BOOK REVIEWS

Lezioni nulla teoria delle funzioni di una variabile complesa. By G. Sansone. Padova, Cedam, 1947. Vol. 1, 8+359 pp. L. 750. Vol. 2, 11+564 pp. L. 950.

These two volumes on the theory of analytic functions are based on lectures given by the author at the University of Florence. As can be expected from the size of this treatise a much wider variety of topics is covered than usual. A brief summary of the contents of the various chapters gives an indication of the material included in each of the two volumes.

Volume 1: Chap. 1, Power series and elementary functions; Chap. 2, Integral theorems of Cauchy, Laurent series; Chap. 3, Factorization theorem of Weierstrass; Chap. 4, Entire functions and the theorem of Picard; Chap. 5, Euler-Maclaurin and Lagrange series, asymptotic series and methods of summability.

Volume 2: Chap. 7, Dirichlet series, Riemann zeta function, hypergeometric series; Chap. 8, Conformal mapping, Riemann's theorem; Chap. 9, Harmonic functions, Dirichlet and Neumann's problems; Chap. 10, Hyperbolic metric and applications to conformal mapping; Chap. 11, Elliptic functions; Chap. 12, Fuchsian functions.

The broadness of the theory of analytic functions has naturally forced the author to restrict himself in his choice of topics and any choice would have invited a criticism. It seems, however, unfortunate that the theory of multivalued functions and Riemann surfaces were left out entirely. Both in his choice of topics and in their treatment the author has followed classical lines. In the opinion of the reviewer the injection at some places of a more modern point of view could have increased the value and interest of the book. The point set topology of the two-dimensional plane is not given as full a consideration as might be desired and in at least two cases the proofs are not complete on this point. But as an over-all judgment the author gives a clear, careful and thorough exposition of the theory. The many references to original papers and later contributions which are included in the text should be particularly valuable to the reader.

F. Bohnenblust

A treatise on set topology. Part I. By R. Vaidyanathaswamy. Madras, Mahadevan, 1947. 6+304 pp. Rs. 16–4.

This book begins with three introductory chapters on sets; topics treated here are algebras of subsets of a set, rings and fields of sets and algebras of partial order. Fundamental ideas concerning sets such as mapping, lattice, Boolean algebra, ideal and measure are included.

With these preliminaries out of the way, the author introduces topological spaces. The next three chapters deal with general topological spaces and some important special classes of spaces. Subjects discussed include Kuratowski's closure function and other topological concepts defined in terms of it, the neighborhood postulates due essentially to Hausdorff and the lattice of closed sets and the lattice of open sets. Concepts such as strengths of topologies, postulates of countability, separation axioms and bicompact and compact spaces are studied.

With the aid of the important properties of the spaces developed in the previous chapters, the book now journeys into a very essential part of the subject, the theory of mappings. The next chapter investigates various kinds of mappings: closed, open and continuous, including homeomorphic. Kolmogorov's theory of resolution spaces, chain resolution, Urysohn's lemma on normality, and Urysohn's extension theorem are treated. This chapter also has an excellent exposition of the important completely regular (Tychonoff) spaces.

The remaining four chapters deal with further structural properties of spaces, sets in spaces, mappings and convergence concepts. The derived set and related subjects are studied. Special topological products and classical theorems on topological products are handled. Convergence in metric spaces is treated; here analyses are made of compactness, completeness and topological completeness. The final chapter is on convergence topology; convergence is considered as a primitive concept given by a system of axioms which is very slightly more specialized than that of Fréchet's L-spaces.

This book has grown out of lectures delivered by the author at the University of Madras for several successive years. For the most part the book is self-contained; the portions demanding extra knowledge involve some elements of analysis, including an acquaintance with measure theory and the theory of transfinite numbers. It seems to the reviewer that a short section on transfinite numbers might be advantageously included so as to present at least the theorems of that subject needed in this text.

Throughout the book, the author emphasizes partial ordering. The author states "partial ordering is an inherent feature of the topological situation." In connection with partial ordering, the author raises two questions (cf. the Preface and p. 134, 1.20). They both admit affirmative answers. For a solution of the first, see the reviewer's Bull. Amer. Math. Soc. Abstract 54-5-212.

Since the book appears to be primarily intended for beginners, it seems that the author should mention and emphasize the methodology and consequences of the axiomatic approach. For instance, mention might profitably be made of the importance of function spaces in modern developments. Indeed, some not-so-bright novice might even be tempted to ask what this is all about. Aside from this general consideration, it seems that certain special aspects of set topology also fail to receive adequate emphasis; for example, the distinction between absolute (intrinsic) and relative concepts is not stressed and the metrization theorem of Urysohn appears as an exercise. Although α and μ ideals are introduced, and μ ideals of the sets in a topological space are studied, the author does not appear to be aware of the further development in the form of H. Cartan's important theory of filters (that is, proper α ideals). Certain classical landmarks fail to have the proper signposts. For instance, epithets such as Tychonoff space, Heine-Borel-Lebesque property, Urysohn's lemma on normality, and first and second axioms of countability do not stand over the corresponding places. The book also lacks an appendix, which can be profitably used especially in a book of this kind. However, some of these criticisms, particularly the last one, are probably premature, since the author plans to publish the second part of this treatise in 1948.

A few misprints and minor errors are observed. Most of the misprints can be easily corrected and are not likely to cause much confusion. Some of the slips are:

The "if" part of ex. 19, p. 58 is valid but not correctly proved; the equalities $H=\text{Int } X$ and $\text{Bd } \overline{F\cap H'}=\text{Bd } X$ are both false.

On p. 111, 1.27 to 1.28 "$S\cap\overline{U}_x$ is the closure $\cdots$ of $\cdots$ $S\cap U_x$" should read "$S\cap\overline{U}_x$ *contains* the closure $\cdots$ of $\cdots$ $S\cap U_x$". This error occurs in several places.

The statement "the extension of f_0 will then give the extension of f" in the last line of p. 158 is not correct unless ∞ is also admitted as a value. However, Theorem 45.1 (Urysohn's extension theorem) is true without considering ∞ as a value; the unbounded case can be disposