McAfee Sims, Instructor, Wiley College, Marshall, Texas;
Mr. Abe Sklar, 4834 North Troy Street, Chicago, Illinois;
Mr. Paul Slepian, Graduate Assistant, Brown University;
Mr. James Celestine Smith, Jr., Instructor, Loyola University of Los Angeles;
Mr. Judson Calkins Smith, Fellow, University of Washington;
Mr. Joseph M. Stein, Manager of Nuclear Engineering, Western Electric Corporation, Atomic Power Division, Homestead, Pennsylvania;
Mr. Warren Bernard Stenberg, Lecturer, University of California, Berkeley, California;
Miss Dorothy May Swan, Instructor, Monticello College, Godfrey, Illinois;
Rev. Robert John Teed, Instructor, Fournier Institute of Technology, Lemont, Illinois;

Miss Helen Jeane Terry, Instructor, University of Idaho;
Associate Professor Samuel Lothrop Thorndike, College of Emporia, Emporia, Kansas;
Mr. Norman Olcott Tiffany, Instructor, Moravian College for Women, Bethlehem, Pennsylvania;
Mr. Howard Gregory Tucker, University of California, Berkeley, California;
Dr. Dennis Peter Vythoulkas, Research Associate, University of Chicago;
Dr. Jack Warga, Mathematician, Reeves Instrument Corporation, New York, New York;
Mr. Paul Weiss, Mathematician, General Electric Company, Syracuse, New York;
Associate Professor Wayne Grant Wild, Buena Vista College, Storm Lake, Iowa;
Assistant Professor George Francis Will, Siena College, Londonville, New York;
Mr. Joseph Boyd Williams, Anderson College, Anderson, Indiana;
Mr. Mack Lester Williams, Teacher, North Texas State College, Denton, Texas;
Assistant Professor Ralph Arthur Willoughby, Georgia Institute of Technology.

It was reported that the following one-hundred fifty persons had been elected as nominees of institutional members as indicated:

University of Alabama: Assistant Professor Wayman L. Strother.
Brooklyn College: Messrs. Anatole Beck, George William Booth, and Alvin Hausner.
Brown University: Messrs. Donald Burke Lehman, Emin Turan Onat, Richard Thorpe Shield, Robert Norman Tompson, and Assistant Professor Harry J. Weiss.
California Institute of Technology: Mr. Don Elton Edmondson.
University of California, Berkeley: Messrs. Julius R. Blum, Edwin O. Elliott, William Robert Gaffey, Paul J. Koosis, Emanuel Parzen, Reese T. Prosser, and Joseph Putter.
University of California, Los Angeles: Mr. Peter Swerling.
Case Institute of Technology: Mr. Harold King Crowder.
University of Chicago: Messrs. Louis Auslander, Isidore Fleischer, Louis A. Kokoris, Bertram Kostant, Hazleton Mirkil, George William Morgenthaler, Ronald J. Nunke, Victor Lenard Shapiro, and Fred Boyer Wright.
City College of New York: Messrs. Jerome Harold Neuwirth and Howard Young.
Columbia University: Messrs. Hugh Gordon, Daniel Kocan, Adolf Nussbaum, Daniel Leo Slotnick, and Clifford Spector.
Cornell University: Messrs Douglas Page Baird, William A. Newcomb, and Steven Orey.
Duke University: Mr. Frank Roland Olson.
Harvard University: Mr. Marion Bayard Folsom, Miss Elsie Louise Goedeke, Professor Wassily W. Leontief, Messrs. Peter Evans Martin, William Francis Reynolds, Richard Steven Varga, and Calvin Hayden Wilcox.
Haverford College: Mr. W. Taylor Putney, III.
University of Illinois: Messrs. Arno Cronheim, Joseph Edward Flanagan, Lester R. Ford, William Alexander Michael, Jr., Charles Brusle Rheams, Kung-Sing Shih, Ray Frederick Spring, and Mrs. Joyce White Williams.
Indiana University: Mr. Byron Howard McCandless.
Institute for Advanced Study: Messrs. Nesmith C. Ankeny, Arrigo Finzi, Ernest Rufener, Jacques Leon Tits, and Leonce Guy Fourès.
Iowa State College of Agriculture and Mechanic Arts: Mr. Leslie D. Gates.
The Johns Hopkins University: Messrs. Solomon Wolf Golomb, Basil Gordon, Justin Gregory MacCarthy, Shirley Nickerson Mills, Jr., Bernard Mushinsky, Charles C. Oehring, Frank Chappell Ogg, John Barr O'Toole, John Thomas Robinson, and Mrs. Helena Long Watts.

Massachusetts Institute of Technology: Messrs. Felix Haas and Jack Kotik.
University of Michigan: Messrs. James Patrick Jans, Joachim Kaiser, Jr., William Jean Byrne, Myrle V. Cross, Jr., William Cassidy Fox, John William Jewett, Charles Christopher Kilby, Jr., Geert C. E. Prins, Ralph Alexis Raimi, Joseph Robert Schoenfield, and Drury William Wall.
University of Missouri: Messrs. Charles H. Cunkle, Joe D. Hankins, and Theral Orvis Moore.
Northwestern University: Messrs. Richard James Driscoll, Glenn Jay Kleinhesselink, and Frank Brooke Sloss.
Ohio State University: Mr. Pat Holmes Sterbenz.
Oklahoma Agricultural and Mechanical College: Mr. John Rolfe Isbell.
University of Oregon: Mr. Chia Kuei Tsao.
University of Pennsylvania: Messrs. John Carl Mairhuber and William Gideon Spohn, Jr.
Princeton University: Messrs. Arthur Charles Allen, Charles Harold Bernstein, Patrick Paul Billingsley, Humberto T. Cardenas, Courtney Stafford Coleman, Richard Henry Crowell, Karel Deleeuw, John Harry Gay, Louis Norberg Howard, Alan Treleven James, Joseph Bernard Kruskal, Jr., Thomas Eugene Kurtz, Solomon Leader, Eugene Alfred Maier, Hartley Rogers, Jr., Herbert Scarf, Oliver King Smith, Rene Thom, Bodo Walter Volkmann, and David M. G. Wishart.
Purdue University: Messrs. James Walter Armstrong, Jack Kenneth Hale, Arnold Herman Koschmann, and Carl James Sinke.
Queens College: Miss Roslyn Braverman.
University of Rochester: Mr. William Andrew Small.
College of St. Thomas: Mr. Richard Paul Goblirsch.
Syracuse University: Messrs. Howard Floyd Becksfort, Charles Robert Hicks, and Adnah Gould Kostenbauder, Jr.
University of Toronto: Messrs. Louis Lorne Campbell, Bruce John Kirby, Kerry Brian McCuthchon, Albert Wallace Walker.
Vanderbilt University: Mr. B. F. Bryant.
University of Washington: Mr. William Charles Guenther.
University of Virginia: Messrs. Roger Durgin Johnson, Jr., and Alexander Morton Maish.
Williams College: Mr. John Henry Hoelzer.
University of Wisconsin: Messrs. Chris Carl Braunschweiger, Robert Cleland Carson, Wayne Russell Cowell, Edmund Harry Feller, Jack Waring Hollingsworth, Donald Wright Miller, Lee Albert Rubel, Earl William Swokowski, and Albert Wayne Wymore.
Yale University: Messrs. John Young Barry, John Courtenay Holladay, Miss Marie Lesnick, Messrs. George Benham Seligman, Robert Barr Smith, and Herbert Aaron Steinberg.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: French Mathematical Society: Professor Roger Apery, University of Caen, Caen, France, Mr. Claude Berge, Centre National de la Recherche Scientifique, Nevilly, France, Mr. Rene Benjamin Chambaud, Consulting Engineer of Raffineries

Francaises de Petrole de l'Atlantique, Paris, France, Mr. Pierre Ernest Dolbeault, Attaché de recherches au Centre National de la Recherche Scientifique, Melakoff, France, Professor Francois Alain Limouzin, Retired, Poitiers, France, Professor Felix Pollaczek, Centre National de la Recherche Scientifique, Paris, France, Professor Yves Rene Thiry, Institut des Hautes Etudes, Tunis, Tunisia, Mr. Robert Gaston Velleneuve, Centre National d'Etudes des Telecommunications, Paris, France; London Mathematical Society: Dr. Jan Kalicki, Visiting Assistant Professor, University of California, Berkeley, and Dr. William Barry Pennington, Benjamin Pierce Instructor, Harvard University; Wiskundig Genootschap: Visiting Assistant Professor Johan H. B. Kemperman, Purdue University.

The following actions taken by mail vote of the Council were reported: election of Professors G. A. Hedlund and R. L. Wilder as members of the Executive Committee of the Council for a period of two years beginning January 1, 1952; acceptance by the Council of an invitation from the North Carolina State College of Agriculture and Engineering to hold a sectional meeting at Raleigh on November 28–29, 1952; and approval of the Council to hold the 1952 Annual Meeting in St. Louis in conjunction with the annual meeting of the AAAS.

The following appointments to represent the Society were reported: Professor C. B. Read at the dedication of Mossman Hall at Southwestern College, Winfield, Kansas, on March 22, 1952, and Professor F. J. Murray at a meeting of the Instrument Society of America on March 24, 1952.

The following additional appointments by the President were reported: Professors A. E. Heins (Chairman), P. Chiarulli, P. Gustafson, G. H. Handelman, L. E. Malver, R. C. Meacham, D. Moskovitz, E. A. Whitman, and W. M. Whyburn as a committee on arrangements for the Fifth Symposium on Applied Mathematics to be held at the Carnegie Institute of Technology, Pittsburgh, June 16–17, 1952; Professors J. S. Frame (Chairman), J. H. Bell, H. M. Gehman, P. Herzog, L. Hatz, E. A. Nordhaus, H. E. Stelson, B. M. Stewart, and J. W. T. Youngs as a committee on arrangements for the Summer Meeting of 1952 to be held at East Lansing, Michigan, on September 3–6, 1952; Professor J. M. Clarkson (Chairman), R. C. Bullock, Jack Levine, C. G. Mumford, J. M. Thomas, and W. M. Whyburn as a committee on arrangements for the meeting to be held at North Carolina State College of Agriculture and Engineering at Raleigh on November 28–29, 1952; Professor R. R. Middlemiss (Chairman), Dr. H. Margaret Elliott, Professors T. L. Downs,

H. M. Gehman, and J. W. T. Youngs as a committee on arrangements for the Annual Meeting to be held in St. Louis on December 27–29, 1952; Professors E. G. Begle (Chairman), C. B. Allendoerfer, A. E. Meder, Jr., G. B. Price, and A. W. Tucker as a committee to consider problems of controversial questions both as to procedures of the Council and as to the membership of the Society; Dr. H. M. MacNeille (Chairman), E. G. Begle, Jekuthiel Ginsburg, A. E. Meder, Jr., and J. V. Wehausen, as a committee to advise the Council and Board of Trustees concerning the problem of exchanges of the Society's publications; Professor Marshall Hall as a member of the committee to select hour speakers for Western Sectional Meetings (committee now consists of J. W. T. Youngs (Chairman), P. R. Halmos, and Marshall Hall); Professors M. H. Heins (Chairman), A. A. Albert, Richard Arens, Dr. Mina Rees, and Professor J. M. Thomas as a committee to nominate officers and members of the Council for 1953; Professor W. T. Martin (Chairman), Dean W. L. Ayres, and Professor J. C. Oxtoby as a committee to consider the advisability of continuing the Society's reprinting program.

The following items were reported for the information of the Council: selection of G. B. Price as Managing Editor of the Bulletin Editorial Committee; R. L. Wilder as Chairman of the Colloquium Editorial Committee; William Feller as Chairman of the Mathematical Reviews Editorial Committee; W. T. Martin as Chairman of the Mathematical Surveys Editorial Committee; G. A. Hedlund as Chairman of the Proceedings Editorial Committee; and G. T. Whyburn as Managing Editor of the Transactions and Memoirs Editorial Committee; as new members of the Policy Committee for Mathematics: S. S. Cairns and J. S. Frame (the committee now consists of J. R. Kline (Chairman), Marston Morse, W. T. Martin, and S. S. Cairns for the American Mathematical Society; H. M. Gehman, Saunders MacLane, and J. S. Frame for the Mathematical Association of America; Henry Scheffé for the Institute of Mathematical Statistics; E. H. C. Hildebrandt for the National Council of Teachers of Mathematics. (The representative of the Association for Symbolic Logic has not yet been appointed.)); acceptance by Antoni Zygmund of an invitation to deliver the Colloquium Lectures at the Summer Meeting of 1953; acceptance by Marston Morse of an invitation to deliver the 26th Josiah Willard Gibbs Lecture at the Annual Meeting of 1952; Committees to Select Hour Speakers have invited Dr. G. E. Forsythe to deliver an address at the Eugene, Oregon, meeting on June 21; Professor E. E. Moise, Summer Meeting at East Lansing; J. Leray, Yale University, New Haven, on October 25, 1952; Professor A. M. Gleason at the Annual Meeting of 1952; Professor

W. L. Chow for the February 1953 meeting in New York City; Professors Emil Artin and W. S. Massey at the April 1953 meeting in New York City.

It was reported that the First General Assembly of the International Mathematical Union was held in Rome, March 6–8, 1952. The United States was represented by Professors M. H. Stone, J. R. Kline, Einar Hille, Saunders MacLane, and G. T. Whyburn. Professor Stone was elected President of the Union.

The Council voted to approve the following dates of meetings of the Society: November 28–29, 1952, Purdue University; February 28, 1953, Hunter College; April 24–25, 1953, New York University. The Council voted to approve the week of August 31 for the time of the Summer Meeting of 1953.

The Council voted to approve the substitution of Professor Irving Kaplansky for Professor Reinhold Baer as a representative of the Society on the Board of Editors of the American Journal of Mathematics for the period June 1952 to September 1953.

The Council voted to approve the report of the Committee on Exchanges, which recommended that the policy of the Society be to discontinue all exchanges, after we have fulfilled our present obligations to the University of Georgia, except that the Executive Editor of Mathematical Reviews be authorized to enter into exchanges of the Society's publications for the benefit of Mathematical Reviews and that the Secretary be authorized to negotiate exchanges in order to further the scientific objectives of the Society, through widening the distribution of our publications.

The Council voted to recommend to the membership of the Society that the by-laws be amended as follows: (1) to eliminate the post of Librarian; (2) to create a separate elected Editorial Board for the Memoirs and an elected Committee on Printing and Publishing; (3) to revise Article IX of the by-laws to eliminate the initiation fee and the reduced rates for early years of membership and to eliminate fractional years of membership; (4) to eliminate obsolete sections of Articles IV and VII.

The Council voted to request the President to appoint a committee to consider the problems involved in nominating officers and members of the Council by petition from the membership of the Society.

The Council referred to the Executive Committee, with power to act, current problems concerning our publications.

The Board of Trustees met Friday evening. The Board voted that institutional dues for the period July 1952 to June 1955 be computed on the basis of a page rate of $5.00. The Board voted to set the page charge at $10 for institutions which are not institutional members.

Abstracts of the papers presented are listed below, those with a "t" after their numbers having been read by title. The joint papers numbered 340, 372, 373, 376, and 381 were read by Dr. Singer, Professor Miller, Mr. Crispin, Dr. Juncosa, and Dr. Lukacs, respectively.

Algebra and Theory of Numbers

319. A. A. Albert: *New power-associative algebras and their derivations.*

It is now known that there exists a central simple commutative power-associative algebra S_p of degree two and dimension $4p$ over a field F of characteristic p. It contains a central simple algebra T_p of degree two and dimension $3p$. The algebra S_p contains a nontrivial idempotent u and is u-stable. However T_p is unstable and so S_p is also unstable. The algebra T_p contains the polynomial algebra $B=F[1, k]$, where $k^p=0$ and the derivation algebra of B is the well known algebra W_p of Witt. We show that the derivation algebra of T_p is the direct sum of W_p and the three-dimensional simple Lie algebra, and that this is the same as the derivation algebra of S_p. We also construct a new class of central simple commutative power-associative algebras of degree two and dimension $(n+3)p^n$ with corresponding central simple subalgebras of dimension $(n+2)p^n$. (Received March 10, 1952.)

320. Joseph Andrushkiw: *Homoreal rings.* Preliminary report.

A ring is real if it is a domain of integrity and zero is not expressible in it as a sum of squares of elements different from zero. Since the field of quotients of a real ring is also real, any real ring can be ordered. A ring R is termed "homoreal" if there exists a real ring $\bar{R}$ homomorphic with it. Dependent upon whether in R either both conditions of reality or only first or only second or none of them are satisfied, four kinds of the homoreal rings can be distinguished. A ring is homoreal, if and only if there exists in it a prime ideal E, different from the null ideal, with the following property: a sum of squares $\sum_1^n e_j^2 \in E$ if and only if all $e_j \in E$. If R is a homoreal ring of the first or second kind, each element of E different from zero is transcendental in the field of quotients of R. Any homoreal ring R of the first kind can be ordered in non-Archimedean way. There exists an order of R in which $me<a$ where $e \in E$, a is an element of any positive residue class of the ordered ring R/E, and m any positive integer. (Received March 10, 1952.)

321t. Leonard Carlitz: *A note on common index divisors.*

Let the prime $p\equiv 1 \pmod 3$ and let C_3 denote the cubic subfield of $Z=k(e^{2\pi i/p})$. By a well known theorem of Hensel, 2 is a common index divisor of C_3 if and only if $p=a^2+27b^2$. In this note we derive several theorems of a similar kind. We quote the following results. Let $p\equiv 1 \pmod 4$ and let C_4 denote the quartic subfield of Z. 1. Then 2 is a common index divisor of C_4 if and only if $p\equiv 1 \pmod 8$. 2. Let $p=a^2+b^2$, $a\equiv 1$, $b\equiv 0 \pmod 2$. Then 3 is a common index of C_4 if and only if (i) $3|b$ for $p\equiv 1 \pmod 8$, (ii) $3|a$ for $p\equiv 5 \pmod 8$. Like results are obtained for C_6, C_8, C_{12}. (Received January 11, 1952.)

322t. Leonard Carlitz: *A theorem of Dickson on irreducible polynomials.*

In connection with the study of modular invariants Dickson (Trans. Amer.

Math. Soc. vol. 12 (1911) pp. 1–18) determined the number of irreducible polynomials $x^3-x^2+ax+b^2$, where a, $b\in GF(p^n)$, $p>2$. In the present paper the author first finds an asymptotic formula for the number of irreducible polynomials of a given degree $Q(x)=x^m+c_1x^{m-1}+\cdots+c_m$, $c_i\in GF(p^n)$, with assigned first coefficient c_1 and last coefficient c_m. Secondly he finds exact formulas connected with the number of irreducibles with assigned first coefficient and last coefficient equal to a square of the field, thus generalizing Dickson's result. Finally he determines the number of irreducibles with assigned first coefficient. (Received January 11, 1952.)

323t. Leonard Carlitz: *Note on irreducibility of the Bernoulli and Euler polynomials.*

The following special results are obtained. 1. Let p denote an odd prime. Then $B_{m(p-1)}(x)$ is irreducible for $1\leqq m\leqq p$; also $B_m(x)$ is irreducible for $m=2^r$ and $m=k(p-1)p^r$, $1\leqq k<p$, $r\geqq 1$. 2. $B_{2m+1}(x)/x(x-1/2)(x-1)$, where $2m=k(p-1)$, $k\leqq p$, is either irreducible or has an irreducible factor of degree $\geqq 2m+1-p$. 3. If $p\equiv 3 \pmod 4$, $r\geqq 1$, then $E_{p^r}(x)/(x-1/2)$ is irreducible. 4. $E_{2p}(x)/x(x-1)$ is irreducible or has an irreducible factor of degree not less than $p+1$. 5. If $B_m^{(k)}$ denotes the Bernoulli number of order k in Nörlund's notation, then $B_{p-1}^{(x)}/x$ is irreducible. (Received March 12, 1952.)

324t. Leonard Carlitz: *Some formulas of Oltramare.*

Dickson (*History of the theory of numbers*, vol. 1, p. 277) reproduces the following congruences due to G. Oltramare: $1+(m!)^4\equiv -2\{(1/3)^2+(1\cdot 5/3\cdot 7)^2+(1\cdot 5\cdot 9/3\cdot 7\cdot 11)^2+\cdots\} \pmod{4m+1}$, $2^5+(m!)^4\equiv -2^6\ \{(3/1)^2+(3\cdot 7/1\cdot 5)^2+(3\cdot 7\cdot 11/1\cdot 5\cdot 9)^2+\cdots\} \pmod{4m+3}$, where the moduli are primes. In the present note it is indicated how many congruences of this sort can be obtained by means of known formulas involving generalized hypergeometric series. (Received February 15, 1952.)

325t. Leonard Carlitz: *Sums of primitive roots of the first and second kind in a finite field.*

Let $e_i\,|\,p^{nm}-1$, $k_i\,|\,p^{nm}-1$, $A(x)\,|\,x^m-1$, where $A_i(x)\in GF[p^n, x]$, and let $a_i(x)$ be the linear polynomial corresponding to $A_i(x)$. Let β_i denote a number of $GF(p^{nm})$ that simultaneously belongs to the exponent e_i and to the polynomial $a_i(x)$ (as defined by Ore). Extending the results of a previous paper we here consider the following problems. 1. The number of solutions $\beta_1, \cdots, \beta_r$ of $\alpha=\alpha_1\beta_1+\cdots+\alpha_r\beta_r$, where $\alpha_i\neq 0$. 2. The number of solutions $\beta_1, \cdots, \beta_r$, $\xi_1, \cdots, \xi_s$ of $\alpha=\alpha_1\beta_1+\cdots+\alpha_r\beta_r+\delta_1\xi_1^{k_1}+\cdots+\delta_s\xi_s^{k_s}(\xi_1\in GF\,(p^{nm}))$. Asymptotic results are obtained in both problems. 3. In the special case $r=1$, s even, a simple exact formula is obtained. (Received February 15, 1952.)

326. P. W. Carruth: *Products of ordered systems.*

Some miscellaneous results concerning ordered products of ordered systems are given (see Day, Trans. Amer. Math. Soc. vol. 58 (1945) pp. 1–43 for definitions of terms involved). For example, it is shown that the only products that are complemented or relatively complemented lattices are cardinal products. Conditions for a product of lattices to be a lattice and for a product of partially ordered sets to satisfy the ascending chain condition are given. In the proofs use is made of results of Day, loc. cit. (Received February 20, 1952.)

327t. Eckford Cohen: *A finite analogue of the Goldbach problem.*

If $m>1$ is an integer with factorization $m=p_1^{\lambda_1}\cdots p_r^{\lambda_r}$, then the elements of the residue class ring R_m can be expressed in the form $p_1^{a_1}\cdots p_r^{a_r}\xi$ where $0\leqq a_i\leqq\lambda_i$, $(\xi, m)=1$. This factorization is unique except for the unit ξ, and the totality of elements of the form $p_i\xi$ are the prime elements of R_m (cf. Vandiver, Trans. Amer. Math. Soc. vol. 13 (1912) p. 293). The following theorem is proved: *There exists a number $g(m)$ such that every number of R_m is expressible as a sum of $g(m)$ primes in R_m if and only if m has at least two distinct prime factors. For such m, the minimum value k of $g(m)$ is given by $k=2$ if m is odd, by $k=3$ if m is even and has at least two distinct odd prime factors or if m is twice an odd prime power, and by $k=4$ if m is of the form $m=2^\lambda p^\mu$, $\lambda>1$, $\mu\geqq 1$, p odd.* Several related theorems are also proved. (Received March 10, 1952.)

328t. Eckford Cohen: *Arithmetic functions in algebraic fields.*

If A is an arbitrary ideal of an algebraic number field F, and K is a field of characteristic 0 containing all the roots of unity, then a single-valued function f is (A, K) arithmetic if, for every integer $\alpha\in F$, $f(\alpha)\in K$, and $f(\alpha)=f(\alpha')$ for $\alpha\equiv\alpha'$ (mod A). It is shown that every such function is uniquely expressible as a sum of exponential functions of the type introduced by Hecke (*Vorlesungen über Zahlentheorie*, 1923, p. 220). It is then shown how exponential formulas for a large class of additive congruence problems in algebraic fields can be constructed. The set of all (A, K) functions forms a commutative semi-simple algebra with multiplication defined by "Cauchy composition" (cf. Carlitz, Duke Math. J. vol. 14 (1947) p. 1121). (Received March 10, 1952.)

329t. Eckford Cohen: *Congruence representations in algebraic number fields.*

Applications of the ideas in the preceding abstract are developed. The arithmetic function of Rademacher (Math. Zeit. vol. 27 (1928) p. 332) and the Hecke generalization of the Gauss sums (cf. preceding abstract) are discussed and applied. In particular, the number of representations of an algebraic integer as a sum of products, relative to an ideal modulus, is found. Explicit formulas for the number of solutions ξ_i of $\rho\equiv\sum_1^s\alpha_i\xi_i^2$ (mod P^λ), where P is an odd prime ideal, $\lambda\geqq 1$, and ρ, α_i are algebraic integers, $(\alpha_i, P)=1$, are derived. From this result it is shown that every algebraic integer can be expressed as a sum of three squares in the ring formed by the residue classes of algebraic integers modulo an arbitrary odd ideal. (Received March 10, 1952.)

330t. Anne C. Davis: *Cancellation theorems for products of order types.* II. Preliminary report.

For notational conventions see Bull. Amer. Math. Soc. Abstract 58-1-77; in addition, "$\alpha\leqq\beta$" means (Fraïssé, C. R. Acad. Sci. Paris vol. 226, p. 1330) that there exists an order-preserving mapping of an ordered set A of type α onto a subset of an ordered set B of type β. Lemma I. *If $\alpha\cdot\beta=\gamma\cdot\delta$, then at least one of the following conditions must hold*: (i) $\beta=\delta$, (ii) *there exist order types μ_1, μ_2, ρ such that $\alpha=\mu_1+\gamma\cdot\rho+\mu_2$*, (iii) *there exists order types μ_1, μ_2, ρ such that $\gamma=\mu_1+\alpha\cdot\rho+\mu_2$*, (iv) *there exist finite order types ν and ϕ, $\nu\neq\phi$, such that $\alpha\cdot\nu=\gamma\cdot\phi$*. Lemma I implies all cancellation theorems of Bull. Amer. Math. Soc. Abstract 58-1-77. A further consequence of Lemma I is Corollary II. *If α is the type of an ordered set without gaps and without first and last elements, and if $\alpha^2=\beta^2$, then $\alpha=\beta$.* Theorem III. *For every positive integer*

n, $\alpha+\alpha \leqq \alpha \leftrightarrow \alpha^n+\alpha^n \leqq \alpha^n$. By use of Lemma I, in conjunction with Theorem III, one obtains Corollary IV. *If* $\alpha^n=\beta^n$, *n a positive integer, and if* $\sim(\alpha+\alpha \leqq \alpha)$, *then* $\alpha=\beta$. It is easily seen that Corollary IV is applicable, in particular, if α, or β, is a scattered type. (Received March 10, 1952.)

331. J. K. Goldhaber: *Special types of linear mappings of algebras.*

Let $\mathfrak{M}$ be a total matric algebra over a field whose characteristic is different from 2, and let Φ be a linear mapping of $\mathfrak{M}$ into $\mathfrak{M}$. Suppose that Φ preserves idempotents and nilpotents of index two, that is, for $A \in \mathfrak{M}$ if $A^2=A$ then $[\Phi(A)]^2=\Phi(A)$ and if $A^2=0$ then $[\Phi(A)]^2=0$. Then either $\Phi(A)=0$ for all $A \in \mathfrak{M}$ or Φ is an automorphism or an anti-automorphism of $\mathfrak{M}$. This theorem does not extend to simple algebras. However, the following theorem is established: Let $\mathfrak{A}$ be a central simple algebra over a field of characteristic different from 2, and let Φ be a linear mapping of $\mathfrak{A}$ into $\mathfrak{A}$ such that Φ maps the identity of $\mathfrak{A}$ into the identity of $\mathfrak{A}$ and such that for $A \in \mathfrak{A}$, A and $\Phi(A)$ have the same characteristic equation, then Φ is either an automorphism or an anti-automorphism of $\mathfrak{A}$. If $\mathfrak{A}$ is a semi-simple separable algebra, then Φ induces either an automorphism or an anti-automorphism on every simple component of $\mathfrak{A}$. (Received March 10, 1952.)

332. Marshall Hall, Jr.: *Subgroups of free products.*

This is a new proof of the theorem of Kurosch that every subgroup of a free product G is itself a free product of factors which are either free groups or conjugates of subgroups of the free factors of G. It depends upon defining a semi-alphabetical well ordering for the elements of G, and in terms of this finding a set of elements generating the subgroup. This set exhibits the free factors explicitly and the proof consists in showing that these are indeed free factors. (Received March 11, 1952.)

333*t*. C. E. Rickart: *Spectral permanence for certain Banach algebras.*

Let A be a commutative Banach algebra which is algebraically embedded in a second such algebra B, and let ϕ be a homomorphism of A into the complex numbers. If the embedding of A in B is an isometry, then Šilov [Uspehi Matematičeskih Nauk N.S. vol. 1 (1946) pp. 48–146] has shown that certain ϕ can be extended to B. It is proved here that, if A is regular in a sense defined by Šilov [Travaux de l'Institut Mathématique Stekloff vol. 21 (1947)], then ϕ can always be extended to B. This implies that elements of A have the same spectra in A as in B and includes a result of Kaplansky [Duke Math. J. vol. 16 (1949) pp. 399–418] concerning the minimal character of the norm in a Banach algebra of all continuous functions on a compact space. Some results are also obtained here for the noncommutative case. For example, let A be a B^*-algebra [cf. Rickart, Ann. of Math. vol. 47 (1946) pp. 528–550] which is algebraically embedded in an arbitrary Banach algebra B. Then elements of A always have the same spectral radii in A as in B. If in addition the involution in A can be extended to B, then elements of A have the same spectra in A as in B. If the norm in B is invariant in A under the involution, then the norm in A cannot be greater than the norm in B for elements of A. (Received March 10, 1952.)

334. Maxwell Rosenlicht: *Generalized Jacobian varieties.*

A semilocal subring of the function field of an algebraic curve defines an equivalence relation among the divisors of the curve (cf. Bull. Amer. Math. Soc. vol. 57 (1951) p. 269). A generalized Jacobian variety corresponding to this equivalence relation can be constructed; this is a commutative algebraic group variety whose points are in a natural one-one correspondence with divisor classes of degree zero on the curve. If two equivalence relations, one of which is stricter than the other, are given on the same curve, then there is a natural rational homomorphism of one generalized Jacobian onto the other. The kernel of this homomorphism is a rational group variety whose structure can be explicitly described. A treatment of the classical generalized Jacobi inversion problem is included in this work. (Received March 10, 1952.)

335. R. D. Schafer: *The Casimir operation for alternative alegbras.*

Let $\mathfrak{A}$ be a semisimple alternative algebra of characteristic 0, and $\mathfrak{B}$ be an irreducible alternative module for $\mathfrak{A}$. For any bilinear mapping $f(a, b)$ of $\mathfrak{A}$ into $\mathfrak{B}$, define $F(a, b, c) = f(a, b)c + f(ab, c) - af(b, c) - f(a, bc)$, $f^+(a, b) = f(a, b) + f(b, a)$. If $F(a, b, c) = F(b, c, a) = -F(b, a, c)$, then there exists a linear mapping $g(a)$ of $\mathfrak{A}$ into $\mathfrak{B}$ such that $f^+(a, b) = a \cdot g(b) + g(a) \cdot b - g(a \cdot b)$, where $a \cdot b$ and $a \cdot v$ denote the operations in $\mathfrak{A}^+$ and $\mathfrak{B}^+$. This lemma, which is proved by means of a Casimir operation Γ associated with any representation (S, T) of $\mathfrak{A}$, has the force of the Wedderburn principal theorem for the Jordan algebra attached to any alternative algebra, and allows for some simplification in the proof of the Wedderburn principal theorem for alternative algebras of characteristic 0. (Received March 12, 1952.)

336*t*. Daniel Shanks: *The quadratic reciprocity law.*

The reciprocity law concerning two odd primes, $P = 2p+1$ and $Q = 2q+1$, is proven directly from the Gauss Lemma in perhaps the simplest and most natural way. Consider the pq numbers: $F(x, y) = xQ - yP$ with $x = 1, 2, \cdots, p$ and $y = 1, 2, \cdots, q$ and assume that there are m solutions of $-p \leqq F < 0$ and n solutions of $0 < F \leqq q$. There is no solution of $F = 0$ and the solutions of $F < -p$ and of $F > q$ may be put into 1-1 correspondence by the symmetrical relations: $x_1 + x_2 = p + 1$ and $y_1 + y_2 = q + 1$. Therefore $m + n \equiv pq$ (mod 2). Since, by the lemma, $(Q/P) = (-1)^m$ and $(P/Q) = (-1)^n$, we thus obtain $(Q/P)(P/Q) = (-1)^{pq}$. Landau [*Aus der elementaren Zahlentheorie*], by considering $F < 0$ and $F > 0$, proves $pq = \sum_{s=1}^{p} [sQ/P] + \sum_{s=1}^{q} [sP/Q]$. By dividing F into twice as many parts, twice as much is proved. The partition and proof given above may also be seen geometrically on a modified Eisenstein lattice. (Received March 12, 1952.)

337. M. L. Tomber: *Lie algebras of type* F.

Let $\mathfrak{L}$ be a Lie algebra over an arbitrary field Φ of characteristic 0, of type F, in the sense that $\mathfrak{L}_\Omega$ is the exceptional simple Lie algebra F_4 where Ω is the algebraic closure of Φ. It is proved that $\mathfrak{L}$ is the derivation algebra $\mathfrak{D}(\mathfrak{J})$ of an exceptional Jordan algebra $\mathfrak{J}$ over Φ and $\mathfrak{D}(\mathfrak{J}_1) \cong \mathfrak{D}(\mathfrak{J}_2)$ if and only if $\mathfrak{J}_1 \cong \mathfrak{J}_2$. The proofs employ the methods of N. Jacobson (Duke Math. J. vol. 5 (1939) pp. 775–783) and the result that any automorphism of F_4 (over Ω) has the form $D \rightarrow S^{-1}DS$ for a unique automorphism S of the exceptional Jordan algebra over Ω. (Received March 11, 1952.)

338. Bodo Volkmann: *On the digits of real numbers.* Preliminary report.

Let $g \geqq 2$ be an integer and $F = (f_1, f_2, \cdots, f_i)$ be any block of digits in the g-adic system. This paper discusses the set $K_{F,g}$ of real numbers in the unit interval in whose g-adic expansion the block F does not occur. If $\mathfrak{P}(F) = \{p_1, p_2, \cdots, p_c\}$ denotes the (possibly empty) set of those integers p with $0 < p < i$ for which the block $(f_1, f_2, \cdots, f_p)$ equals the block $(f_{i-p+1}, f_{i-p+2}, \cdots, f_i)$, let $\gamma_{F,g}$ be the greatest real root of the equation $\xi^i - g\xi^{i-1} + \sum_{y=1}^{c} \xi^{i-p_y} - g\sum_{y=1}^{c} \xi^{i-p_y-1} + 1 = 0$, the sums being zero if $\mathfrak{P}(F)$ is empty. The Hausdorff dimension of $K_{F,g}$ turns out to be $\log \gamma_{F,g}/\log g$. This result was proved by E. Best (J. London Math. Soc. (2) vol. 47 (1942) pp. 436–454) in the special case $i=1$, $g\geqq 2$ and by the author (forthcoming in J. Reine Angew. Math.) in the special case $g=2$, $i\geqq 2$, $F=(1, 1, \cdots, 1)$. (Received March 11, 1952.)

Analysis

339. A. C. Allen: *The closure of the translations of sets of functions of L_1, and certain applications to harmonic functions.* Preliminary report.

N. Wiener (*The Fourier integral*, chap. 2) gave a series of theorems on the closure of the translations of a function of L_1, the significant class of functions being considered being those whose Fourier transforms do not vanish for any real values of the argument. In this paper the author gives analogous theorems for $n\times n$ matrices whose elements are functions of L_1; "translation" being interpreted as "folding" with a certain translation-matrix; and the significant class of function-matrices considered being those in which the determinant, whose elements are the Fourier transforms of the elements of the matrix, does not vanish for any real value of the argument. As an application of the above, certain generalizations of the Phragmén-Lindelöf type theorem for harmonic functions and of Montel's theorem for bounded analytic functions are derived. The methods used, however, are applicable to a wider class of functions than those which satisfy Laplace's equation, and so have an advantage over methods of proof which depend on conformal mapping. (Received March 12, 1952.)

340. Warren Ambrose and I. M. Singer: *L_2-matrices.*

It is shown that some H-systems are matrix-like. (Received March 12, 1952.)

341. J. H. Barrett: *Differential equations of nonintegral order.*

Let α be a complex number; n the integer such that $0<R\alpha+n\leqq 1$; $f(x)$ a real function, a a real number. $I\{\alpha, a, x, f\}=\int_a^x f(t)((x-t)^{\alpha-1}/\Gamma(\alpha))dt$, $R\alpha>0$; $I\{\alpha, a, x, f\}=D_x^n I\{\alpha+n, a, x, f\}$, $R\alpha\leqq 0$. For $(x-t)^{\alpha-1}$ use the principal value. This is called the Riesz-Holmgren transform. An extension of a result of Hardy's (Messenger of Mathematics vol. 47 (1918) pp. 145–150) is if $X>a$, $f(x)$ is Lebesgue summable on $a\leqq x\leqq X$, $R\alpha>0$, $R\beta>0$, then $I\{\alpha, a, x, I(\beta, a, x, f)\}=I\{\alpha+\beta, a, x, f\}$ and $D_x I\{\alpha+1, a, x, f\}=I\{\alpha, a, x, f\}$ almost everywhere on $a\leqq x\leqq X$. Let $f(x)$ be a Lebesgue summable function on $0<x\leqq X$. $y(x)$ is called a solution of $I\{-\alpha, 0, x, y\}+y(x)=f(x)$, $R\alpha>0$, $0<x<X$, provided that $y(x)$ is Lebesgue summable, $I\{1-\alpha, 0, x, y\}$ exists and is absolutely continuous in x, and $y(x)$ satisfies the equation a.e. on $0<x<X$. The differential equation is transformed into an integral equation, which is solved by means of an inverse of the operator: $I\{0, 0, x, y\}-I\{\alpha, 0, x, y\}$. Let $K_0, K_1, \cdots, K_{n-1}$, where $0<n-R\alpha\leqq 1$, be real numbers; then there is a unique solution satisfying the boundary conditions: $I\{n-\alpha-p, 0, 0^+, y\}=K_p$, $p=0, 1, \cdots, n-1$. Let $U_q(x)$ be the solution where $K_p=0$, $p\neq q$, and $K_q=1$ and $f(x)\equiv 0$. The solution of the nonhomogeneous equation is $y(x)=\sum_{i=0}^{n-1} K_i U_i(x)+\int_0^x f(t)U_{n-1}(x-t)dt$, $x>0$. (Received March 12, 1952.)

342t. E. F. Beckenbach: *A property of mean values of an analytic function.*

Let $w=f(z)$ be analytic in the circle $|z|<1$, and let the length of the map of each radius of the circle be $\leqq 1$. Bounds are obtained for the lengths of the images of concentric circles $|z|=\rho$, $0<\rho<1$. This extends a result of Nehari (C. R. Acad. Sci. Paris vol. 208 (1939) pp. 1785–1787). (Received February 29, 1952.)

343t. Stefan Bergman: *The coefficient problem in the theory of linear partial differential equations.*

Let ψ be a solution of $\psi_{xx}+\psi_{yy}+F\psi=0$, $\psi_{xx}\equiv\partial^2\psi/\partial x^2, \cdots$, where F is an entire function. Many relations between the properties of the subsequence $\{a_{m0}\}$, $m=0, 1, 2, \cdots$, of coefficients of a real solution $\psi(x, y)=\sum_{m,n} a_{mn}z^m\bar{z}^n$, $z=x+iy$, $\bar{z}=x-iy$, on one side and location and character of singularities of ψ on the other are independent of F. (See, e.g., Trans. Amer. Math. Soc. vol. 57, p. 299 ff.). If $g(z)=\sum_m a_{m0}z^m$ is a function element of an algebraic function, then the ψ can be represented in the form $\psi=\text{Re}(\sum_{n=1}^{\infty} Q^{(n)}g_n)$ where $Q^{(n)}$ are functions which depend only upon F, and g_n, $n=0, 1, 2, \cdots$, are functions which can be represented in a closed form using θ-functions, their derivatives and integrals of the first kind defined on the Riemann's surface of $g(z)$. Similar theorems are derived for solutions of differential equations $\sum_{\nu=1}^{3} (\partial^2\psi/\partial x_\nu^2)+F(r^2)\psi=0$, where $F(r^2)$ is an entire function of $r^2=\sum_{\nu=1}^{3} x_\nu^2$. (Received April 25, 1952.)

344. Jerome Blackman: *Some notes on the backward heat equation.*

Consider the classical solution of the heat equation on the infinite rod with initial condition $\phi(x)$, given by $u(x, t)=\int_{-\infty}^{\infty}\phi(s) \exp (x-s)^2/4t ds$. If $\phi(s)=O(\exp s^2)$, this integral may diverge for a certain $t=T$ but converge for $t<T$. The following problems are investigated: (1) Can lim $u(x, t)$ exist as $t\rightarrow T-$? (2) What properties must this limit have if it exists? The analytic character of the limit is demonstrated; an example and a method of determining a class of examples is given. The methods are applicable to the solution of the backward heat equation $u_{xx}+u_t=0$ for the infinite rod, and demonstrate the relation between uniqueness theorems for the heat equation and uniqueness theorems for certain types of summability of trigonometrical integrals. (Received March 5, 1952.)

345t. R. P. Boas: *Growth of analytic functions along a line.*

Results reported earlier (Bull. Amer. Math. Soc. Abstract 58-2-156) are improved as follows. Let $\epsilon(x)$ be positive, increasing, $\epsilon(x)=o(x)$, $\log x=o(\epsilon(x))$, and let $\epsilon(x)$ satisfy some minor conditions of smoothness and monotonicity; let the increasing sequence $\{\lambda_n\}$ of real numbers satisfy $|\lambda_{n+1}-\lambda_n|\geqq\delta>0$ and $|n-\lambda_n|\leqq\epsilon(n)$. Then if $f(z)$ is of exponential type less than π in the right half-plane, $\log |f(\lambda_n)|\leqq O(\epsilon(\lambda_n))$ implies $\log |f(x)|\leqq O(\epsilon(x))$. If in addition $(*)\int^{\infty}x^{-2}\epsilon(x)dx$ converges, if $f(z)$ is an entire function of exponential type less than π, and if $\log |f(\pm\lambda_n)|\leqq -B\epsilon(\lambda_n)$ with a sufficiently large B, then $f(x)$ is bounded. If $f(z)$ is entire and of zero exponential type, (*) converges and $\{f(\pm\lambda_n)\}$ is bounded, $f(z)$ is a constant; if $|n-\lambda_n|\leqq n^\rho$ and $\log |f(\pm\lambda_n)|\leqq A\lambda_n^\rho$, $0<\rho<1$, then $f(z)$ is of order at most ρ. The first result for functions of zero type improves a result of Levinson as far as concerns the size of $\epsilon(n)$ [*Gap and density theorems*, New York, 1940, pp. 129–130]. (Received March 24, 1952.)

346t. F. E. Browder: *Fundamental solutions in the large for linear elliptic equations of arbitrary order.*

Let K be a suitably differentiable linear elliptic differential operator on a bounded domain D of E^n. If the Dirichlet problem for K on D has at most one solution with zero boundary values, then there exists a fundamental solution $e(x, z)$ of K on D. If K is positive, self-adjoint, and bounded at infinity, a similar result may be established for unbounded domains. Differentiability properties as related to the differentiability of the coefficients of K and the relations between various fundamental solutions are studied. (Received March 12, 1952.)

347. F. E. Browder: *Green's function and the kernel function for linear elliptic equations of arbitrary order.*

Let K be a suitably differentiable linear elliptic differential operator on a bounded domain D of E^n. Then there exists a Green's function for K on D if and only if the Dirichlet problem for K on D with zero boundary values has only the zero solution. If K is positive, self-adjoint, and bounded at infinity, this result may be extended to arbitrary unbounded domains. If K is strongly positive and self-adjoint, there exists for each Poincaré domain D (Courant-Hilbert, vol. II, chap. VII) a kernel function $k(x, z)$ defined in terms of a complete family of solutions of the equation $Ku=0$ in D. In the case of second order equations in normal form, $k(x, z)$ reduces to the Bergman-Schiffer kernel function. The solution of the Dirichlet problem with boundary value function g can be represented in a simple closed form in terms of $k(x, z)$. (Received March 12, 1952.)

348*t*. Y. W. Chen: *Existence of minimal surfaces with given period and with given conditions on the boundary and at infinity.*

Let $z(u, v)=\Re[a\beta(1+\beta^2)^{-1/2}(e^{i\tau}w^{-1}+e^{-i\tau}w)+2Ki \log w]$ for $|w|=|u+iv|\leqq 1$; the real constants K and $\beta\geqq 0$ are given while a and τ are to be determined later. A simple closed Jordan curve C is given in the x, y plane. In order to satisfy the condition that $\partial z/\partial x\to\beta$, $\partial z/\partial y\to 0$ at infinity of the x, y plane which corresponds to $w=0$, one puts $x(u, v)=\Re[a(1+\beta^2)^{-1/2}(e^{i\tau}w^{-1}-e^{-i\tau}w)]+X(u, v)$ and $y(u, v)=\Re[-ai(e^{i\tau}w^{-1}+e^{-i\tau}w)+2K\beta(1+\beta^2)^{-1/2}\log w]+Y(u, v)$. The problem is to find the constants a, τ and two regular harmonic functions $X(u, v)$, $Y(u, v)$ in $|w|<1$ with the following properties: (1) X, Y maps the positive oriented circle $|w|=1$ continuously and monotonically into the negative oriented curve C, (2) $x(u, v)$, $y(u, v)$, $z(u, v)$ represents a minimal surface. The problem is analogous to the classical problem of finding by conformal transformations flows of incompressible fluids around a profile C, with given circulation K and uniform speed β at infinity. Here the minimal surface takes the place of the conformal mapping. The author uses the direct method of the calculus of variations to establish the existence proof for which a sufficient condition (*) is needed to guarantee the nondegeneracy of the limit functions. Various applications of the condition (*) are made to study the ranges of the values of K and β for which the problem has a solution. The main point of the paper is to construct the integral expression to be minimized. (Received February 18, 1952.)

349*t*. E. A. Coddington: *On ordinary self-adjoint differential operators.* I.

Let L be the formal operator $L=p_0(d/dx)^n+p_1(d/dx)^{n-1}+\cdots+p_n$, where the p_i are complex-valued functions of class C^{n-i} on an open (possibly unbounded) interval (a, b), and $|p_0|\neq 0$. Assume $L=L^+$, where $L^+=(-1)^n(d/dx)^n(\bar{p}_0\cdot)+(-1)^{n-1}(d/dx)^{n-1}(\bar{p}_1\cdot)+\cdots+\bar{p}_n$. Let $\mathfrak{D}$ be the set of all $u\in\mathcal{L}^2(a, b)$ having continuous derivatives up to order $n-1$, $u^{(n-1)}$ absolutely continuous on every closed subinterval, $Lu\in\mathcal{L}^2(a, b)$. Define T by $Tu=Lu$, for $u\in\mathfrak{D}$. For $u, v\in\mathfrak{D}$, let $[uv](a)=\lim [uv](x)$, $x\to a$, where $[uv](x)$ is the skew-hermitian semi-bilinear form appearing in Green's formula. Similarly define $[uv](b)$, and let $\langle uv\rangle=[uv](b)-[uv](a)$. If $\mathfrak{D}_0=\{u\in\mathfrak{D}\,|\,\langle uv\rangle=0 \text{ for all } v\in\mathfrak{D}\}$, define T_0 by $T_0u=Lu$, for $u\in\mathfrak{D}_0$. The operator

T_0 is closed, symmetric, and $T_0^* = T$. For any fixed c, $a<c<b$, and any complex number l, $\Im l \neq 0$, let $\mathfrak{E}_a(l) = \{u \mid Lu = lu$ and $u \in \mathcal{L}^2(a, c)\}$. Similarly define $\mathfrak{E}_b(l)$. Then $\dim \mathfrak{E}_a(l) \geqq \nu$, $\dim \mathfrak{E}_b(l) \geqq \nu$, where $n = 2\nu$ or $2\nu+1$ according as n is even or odd. If $\tau(l) = \dim \mathfrak{E}_a(l) + \dim \mathfrak{E}_b(l) - 2\nu$, then the deficiency indices (ω^+, ω^-) of T_0 satisfy $\tau(l) + 2\nu - n \leqq \omega^+ \leqq \tau(l)$ $(\Im l > 0)$, $\tau(l) + 2\nu - n \leqq \omega^- \leqq \tau(l)$ $(\Im l < 0)$. (Received March 12, 1952.)

350*t*. E. A. Coddington: *On ordinary self-adjoint differential operators. II.*

Suppose the deficiency indices (ω^+, ω^-) of the formally self-adjoint differential operator L are both equal to ω (cf. the preceding abstract). If $\alpha_1, \cdots, \alpha_\omega$ are ω functions in $\mathfrak{D}$ which are linearly independent mod $\mathfrak{D}_0$, the boundary conditions $\langle u\alpha_j \rangle = 0$ $(j = 1, \cdots, \omega)$ for $u \in \mathfrak{D}$ are self-adjoint if $\langle \alpha_j \alpha_k \rangle = 0$ $(j, k = 1, \cdots, \omega)$. Two sets of ω self-adjoint boundary conditions are equivalent if they determine the same subset of $\mathfrak{D}$. There is a 1-1 correspondence between self-adjoint extensions of T_0 and equivalence classes of ω self-adjoint boundary conditions. In case $\omega = 0$, T_0 is self-adjoint, and has a spectral representation $T_0 = \int \lambda dE(\lambda)$. In this case let δ be a closed interval interior to (a, b), and let $\mathfrak{D}_\delta$ be the set of all $u \in \mathcal{L}^2(a, b)$ having continuous derivatives up to order $n-1$ on δ, $u^{(n-1)}$ absolutely continuous on δ, $Lu \in \mathcal{L}^2(\delta)$, and u satisfies n self-adjoint boundary conditions on δ. Define H_δ by $H_\delta u(x) = Lu(x)$, $x \in \delta$, and $H_\delta u(x) = 0$, $x \notin \delta$. Then H_δ is self-adjoint, $H_\delta = \int \lambda dE_\delta(\lambda)$, and if $\delta \to (a, b)$, $E_\delta(\lambda) \to E(\lambda)$, if λ is not an eigenvalue of T_0. This result implies the uniqueness of the (suitably normalized) matrix appearing in the generalized Plancherel theorem for T_0. (Received March 12, 1952.)

351. R. M. Cohn: *Essential singular manifolds of difference polynomials.*

It is shown that 0 is not an e.s.m. (essential singular manifold) of the difference polynomial $yy_2^2 + y_1^2 y_3 - \lambda y_1 y_2$ if $\lambda = 1$, but is an e.s.m. if λ is an element transformally transcendental over the difference field consisting of the rational numbers. Hence there can be no criterion based on the degrees and orders of the terms only for determining whether or not 0 is an e.s.m. of a difference polynomial. This is an unexpected result since the low-power theorem of Ritt provides a criterion of this sort for differential polynomials. It is also shown that such a criterion exists for difference polynomials of second order and for other special classes of difference polynomials. (Received February 25, 1952.)

352. Philip Cooperman: *The multiplier rule for a partial differential equation in two dependent and two independent variables as side condition.*

The object of this paper is to give a proof of the Lagrangian multiplier rule for the case of a general partial differential equation in two dependent and two independent variables as side condition. The method used depends on the fact that the differential equations resulting from the variation of the dependent variables are greater in number than the number of multipliers. Hence, when these equations are regarded as equations in the multipliers, the existence of the multipliers depends on the satisfaction of compatibility requirements. These requirements turn out to be that the dependent variables satisfy certain relations, independent of the multipliers. If there are functions satisfying these relations, as well as the side and boundary conditions, then the multipliers are given by explicit differential expressions in the dependent variables. (Received February 18, 1952.)

353. R. B. Davis: *A new boundary value problem for third-order partial differential equations of composite type.*

Let G be a simply-connected region, with boundary B, for which the classical Green's function (for Laplace's equation) exists and satisfies a certain inequality. Consider the equation (reduced to the canonical form established in a previous paper) (1) $(\Delta u)_s+au_{ss}+bu_{st}+cu_s+du_t+eu=f$, where $a, b, \cdots, f\in C^1$ on G. (The s-curves are the characteristics.) Let $B'\subset G$ extend from B to B, cutting every characteristic in G exactly once, and being tangent to none. Let $g\in C$ on B, and $h\in C^3$ on $G+B$. (There is one further smoothness hypothesis on B'.) The Fredholm alternatives are established for the problem: $u\in C^3$ on G (and C on $G+B$), u satisfies (1) in G, $u=g$ on B, $\Delta u=h$ on B' (i.e., either the problem stated has a unique solution, or else the homogeneous problem $f=g=h=0$ has nonzero solutions). Regions are studied for which the inequality on the Green's function is satisfied. The methods are those of E. E. Levi and the preceding paper of the author. (Received March 10, 1952.)

354. Avron Douglis: *An elementary approach to Cauchy's problem.* Preliminary report.

Any solution u of a linear hyperbolic equation $Lu=\sum_{i,j=1}^{n} a_{ij}(x)\partial^2u/\partial x_i\partial x_j+\sum_{j=1}^{n} b_i(x)\partial u/\partial x_i+c(x)u=f(x)$ in an even number $n\geqq 4$ of independent variables is characterized by an integral relation of Volterra type in terms of its Cauchy data on a space-like initial manifold I. Let C^P: $\phi(x)=0$ designate the characteristic conoid with vertex P determined by the characteristic equation $\sum_{i,j=1}^{n} a_{ij}(\partial\phi/\partial x_i)(\partial\phi/\partial x_j)=0$; let I^P be the intersection of C^P with I, and let A^P be the part of C^P intercepted between P and I^P. Then $u(P)$ can be represented, namely, as the sum of two integrals, the first involving Cauchy data of u and a finite number of their derivatives over the domain of integration I^P, the second involving the solution u over the domain of integration A^P. (Both integrals also contain the coefficients a_{ij}, b_i, c, f and a finite number of their derivatives.) This representation was obtained by a procedure, suggested by Beltrami's method of solving the Cauchy problem for the wave equation, which is based upon integration (in the ordinary sense) over A^P of expressions of the form $F(x)\cdot Lu$. The result is analogous to the integral representations obtained by the method of characteristics for the solutions of hyperbolic equations in two independent variables and bears, like these, on numerous questions relating to initial value problems. (Received April 1, 1952.)

355. Abolghassem Ghaffari: *The corresponding Riccati equations of the hodograph equations.*

The purpose of this paper is to give the corresponding Ricatti equations of the hodograph equations of the compressible subsonic flow (1) $P\psi_\zeta=\phi_\theta$, $Q\phi_\zeta=-\psi_\theta$ and to investigate their solutions. If one defines $t=\int_\zeta^{\zeta_s}(PQ)^{-1/2}d\zeta$ and sets $R=(PQ^{-1})^{1/2}$, then the equations (1) may be written in the simplified form (2) $R^{-1}\phi_t=\psi_\theta$, $R\psi_t=-\phi_\theta$. The definitions of ϕ, ψ, ζ, ζ_s, θ, and the expressions of P and Q are given in a previous paper (Bull. Amer. Math. Soc. vol. 58). The first order equations (1) now become (3) $\phi_{tt}+S\phi_t+\phi_{\theta\theta}=0$, $\psi_{tt}-S\psi_t+\psi_{\theta\theta}=0$, where $S=-R^{-1}R'$, where primes denote differentiation with respect to t. The advantage of working with the equations (2) or (3) is that *one has only to consider the one function R and its logarithmic derivative S as* *functions of t instead of dealing with the two unrelated functions P and Q.* Writing ϕ and ψ in the form $\phi = R^{1/2}e^{mt}F_m \cos(m\theta+\epsilon_m)$, $\psi = -R^{-1/2}e^{mt}G_m \sin(m\theta+\epsilon_m)$ and inserting these expressions in equations (3), one gets (4) $F_m''+2mF_m'-\lambda F_m=0$, $G_m''+2mG_m'+\mu G_m=0$, where $\lambda=(1/2)S'+(1/4)S^2$, $\mu=(1/2)S'-(1/4)S^2$. If one puts $F_m'/F_m=f_m$, $G_m'/G_m=g_m$, then *the following Riccati equations for* f_m *and* g_m, (5) $f_m'+f_m^2+2mf_m=\lambda$, $g_m'+g_m^2+2mg_m=-\mu$, *are obtained. It is shown that* $S'\geqq 3S^2$, *and also that* f_m *and* g_m *verify the conditions* $0<f_{m+k}<f_m<f_{m-k}$, $g_{m-k}<g_m<g_{m+k}<0$. (Received March 5, 1952.)

356. Samuel Goldberg: *The convergence of sequences determined by nonlinear difference equations.* Preliminary report.

Let two real numbers x_0 and x_1 be given and suppose that the terms of the sequence (x_n), $n=2, 3, \cdots$, are determined recursively by the second order nonlinear difference equation with constant coefficients: $x_{n+2}=ax_{n+1}+bx_n^2+cx_n+d$. The problem under consideration is the determination of those values of x_0 and x_1 for which the sequence (x_n) has a finite limit. The special case $a=0$, with the sequence (x_{2n}), is the problem of the iteration of the function bx^2+cx+d. A number of special cases are considered where, with suitable restrictions on the coefficients, the set of values of x_0 and x_1 are given for which $\lim x_n$ exists. The couple x_0, x_1 is said to initiate a finite path to the limiting value L if the terms of the corresponding sequence (x_n) are, from some point on, all equal to L. Conditions are given under which convergence of (x_n) can occur only in this special way. (Received March 11, 1952.)

357. J. W. Green: *On the spherical means of α-potentials.*

By an α-potential is meant in the present paper a potential of the sort investigated by M. Riesz, O. Frostman, and others, based upon the function $1/r^\alpha$, $1\leqq\alpha<3$, instead of the Newtonian $1/r$. Only positive mass distributions are considered. An α-potential, $u(P)$, is subharmonic outside the mass, but in general neither subharmonic nor superharmonic on it. Frostman has shown that it satisfies an inequality analogous to the super-mean property; namely, $A(u, P, r)\leqq A(\alpha)u(P)$, where $A(u, P, r)$ is the volume average of u over the sphere of radius r and center P, and $A(\alpha)$ is a number depending only on α, but greater than 1 for $\alpha>1$. In the present paper, the factor $A(\alpha)$ is studied in some detail, bounded above and below, and its asymptotic behavior near $\alpha=3$ found. Among other inequalities it is shown that $3(3/\alpha)^\alpha\{1/(3-\alpha)+2^{-1}\log(1-\alpha/3)\}\leqq A(\alpha)\leqq 3/(3-\alpha)$, and so $A(\alpha)\sim 3/(3-\alpha)$. The corresponding factor in the case of spherical surface means is determined exactly. (Received March 10, 1952.)

358. George Klein: *On a polynomial inequality of Zygmund.*

The principal result of this paper is that if $S_n(x)$ is a trigonometrical polynomial of order n, then $\|S_n'\|_p\leqq A_p n\|S_n\|_p$ for $0<p<1$, A_p being a constant depending only on p, and where $\|S_n\|_p=\{(1/2\pi)\int_0^{2\pi}|S_n(x)|^p dx\}^{1/p}$. This is an extension of the result for $p\geqq 1$, where $A_p=1$, due to Zygmund (Proc. London Math. Soc. (2) vol. 34 (1932) pp. 394–400) whose proof yields the theorem for $p>1/2$. Bernstein's well known inequality is the case $p=\infty$. The theorem for power polynomials corresponding to Bernstein's theorem is due to A. Markoff and can be obtained from the result of Hille, Szegö, and Tamarkin (Duke Math. J. vol. 3 (1937) pp. 729–739) that for $p\geqq 1$, $\|f_n'\|_p\leqq B_p n^2\|f_n\|_p$. Here f_n is a polynomial of degree n, B_p depends only on p, and $\|f_n\|_p=\{(1/2)\int_{-1}^1|f_n(x)|^p dx\}^{1/p}$. This result is also extended to the cases $p>0$. (Received March 11, 1952.)

359t. Walter Leighton: *On a function of Ramanujan.*

The author of this note presents an alternate proof that the function of Ramanujan $(1-x-x^4+x^7+x^{13}- \cdots)/(1-x^2-x^3+x^9+x^{10}- \cdots)$ has the unit circle as a natural boundary. (Received March 12, 1952.)

360. L. F. Markus: *Global structure of ordinary differential equations in the plane.*

Two real differential systems $\mathfrak{S}_i$: $\dot{x}=f_i(x, y)$, $\dot{y}=g_i(x, y)$, $i=1, 2$, with $f_i, g_i \in C^{(1)}$ and with isolated critical points $(f_i^2+g_i^2=0)$ in the plane E are o-equivalent in case there exists an orientation preserving homeomorphism of E onto itself which carries the solution curve family of $\mathfrak{S}_1$ onto that of $\mathfrak{S}_2$. The topological analysis of a differential system $\mathfrak{S}$ is effected by certain intrinsic curves known as separatrices. A separatrix S is essentially a solution curve of $\mathfrak{S}$ which cannot be embedded in a plane neighborhood bounded by exactly two solution curves of $\mathfrak{S}$ plus the limit sets of S, and which admits a transversal. The separatrices have two important properties: firstly, they decompose $\mathfrak{S}$ in E into canonical regions each of which admits a transversal and is o-equivalent to one of three elementary types, and secondly, they furnish a complete set of invariants for the o-equivalence classes of plane differential systems. The first property permits the extension of many local theories (e.g., existence of an integral) of differential equations to global ones. The second property unifies the classical function-theoretic descriptions of differential equations. (Received March 13, 1952.)

361t. M. H. Protter: *On the solution of hyperbolic equations by the method of finite differences.*

Consider the mixed initial-boundary value problem for the hyperbolic differential equation (1) $au_{xx}-u_{yy}+bu_x+cu_y+du=0$, $a=a(x, y)>0$. Courant, Friedrichs, and Lewy showed that the solution of the difference equation $au_{x\bar{x}}-u_{y\bar{y}}+bu_x+cu_y+du=0$ converged to the solution of (1) (in the case of the Cauchy problem) under a certain restrictive hypothesis on the ratio of the mesh sizes in the x and y directions. By different selections of the difference operator it is possible to prove convergence under less restrictive hypotheses on the ratio of the mesh sizes. Let h, k be the mesh sizes in the x and y directions, respectively. One of the operators that can be considered in this way is the von Neumann operator which replaces $u_{x\bar{x}}(x, y)$ by $\alpha u_{x\bar{x}}(x, y+k)+(1-2\alpha)u_{x\bar{x}}(x, y)+\alpha u_{x\bar{x}}(x, y-k)$ and which leaves $u_{y\bar{y}}$ unchanged. (Received March 7, 1952.)

362. J. H. Roberts: *A nonconvergent iterative process.*

In studying a certain heat transfer problem, Mann and Wolf were led to a consideration of the integral equation $y(t)=\int_0^t (G[y(x)]/[\pi(t-x)]^{1/2})dx$, where $G(y)$ is continuous and strictly decreasing and $G(1)=0$. On a fixed closed interval $[0, T]$ let $y_0(x)=0$, and let $y_1(x)$, $y_2(x)$, $\cdots$ be determined from the equation by the usual iterative process. Under the additional assumption that $|\Delta G/\Delta y|$ is bounded they proved that the sequence $y_0, y_1, y_2, \cdots$ converges uniformly to a solution. The present paper shows by a counter example that the assumption that G satisfies a Lipschitz condition is not superfluous. On the other hand the assumption is not necessary, since the desired result follows for all convex G. (Received April 25, 1952.)

363. David Rosen: *On the behavior of automorphic functions near the boundary.*

The Fuchsian groups $\Gamma(\lambda)$ generated by $z'=z+\lambda$, $z'=-1/z$ are considered, where $\lambda>0$. It is known, when $\lambda<2$, that $\Gamma(\lambda)$ is properly discontinuous if and only if $\lambda=2\cos\pi/q$, $q=\text{integer}\geqq 3$; then $\Gamma(\lambda)$ is of the first kind. Otherwise, for arbitrary $\lambda>2$, $\Gamma(\lambda)$ is of the second kind. Specialized continued fractions, called λ-fractions, are introduced whose first term is λr_0, $r_0=\text{integer}$; the general term is $e_i/r_i\lambda$, where $e_i=\pm 1$, $r_i=\text{positive rational integer}$ $(i=1, 2, \cdots)$. For $\lambda>2$, every λ-fraction converges to a real number, whereas for $\lambda=2\cos\pi/q$, a sufficient condition for convergence is that $r_i+e_{i+1}<1$ occurs for no more than $[(q-1)/2]-1$ consecutive values of i. It is shown that a substitution $z'=(az+b)/(cz+d)\in\Gamma(\lambda)$ if and only if a/c is a finite λ-fraction. Also, a/c is a parabolic point if and only if it is a finite λ-fraction where a, c are polynomials in λ. The set of limit points of $\Gamma(\lambda)$ is the real axis when $\Gamma(\lambda)$ is of the first kind; as expected, every real number has a λ-fraction representation whose elements are computed by a "nearest integer" algorithm explicitly given. When $\Gamma(\lambda)$ is of the second kind, $\lambda>2$, the limit points form a perfect nowhere dense set; the real numbers on this set, and only those, have λ-fraction representations. On approach to a parabolic point in a Stolz angle, an automorphic function on $\Gamma(\lambda)$ has a unique limit. If the point is an infinite λ-fraction, an automorphic function does not admit a unique limit even on vertical approach. Similar results hold for automorphic forms. (Received March 10, 1952.)

364*t*. I. M. Sheffer: *On certain entire functions.*

An analytic function $f(z)$ will be said to have property $\mathcal{J}$ at a point z_1 if the sequence of derivatives $\{f^{(n)}(z_1)\}$, $n=0, 1, \cdots$, takes on only a finite number of distinct values. Entire functions of form (1) $f(z)=Q(z)+\sum_{j=0}^{m-1} A_j \exp\{\omega^j z\}$, where Q is a polynomial and $\omega=\exp\{2\pi i/m\}$, have property $\mathcal{J}$ at all points. Some theorems are given in which a function $f(z)$, having property $\mathcal{J}$ at one point and satisfying supplementary conditions (e.g., having property $\mathcal{J}$ at a second point), is shown to be a function of form (1). These theorems link up with a result of Szegö on power series whose coefficients take on only a finite number of distinct values. (Received February 14, 1952.)

365*t*. Seymour Sherman: *On the roots of a transcendental equation.*

N. D. Hayes [*Roots of the transcendental equation associated with a certain difference-differential equation*, J. London Math. Soc. vol. 25 (1950) pp. 226–232] has given necessary and sufficient conditions on real a_1 and a_2 that the zeros of $\tau(s)\equiv se^s-a_1e^s-a_2$ lie to the left of $R(s)=0$. In a physical application where $a_1<0$ and $a_2<0$, D. F. Gunder and D. R. Friant [*Stability of flow in a rocket motor*, Journal of Applied Mechanics vol. 17 (1950) pp. 327–333] give an equivalent condition. It is the purpose of this note to illustrate the idea of H. I. Ansoff and J. A. Krumhansl, *A general stability for linear oscillating systems with constant time lag*, Quarterly of Applied Mathematics vol. 6 (1948) pp. 337–341, by applying it to the case where a_1 and a_2 may be complex. (Received February 11, 1952.)

366. Annette Sinclair: *The zeros of an analytic function of arbitrarily rapid growth.*

Suppose S_1, S_2, $\cdots$ is an infinite sequence of simply connected regions whose closures are nonintersecting and whose only sequential limit point is the point at infinity. In this paper it is proved that, for any preassigned sequence of positive constants $\{M_i\}$, there exists a nonvanishing integral function $f(z)$ such that $|f(z)| > M_i$ when $z \in S_i$. More generally, necessary and sufficient conditions are obtained on a "Q_R-set" S and a set E in $C(S)$ in order that to every function $M(z)$ which is component-wise bounded on S, there corresponds a function $f(z)$ analytic and nonvanishing in $C(E)$ and such that $|f(z)| > |M(z)|$ when $z \in S$. (A set S is a "Q_R-set" if (1) its components are closed regions, and (2) its sequential limit points lie in $C(S)$. Actually, the theorem is proved with a weaker condition substituted for (1).) (Received March 14, 1952.)

367*t*. D. B. Sumner: *A convolution transform admitting an inversion formula of integro-differential type.*

The inversion operator $E(D)$ for the convolution transform $f(x) = \int_{-\infty}^{\infty} G(x-t)\phi(t)dt$ is given by $[E(s)]^{-1} = \int_{-\infty}^{\infty} \exp(-st)G(t)dt$. It is shown by means of the example $G(x) = [\exp(-2x) + 2 \cos \pi\beta \cdot \exp(-x) + 1]^{-1}$, $0 < \beta < 1$, that cases where $E(s)$ is meromorphic arise from kernels little more complicated than that of Stieltjes' transform, and that the order in which the differentiating factor and the integrative factor of the inversion operator are applied is material. The two factors $\sin \pi D/\pi$ and $\sin \pi\beta/\sin \pi\beta(1-D)$ of the invertor are represented as integrals, and the methods of the complex variable are employed. (Received January 25, 1952.)

368. Albert Wilansky: *The inset of a summability matrix.*

Let A be a summability matrix, a_k the column limit $\lim_n a_{nk}$. The inset $I(A)$ is the set of sequences summable A such that $\sum a_k x_k$ converges. If $I(A) = (A)$ (the field of A), say A has maximal inset. If every B with $(B) = (A)$ has maximal inset, say A has PMI (propogation of maximal inset). The regular matrix ($a_{nn} = a_{n,n-1} = 1/2$, $a_{nk} = 0$ otherwise) does not have PMI. The Cesaro and Riesz matrices have PMI. All these matrices are of type M and their fields cannot be distinguished by previously known invariants. A co-regular matrix with maximal inset has an equipotent regular matrix. There is a co-regular normal matrix such that no equipotent matrix has maximal inset. Hence there is no equipotent regular matrix. (Example is due to K. Zeller). The regular normal matrix $a_{nk} = s_n t_k$ has PMI. (Received March 7, 1952.)

369*t*. František Wolf: *Spectral decomposition of a class of operators in Banach space.* Preliminary report.

The spectral decomposition concerned is of a generalized type $f(A) = \int_{\sigma(A)} f(\lambda) d^n E(\lambda)$ where (i) $f(\lambda)$ is of class C^n in an open set containing $\sigma(A)$, (ii) the integral is essentially Bochner's (Bochner, *Fouriersche integrale*, 1932, also F. Wolf, *(C, k) summability of trigonometric integrals*, University of California Press, 1946 and L. Schwartz, *Théorie des distributions*). The class of operators with real bounded spectrum can be characterized in different ways: (i) the resolvent $R_\lambda = O((\lambda - \bar{\lambda})^{-n+2})$, (ii) the polynomials $f(A)$ as functions of f prove to be continuous under the topology of C^n (cf. L. Schwartz). An equivalent class can be characterized by $\|A^k\| \leqq Mk^{n-1}$, $k = 0, 1, -1, \cdots$. This is an extension of E. R. Lorch's class of "weakly almost periodic" operators. It includes nilpotent operators of order not more than $n-1$. The generalized "decomposition of the identity" $E(\lambda)$ is outside of $\sigma(A)$ a polynomial of order $n-1$ and $\int f(\lambda) d^n E(\lambda) \cdot \int g(\lambda) d^n E(\lambda) = \int f(\lambda) g(\lambda) d^n E(\lambda)$ which yields a corresponding orthogonality property for $E(\lambda)$. An eigenspace S_{λ_0}, defined as the annihilator of $g(A)$ where $\lambda \neq \lambda_0 \rightarrow g(\lambda) \neq 0$ and $g(\lambda_0) = g'(\lambda_0) = \cdots = g^{(n-1)}(\lambda_0) = 0$, is such that $A - \lambda_0 I$ is nilpotent in S_{λ_0}. If $h'(\lambda_0) = \cdots = h^{(n-1)}(\lambda_0) = 0$, then $h(A)$ is a scalar in S_{λ_0}. There exists an expanding family of manifolds $M(\lambda)$ such that $\sigma(A | M(\lambda_0))$ is exactly $\sigma(A)\cap\{\lambda|\lambda\leqq\lambda_0\}$ and $M(-\infty)=(0)$, $M(\infty)$ is the whole space. (Received March 13, 1952.)

APPLIED MATHEMATICS

370t. H. G. Bergmann: *The boundary layer problem for certain nonlinear ordinary differential equations.*

This paper considers a problem similar, but simpler in type, to that arising in the theory of buckling of circular plates. One has a pair of nonlinear ordinary differential equations of second order depending upon a parameter, (i) $p_{xx}(x)=q^2(x)/2$, (ii) $kq_{xx}(x)+p(x)q(x)=0$, $-1\leqq x\leqq 1$, with the associated boundary conditions $p(-1)=p_1$, $p(1)=p_2$, $q_x(-1)=q_x(1)=0$. When the parameter k approaches zero as a limit, the solutions p and q approach limit functions whose existence and uniqueness we prove by variational methods. The nonuniform convergence in the "boundary layer" is studied by an appropriate stretching process. The limit functions satisfy the limit equations, but no longer satisfy all the boundary conditions. Since q_{xx} disappears in the limit equation, it is reasonable that the boundary condition on q should no longer be met; the interesting result is that although the limit equation remains of the same order in p, the boundary condition here may be lost. The paper proves that if p_i is negative, the limit function does indeed assume this value; but if p_i is positive, the limit function assumes instead the value $-.47271p_i$. (Received February 13, 1952.)

371. Jack Kotik: *Linear water waves and the equation of the vibrating beam.* Preliminary report.

We consider two-dimensional surface waves on an ocean of infinite depth. The existence problem has been proposed for the following data: given the surface elevation and slope at $x=0$ for all values of the time, $-\infty<t<\infty$. It is shown (with inessential rigor restrictions) that this problem is impossible unless the prescribed functions are analytic, and (with essential hypotheses) that if the solution exists it is unique, as follows: in deep water the surface condition can be differentiated to yield $\phi_{tttt}+\phi_{xx}=0$, which is formally the equation for the transverse vibrations of a beam. An integral representation is found for the solution from which the analyticity in t and uniqueness follow. The main difficulty is that the source solutions do not die out at infinity but only oscillate with increasing rapidity. (Received March 11, 1952.)

372. K. S. Miller and F. J. Murray: *Error analysis for differential analyzers.*

The machine solution of a system of differential equations differs from the true solution by satisfying somewhat different differential equations and by sporadic perturbations β during the solution process. The machine differential equations are considered to involve parameters α and λ, such that setting α and λ zero yields the correct equations. λ errors raise the order of the system, α errors do not. For no λ errors, the machine solutions, $u_i(x, \alpha, \beta)$, of practical systems (including nonlinear systems, with broken region of analyticity) depend analytically on α and β. The α and β partial derivatives for each order satisfy a linear system of differential equations, with the same homogeneous part. Thus the Taylor series for u_i yields the solution error accurately without "linearizing" the system. This error analysis for α and β applies to both continuous and digital machines. When λ errors are present, the solution error is analyzed into a long range part, analytic in λ, and a transient part not analytic in λ. The methods thus indicated are combined to yield a discussion of the joint effect of all error types. (Received March 7, 1952.)

373. K. M. Siegel, J. W. Crispin, and R. E. Kleinman: *The zeros of the associated Legendre function of order one and nonintegral degree.*

The consideration of certain problems related to the scattering of electromagnetic waves involves the determination of those values of the degree for which the associated Legendre function with a fixed real argument between -1 and 0 will vanish. This paper presents a method which will yield numerical results more easily than any previously reported method. This method involves expanding $P'_\nu(x_0)$ in a Taylor series about integer and half-integer values of ν, terminating the series after three terms, and equating the expansion to zero. The solution of the resulting algebraic equation leads to the zeros, which are the values of ν that are desired. Although the coefficients in this expansion are themselves infinite series, they are rapidly convergent, and thus can be readily evaluated. The method has been applied to the cases $x_0=\cos 165°$ and $x_0=\cos 170°$. In addition, the method provides a means of obtaining quantitative results for the normalizing factors $\int_{x_0}^1[P_{n_i}(x)]^2dx$ and $\int_{x_0}^1[P'_{n_i}(x)]^2dx$ with the n_i such that $P'_{n_i}(x_0)=0$. (Received March 5, 1952.)

374*t*. R. L. Sternberg and Hyman Kaufman: *A general solution of the two-frequency modulation product problem.* I.

Consider an arbitrary modulator whose output versus input characteristic $Y=Y(X)$ is a continuous function of X on a finite closed interval $[-a, a]$. Let the input be $x(t)=P\cos(pt+\theta_p)+Q\cos(qt+\theta_q)$, $0<P<P+Q\leqq 2P\leqq a$. Then by extending the method of W. R. Bennett (*The biased ideal rectifier*, Bell System Technical Journal vol. 26 (1947) pp. 139–169) it is shown that the double Fourier series coefficients of the output $y(t)=Y(x(t))$ can be approximated to within an arbitrary 2ϵ by simple linear combinations of the values of four new functions which were introduced by Bennett and are independent of the modulator characteristic, while the double Fourier series having these approximate values as coefficients converges and differs from the output $y(t)$ by less than ϵ for all t. Tables of the four new functions mentioned are in preparation for Part II of the present paper. (Received February 29, 1952.)

375. H. F. Weinberger: *Upper and lower bounds for torsional rigidity.*

By an extension to multiply-connected domains of a minimum principle of J. B. Diaz (Proceedings of the Symposium on Spectral Theory and Differential Problems, Oklahoma A. and M., 1951, p. 289), the following is shown. If a region with torsional rigidity S be split into disjoint subregions with rigidities $S_1, \cdots, S_n$, then $S\geqq S_1+\cdots+S_n$. This is a lower bound for S or, by transposing, an upper bound for S_1. Of particular interest is the region S_1 consisting of the simply-connected region S with holes $S_2, \cdots, S_n$. For the hollow square with sides $2b$ and $2a$, the upper bound $2.2496\,(b^4-a^4)$ is so obtained. Good lower bounds for certain regions with holes are obtained as follows. If S_1 consists of the simply-connected region S with a hole S_2 and if the torsion function of S has a level line entirely outside S_2 but inside a larger region S'_2, then $S_1\geqq S-S'_2$. For a regular polygon with similar hole, a circle circumscribed about the hole serves as S'_2. In particular, for the hollow square $S\geqq 2.2496b^4-2\pi a^4$. For small holes, the upper and lower bounds are very near to each other. (Received March 12, 1952.)

376. D. M. Young and M. L. Juncosa: *On the convergence of solutions of difference equations to solutions of the heat equation.*

Let $u(x, t) = \sum_{n=1}^{\infty} a_n \sin n\pi x \exp(-n^2\pi^2 t)$ be a Fourier series solution of the heat equation $u_{xx} = u_t$ in R: $0 < x < 1$, $t > 0$, with boundary conditions $u(+0, t) = u(1-0, t) = 0$, $t > 0$, and initial condition $u(x, +0) = f(x)$, $0 < x < 1$. For points (x, t) of R such that Mx and M^2t/r^2 are integers, M an integer, let $U_M(x, t)$ satisfy the difference equation $U_M(x, t+k) - U_M(x, t) = r\{U_M(x+h, t) + U_M(x-h, t) - 2U_M(x, t)\}$, where $h = 1/M$, $k = rh^2$. Let $\overline{U}_M(x, t)$ be obtained from $U_M(x, t)$ by bilinear interpolation. By the use of a generalization of Fejér's theorem on summability $(C, 1)$ of Fourier series (D. Jackson, *The theory of approximation*, p. 66) the authors prove that if $f(x)$ is continuous in $0 \leqq x \leqq 1$ except for a finite number of finite jumps, if the Fourier series for $f(x)$ converges, and if $0 < r \leqq 1/2$, then as $M \to \infty$, $\lim \overline{U}_M(x, t) = u(x, t)$, uniformly for $t \geqq t_0 > 0$ and for $0 \leqq x \leqq 1$. This generalizes previous results; for instance Leutert (Journal of Mathematics and Physics vol. 30 (1952) pp. 245–251) has proved convergence for $0 < r \leqq 1/4$. On the other hand, Hildebrand, Bull. Amer. Math. Soc. Abstract 58-2-216, proved convergence for $0 < r \leqq 1/2$, assuming bounded variation for $f(x)$. (Received March 12, 1952.)

Geometry

377*t*. T. Minagawa and Tibor Radó: *Infinitesimal rigidity of nonconvex surfaces.*

The theorem stated in a previous communication (T. Radó, *Rigidity theorems for non-convex surfaces*, Bull. Amer. Math. Soc. Abstract 58-4-378) remains valid if the asymptotic condition relative to the boundary curve is dropped. At the same time, differentiability assumptions can be reduced to the point where the results appear in their natural setting. (Received February 11, 1952.)

378*t*. Tibor Radó: *Rigidity theorems for nonconvex surfaces.*

Let R be a simply-connected piece of surface with boundary C. Then R will be termed i.r.b. (infinitesimally rigid for point-wise fixed boundary) if R does not admit of nonidentical infinitesimal deformations leaving the boundary fixed point-wise. Now let S be a simply connected surface satisfying one of the following three conditions. (1) The Gauss curvature K of S is positive. (2) $K < 0$, and the asymptotic lines can be taken as parameter curves for S in the large. (3) $K \equiv 0$, and the generators and their orthogonal trajectories can be taken as parameter lines for S in the large, and there are no "flat points" present. *Theorem.* If R is a simply connected piece of S whose boundary C has aysmptotic direction at a finite number of points at most, then R is i.r.b. *Remarks.* (a) Case (1) is, essentially, known, and is stated merely for completeness. (b) Analyticity is not assumed. (c) The result has a bearing on the problem of flexible, inextensible membranes. (Received January 28, 1952.)

Logic and Foundations

379. Ilse L. Novak: *On direct unions of algebras.* Preliminary report.

The terminology used here is taken from J. C. C. McKinsey, J. Symbolic Logic vol. 8 (1943) pp. 61–76. Given an algebra Γ which is known to be a direct union of algebras Γ_i, where i ranges over a given index set I, and a sentence S which holds in Γ. One wishes to determine what conditions this imposes on the factors of Γ. For certain classes of sentences it is possible to obtain equivalent conditions which hold for all index sets I. If all the factors are isomorphic, more general results can be obtained. If I is finite, one can always find equivalent conditions. (Received March 13, 1952.)

Statistics and Probability

380t. J. C. Kiefer and Jacob Wolfowitz: *Stochastic estimation of the maximum of a regression function.*

Let $F(y|x)$ be a family of distribution functions which depend on a real parameter x, let $\int ydF(y|x)=M(x)$, and suppose that $\int(y-M(x))^2dF(y|x)\leqq S<\infty$. $M(x)$ is strictly increasing for $x<\theta$ and strictly decreasing for $x>\theta$. Let $\{a_n\}$ and $\{c_n\}$ be sequences of positive numbers such that $c_n\to 0$, $\sum a_n=\infty$, $\sum a_nc_n<\infty$, $\sum a_n^2/c_n^2<\infty$. Let z_1 be an arbitrary number and define, for all n, $z_{n+1}=z_n+a_n((y_{2n}-y_{2n-1})/c_n)$. Here y_{2n-1} and y_{2n} are independent chance variables with respective distributions $F(y|z_n-c_n)$ and $F(y|z_n+c_n)$. Under weak regularity conditions on $M(x)$ the authors prove that z_n converges stochastically to θ. (Received February 14, 1952.)

381. Eugene Lukacs and Otto Szász: *Analytic characteristic functions.*

We give here a proof of the following theorem: If a characteristic function $\phi(z)$ is analytic in a neighborhood of the origin, then it is also analytic in a horizontal strip and can be represented in this strip by a Fourier integral. This strip is either the whole plane or has one or two horizontal boundary lines. The purely imaginary points on the boundary of the region of convergence (if this region is not the whole plane) are singular points of $\phi(z)$. A number of properties of analytic characteristic functions follow from this theorem: (1) In either half-plane, the singularity nearest to the real axis is located on the imaginary axis. (2) There are no zeros on the segment of the imaginary axis located inside the strip of analyticity. (3) The zeros and the singular points of $\phi(z)$ are located symmetrically with respect to the imaginary axis. (4) If the characteristic function of an infinitely divisible law is analytic, then it has no zeros inside its strip of convergence. (Received March 5, 1952.)

382t. Jacob Wolfowitz: *On the stochastic approximation method of Robbins and Monro.*

Let $H(y|x)$ be a family of distribution functions which depend on the parameter x. Let $M(x)=\int ydH(y|x)$, and suppose $|M(x)|<C<\infty$ and $\int y^2dH(y|x)<S<\infty$. Let $\{a_n\}$ be a sequence of positive numbers such that $\sum a_n=\infty$, $\sum a_n^2<\infty$. Suppose that either (I) $M(x)\leqq\alpha-\delta$ for $x<\theta$, $M(x)\geqq\alpha+\delta$ for $x>\theta$ or (II) all the following hold: (1) $M(x)<\alpha$ for $x<\theta$, $M(\theta)=\alpha$, $M(x)>\alpha$ for $x>\theta$, (2) for some positive δ, $M(x)$ is strictly increasing if $|x-\theta|<\delta$ and (3) $\inf_{|x-\theta|\geqq\delta}|M(x)-\alpha|>0$. Let x_1 be arbitrary and define recursively $x_{n+1}=x_n+a_n(\alpha-y_n)$, where y_n is a chance variable with the distribution $H(y|x_n)$. Then x_n converges stochastically to θ. This generalizes results of Robbins and Monro, Ann. Math. Statist. vol. 22 (1951) pp. 400–407. (Received February 25, 1952.)

Topology

383. R. D. Anderson: *On extending the domain of a monotone interior mapping.*

The author shows that if M, imbedded in S^n, is a closed set of dimension not greater than $n-2$ and f is a monotone interior mapping of M onto a space Y, then there exists a monotone interior mapping F of S^n onto a space in which Y is imbedded such that $F(S^n-M)$ is homeomorphic to S^n-M and for any point y in Y, $F^{-1}(y)$ is $f^{-1}(y)$. (Received March 12, 1952.)

384*t*. H. W. Becker: *Nonplanar graphs and passive circuits*. Preliminary report.

A passive circuit is a subgraph with two terminal-nodes. A graph which can be mapped on a sphere can be mapped on a plane, but this is not necessarily true of p.c., hence the distinction between planar, global, and nonglobal p.c. Kuratowski's theorem, that all nonplanar graphs contain one of two elementary n.g., the triple wye, or pentagon-pentacle, has this analogy: all nonplanar p.c. contain one of three el. n.p.c., the X (Bloch), Δ (Carvallo), or Y (el. sesquispheric) bridges. In any bridge, all terminal and unambiguous boundary branches are nonbridgers (have no negative terms in the transfer conductance). But in a nonplanar bridge, there is no unique pair of boundary meshes. Consequently n.p.c., like n.g., have no topological duals—but have algebraic duals with square first partial derivatives, like physical p.c., Proceedings of the American Mathematical Society vol. 1 (1950) p. 316. For $9 \leqq n \leqq 13$, the fundamental n.g. $(a)_n^* = 1, 2, 3, 10, 29$ (a majority of a_{13}^*), and the n.g. $(a)_n = 1, 4, 21, 130, 828$. For $8 \leqq n \leqq 12$, the fundamental n.p.c. $(b)_n^* = 1, 4, 12, 48, 200$ (a majority of b_{12}^*), and the n.p.c. $(b)_n = 1, 10, 90, 766, 6251$. So far, $(b)_n \sim 1.5 \cdot 8^{n-8}$. These enumerations would all be doubled, admitting algebraic duals. The first graph and bridge-bridges such that neither they nor their duals are physical are $\langle a\rangle_{16}^{**} = 1$, $\langle b\rangle_{15}^{**} = 4$. Reference is made to classic papers of Hassler Whitney. (Received March 21, 1952.)

385*t*. H. W. Becker: *The enumeration of series-parallel-bridge graphs*. Preliminary report.

R. M. Foster classified s.p.b. graphs by rank and nullity (Transactions of the American Institute of Electrical Engineers, 1932, p. 309) and formulated the atlas of s.p. graphs C_n (International Congress of Mathematicians, 1950). Enumeration of all nonseparable n-branch s.p.b. graphs a_n starts with the fundamental graphs a_n^*, called pure active bridges, Bull. Amer. Math. Soc. vol. 54 (1948) p. 76, and proceeds by the methods there applied to passive s.p.b. circuits b_n. Topological binomial coefficients and products reduce to ordinary ones only for networks of maximum assymmetry, where the branch diversity equals the number of branches; there are no such graphs, $n<12$. Thus the topological power $2^4 = 5, 6, 7, 8, 9, 10, 12$, or 16 according to the symmetries involved. For $10 \leqq n \leqq 13$, the fundamental bridge-bridge graphs $a_n^{**} = 1, 2, 3, 11$; for $6 \leqq n \leqq 13$, $a_n^* = 1, 0, 1, 3, 4, 7, 22, 51$, whence $a_n = 1, 2, 11, 47, 197, 837, 3689, 16456 > 2C_{13}$. With the aid of a table of $s_{n,d}$, s.p. circuits classified by diversity, the analogous $a_{n,d}$ is tabulated. This achieves the cross-check $b_{n-1} = \sum d \cdot a_{n,d}$. For $5 \leqq n \leqq 12$, $b_n = 1, 6, 37, 195, 1003, 5034, 25231, 127217$. Nonplanarities few, $b_n \sim 1.6 \cdot 5^{n-5}$, additional light on the Riordan-Shannon problem. Journal of Mathematics and Physics vol. 21 (1942) p. 83. The previous procedure started from $b_n^* = 1, 0, 2, 5, 12, 34, 113, 388$, $5 \leqq n \leqq 12$. The two different procedures enforce a mutual correction and corroboration, in a census exceedingly prone to error. (Received March 21, 1952.)

386. A. L. Blakers: *On the realization of homotopy sequences*.

Let $\mathfrak{S} = (\cdots \to A_n \to B_n \to C_n \to A_{n-1} \to \cdots \to A_1 \to B_1 \to C_1 \to 1)$ be a sequence of sets and single-valued functions, subject to the conditions: (1) All sets are groups, except possibly C_1, which is a set with a unit element. (2) All groups to the left of C_2 are abelian. The groups C_2, A_1, B_1 need not be abelian. (3) All functions except possibly j_1: $B_1 \to C_1$ are homomorphisms. (4) The sequence is exact. In addition if i_1: $A_1 \to B_1$, b_1, $b_1' \in B_1$, and $j_1(b_1) = j_1(b_1')$, then $b_1' b_1^{-1} \in i_1(A_1)$. (5) A_1 acts as a group of operators on all groups of $\mathfrak{S}$ and the homomorphisms of $\mathfrak{S}$ are operator homomorphisms. A_1 acts on itself by inner automorphisms, and if k_2: $C_2 \to A_1$, c_2, $c_2' \in C_2$, then $(k_2(c_2))c_2' = C_2 C_2' C_2^{-1}$. (6) B_1 acts as a group of operators on all groups B_n, $n \geqq 1$; on itself by inner automorphism. (7) The operators from A_1, B_1, on B_n, $n \geqq 1$, are related by $a_1(b_n) = (i_1(a_1))b_n$ $(b_n \in B_n,\ a_1 \in A_1)$. The algebraic properties of $\mathfrak{S}$ are all shared by the homotopy sequence of any 0-connected pair. The main theorem proved about $\mathfrak{S}$ is that it can be realized as the homotopy sequence of a 0-connected C. W. complex-pair. More specifically, there is such a pair (K, L) and an operator isomorphism of the homotopy sequence of (K, L) onto $\mathfrak{S}$. Moreover, all Whitehead products in (K, L) are trivial. (Received March 6, 1952.)

387. M. L. Curtis: *The Poincaré-Lefschetz duality theorem for closed Brouwer manifolds.*

Let S be a closed connected n-manifold (not assumed to be triangulable) and G, I be, respectively, an arbitrary abelian group and the group of integers. Let $\mathcal{H}$, H', H be a cohomology theory and two homology theories with coefficient groups G, I, G. Assume there exists a cap product $\frown$ pairing $\mathcal{H}$ and H' to H. If $H_n'(S)$ is isomorphic with I and z_n is a generator of $H_n'(S)$, let $\phi_r: \mathcal{H}^r(S) \to H_{n-r}(S)$ be defined by $\phi_r(u) = u \frown z_r$. This paper is devoted to showing that under these circumstances each ϕ_r $(r = 0, 1, \cdots, n)$ is an isomorphism onto. (Received March 12, 1952.)

388t. S. P. Diliberto: *Existence of cubical maps.* Preliminary report.

Let K and L be finite Euclidean polyhedra which are subdivided into cubes, and let $f: |K| \to |L|$ be a continuous map. It is shown that there exist cubical maps f_ϵ approximating f (paralleling the existence of simplicial maps when K and L are cut into simplices): for $\epsilon > 0$, there exist subdivisions $\overline{K}$ and $\overline{L}$ and a continuous map $f_\epsilon: |\overline{K}| \to |\overline{L}|$ such that (i) dist $(f, f_\epsilon) < \epsilon$, (ii) f_ϵ is ϵ-homotopic to f, (iii) if $|A^p|$ is a p-cube of $\overline{K}$, there exists a q-cube $(q \leqq p)$ $|B^q|$ of $\overline{L}$ such that $f(|A^p|) = |B^q|$, (iv) f_ϵ induces a natural map $C_{f_\epsilon}: C_p(\overline{K}) \to C_p(\overline{L})$ where $\partial C_{f_\epsilon} = C_{f_\epsilon}\partial$. By standard methods one obtains subdivisions $\tilde{K}$ and $\overline{L}$ and a *vertex* map f_ϵ satisfying the nearness properties (i), (ii) and such that if a_i are vertices of $|A^p| \in \tilde{K}$, there exists $|B^q| \in \overline{L}$ such that $f(a_i) \in |B^q|$ (q possibly greater than p). $\overline{K}$ is the $2m \cdot n$th subdivision of $\tilde{K}$, $m = \dim K$ and $n = \dim L$. f_ϵ is extended to $\overline{K}$ so as to have properties (iii) and (iv). (Received March 17, 1952.)

389. Mary E. Estill: *A primitive dispersion set of the plane.*

R. L. Wilder, in his book, *Topology of manifolds*, makes the following definitions. "If M is a connected set and D a subset of M such that $M - D$ is totally disconnected, then D may be called a dispersion set of M. If no proper subset of D is a dispersion set of M, then let us call D a primitive dispersion set of M." And Wilder raises the question of the existence of a primitive dispersion set of the plane. In this paper an example is given of a primitive dispersion set of the plane. (Received January 28, 1952.)

390t. R. H. Kyle: *Calculation of invariants of a quadratic form.*

It is convenient to have methods of calculating the invariants of a quadratic form which do not involve a preliminary reduction to canonical form. Let f be a quadratic form with integer coefficients, and p be an odd prime. Let the elementary divisors of the matrix of f be $\tau_i = \tau_i' p^{d_i}$, with $d_i = e_k$ when $r_{k-1} < i \leqq r_k$, and $e_k < e_{k+1}$. Put $s_j = \sum_1^j e_k(r_k - r_{k-1})$. Let $\{f^{(r)}\}$ denote the collection of subforms of f obtained by dropping all but r variables of f. Then there exists at least one subform g in $\{f^{(r_k)}\}$ such that $|g|p^{-s_k}$ is an integer prime to p. Moreover if g, h are two such subforms, the Legendre symbols $(|g|p^{-s_k}/p)$, $(|h|p^{-s_k}/p)$ are equal and define a unit $\epsilon_k(p)$. The $\epsilon_k(p)$ are invariants of the class of f, and are calculable without reduction to canonical form. The r_k, e_k, $\epsilon_k(p)$ provide a complete set of local invariants of f at odd primes. The invariants $(|f_k|/p)$ defined in B. W. Jones, *The arithmetic theory of quadratic forms*, are given by $(|f_k|/p) = \epsilon_k(p)\epsilon_{k-1}(p)$ and the Minkowski units $c_p(f)$ in turn are given by $c_p(f) = (-1/p)^{[q/2]} \cdot \prod'(|f_k|/p)$ where $q = \sum'(r_k - r_{k-1})$; $\sum', \prod'$ being taken over those k such that e_k is odd. (Received March 12, 1952.)

391t. R. H. Kyle: *Embeddings of the Moebius band in S^3.*

A rectangle $ABCD$ is embedded in S^3 with AD identified with CB. The boundary $ABCD$ of the Moebius band thus constructed forms a knot l, and the line joining the midpoints of AD and BC forms a knot k called the center knot of the band. The linking number $\nu(k, l)$ gives the number of twists. It is shown that given an oriented knot k and an odd integer ν, an oriented knot l can be constructed in a well-defined way, so that l is the boundary of a Moebius band having k for its center knot and ν twists. Conversely if two Moebius bands M, M' have boundaries which are equivalent to each other but not to a circle, then there exists a semilinear mapping of S^3 onto itself sending M' onto M. Thus distinct pairs (k, ν) define distinct knots except in the special case (circle, ± 1), and the invariants of k are also invariants of l. This gives a complete classification of a subclass of the parallel knots, analogous to H. Seifert's classification of doubled knots in *Schlingknoten*, Math. Zeit. vol. 52 (1949). (Received March 12, 1952.)

392. R. H. Kyle: *The quadratic form of a knot.*

The quadratic form of a projection of a knot was defined by L. Goeritz, *Knoten und quadratische Formen*, Math. Zeit. vol. 36 (1932). It was shown that the Minkowski units $c_p(f)$ of this form are invariant under combinatorial transformations, but the question of topological invariance remained unsettled. H. Seifert, in *Der Verschlingungsinvarianten der zyklischen knotenüberlagerungen*, Abh. Math. Sem. Hamburgischen Univ. vol. 11 (1935), exhibited the connection between the quadratic form and the two-sheeted covering of the knot. He gave an example of two knots which can be distinguished by the linking invariants of this covering but not by the Minkowski units, and he asked whether an example of the opposite case could be found. A negative answer to Seifert's question is given by expressing $c_p(f)$ as a product of certain of the linking invariants. The topological invariance of the Minkowski units is an immediate consequence of the topological invariance of the linking numbers. (Received March 12, 1952.)

393t. J. C. Moore: *A new proof of the Blakers and Massey triad theorem.*

The theorem states that if $(X; A, B)$ is a triad such that $X = A \cup B$, A and B are open in X, A, B, and their intersection are arcwise connected, (X, A) is m-connected, $m \geqq 2$, (X, B) is n-connected, $n \geqq 2$, then the triad $(X; A, B)$ is $(m+n)$-connected (Bull.

Amer. Math. Soc. Abstract 57-2-164). Let $x_0 \in A \cap B$. Let D be the space of paths in X which start at x_0 and end in A; let E be the space of paths in X which start at x_0 and end in B, and let $Y = D \cup E$. Then the triads $(X; A, B)$ and $(Y; D, E)$ have isomorphic triad homotopy groups, and Y has trivial homology groups. Therefore it is sufficient to prove the theorem in the special case where X has trivial homology. In this case A is $(m-1)$-connected, and B is $(n-1)$-connected. Let $C = A \cap B$, and let Δ be the diagonal of $A \times B$. Δ is homeomorphic with C. $H_q(A \times B, \Delta) = \{0\}$ for $q \leqq m+n-1$ by the Künneth theorem. Therefore $\pi_q(A \times B, \Delta) = \{0\}$ for $q \leqq m+n-1$. These facts imply $\pi_q(X; A, B) = \{0\}$, $q \leqq m+n$, by standard methods. (Received January 28, 1952.)

394t. J. C. Moore: *On the homotopy groups of spheres.*

Using the concept of generalized covering space of G. W. Whitehead (*Fibre spaces and the Eilenberg homology groups*, Bull. Amer. Math. Soc. Abstract 58-2-267) it is proved that for p a prime the p-primary component of $\pi_{2p}(S^3)$ is a cyclic group of order p, and that $\pi_q(S^3)$ has no elements of order p for $2p < q < 4p-3$. Let Z_p denote a cyclic group of order p. Then using the relative Hurewicz theorem and the results of J. P. Serre (Ann. of Math. vol. 54 (1951) pp. 425–505) it is proved that if $E^2: \pi_q(S^n) \rightarrow \pi_{q+2}(S^{n+2})$ is the double suspension homomorphism, then for n odd the induced homomorphism $E^2_p: \pi_q(S^n) \otimes Z_p \rightarrow \pi_{q+2}(S^{n+2}) \otimes Z_p$ is an isomorphism for $q < (n+1)p-3$. Combining these results with those of Serre, it is proved that for n odd $\pi_q(S^n) \otimes Z_p = \{0\}$ for $n+2p-3 < q < n+4p-6$ and the p-primary component of $\pi_{n+2p-3}(S^n)$ is a cyclic group of order p. It is also proved that for n even and $p \neq 2$, $\pi_q(S^n) \otimes Z_p = \{0\}$ if $q \neq 2n-1$ and $n+2p-3 < q < \min\{2n+2p-4, n+4p-6\}$, and the p-primary component of $\pi_{n+2p-3}(S^n)$ is a cyclic group of order p. (Received February 7, 1952.)

395t. J. C. Moore: *On the homotopy of the union of two spheres with a point in common.*

Let $x_0 \in S^n$. Then $S^n \vee S^n$ is defined to be the subset $S^n \times \{x_0\} \cup \{x_0\} \times S^n$ of $S^n \times S^n$. Let X be the space of paths in $S^n \times S^n$ which start at (x_0, x_0), and end in $S^n \vee S^n$. $H_q(X)$ is a free abelian group with k generators for $q = (k+1)n-k$, $k \geqq 1$, and trivial for other values of $q > 0$. Then it is proved that $\pi_q(X) = \pi_q(S^{2n-1}) \cdot \pi_q(S^{3n-2} \times S^{3n-2}) \otimes \pi_q(S^{4n-3} \times S^{4n-3} \times S^{4n-3})$ for $q \leqq 5n-6$. This is used to extend some results of G. W. Whitehead on the distributive law for the composition operation (Ann. of Math. vol. 51 (1950) p. 215). (Received February 1, 1952.)

396t. J. C. Moore: *On the relative Hurewicz theorem.*

The formulation of the Hurewicz theorem of J. P. Serre (Ann. of Math. vol. 54 (1951) p. 491) is extended to the relative case. The theorem then reads that if (X, A) is a pair such that X and A are connected, simply connected, uniformly locally connected spaces whose homology groups are finitely generated, k is a field, and $H_q(X, A; k) = \{0\}$ for $0 \leqq q < n$, then $\pi_q(X, A) \otimes k = \{0\}$ for $0 \leqq q < n$, and $\pi_n(X, A) \otimes k$ is isomorphic to $H_n(X, A; k)$. The theorem is then applied to prove that if $E: \pi_q(S^n) \rightarrow \pi_{q+1}(S^{n+1})$ is the suspension homomorphism for n odd, and if $\alpha \in \pi_q(S^n)$ is an element such that $E(\alpha) = 0$, then α has order a power of 2. The Blakers and Massey triad theorem is extended to read that if $(X; A, B)$ is a triad such that X, A, B, and $A \cap B$ are connected, simply connected, uniformly locally connected spaces with finitely generated homology groups, (X, A) and (X, B) are 2-connected, k is a field, $H_q(X, A; k) = \{0\}$ for $q \leqq m$, and $H_q(X, B; k) = \{0\}$ for $q \leqq n$, then $\pi_q(X; A, B) \otimes k = \{0\}$ for $q \leqq m+n$. (Received February 6, 1952.)

397*t*. Moses Richardson: *Solutions of irreflexive relations*. Preliminary report.

The concept of solution of an irreflexive binary relation is defined in J. von Neumann and O. Morgenstern, *Theory of games and economic behavior*, Princeton, 1944, where it is proved that if the relation is strictly acyclic, a solution must exist. In a previous note (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 113–116), the present author proved that if the graph of the relation contained no odd cycles modulo 2, solutions exist. The assumption of asymmetry in this note was unnecessary. In the present paper, the investigation of various sufficient conditions for the existence of solutions of irreflexive relations, in which no assumption is made as to symmetry or transitivity, is continued. In particular, it is proved that if the graph is finite and contains no odd oriented cycles, then solutions exist. This result improves on the previous one since intransitivity is no longer required, thus making the result applicable to the theory of games. The proof uses a new method but still employs the theory of linear graphs. Still weaker but more complicated assumptions, as well as extensions to infinite systems, are studied. (Received February 14, 1952.)

398*t*. P. M. Swingle: *Densely looped indecomposable connexes*.

Let S be the plane. C is a densely looped indecomposable connexe if C is connected and for each pair R, R' of mutually exclusive regions where $C\cdot R\neq 0\neq C\cdot R'$, one of these, R say, is such that its boundary plus $S-C$ contains a continuum which bounds a domain and separates $C\cdot R'$. There exists in S a compact indecomposable connexe (and a compact indecomposable continuum) which is not a densely looped indecomposable connexe. Each composant of a compact indecomposable continuum is itself a densely looped indecomposable connexe. Let C be a densely looped indecomposable connexe contained densely in a domain of S and N be a finite set of limit points of C. Then $C+N$ is a densely looped indecomposable connexe. Let D be a compact connected domain in S, C be a connected subset of D, K' be a composant of an indecomposable connexe in $D\cdot(S-C)$ where both K' and D and C and D have the same closure; then C is a densely looped indecomposable connexe. (Received March 10, 1952.)

399*t*. C. T. Yang: *On cohomology theories*.

It is proved that for fully normal spaces the Alexander-Kolmogoroff cohomology theory agrees with the unrestricted Čech cohomology theory for arbitrary coefficient groups. A natural isomorphism between these two kinds of cohomology groups is actually constructed. The following corollaries result: (1) For compact Hausdorff spaces the Alexander-Kolmogoroff cohomology theory agrees with the Čech cohomology theory (Spanier, Ann. of Math. (1948)). (2) The homotopy axiom for the Alexander-Kolmogoroff cohomology theory holds for fully normal spaces and hence all of the Alexander-Kolmogoroff cohomology groups of convex subsets of linear metric spaces are trivial. (3) The extension, reduction, and hence map excision theorems hold for the unrestricted Čech cohomology theory. (4) If a fully normal space has Lebesgue dimension at most n, then all of its Alexander-Kolmogoroff cohomology groups in dimensions above n vanish. (Received February 22, 1952.)

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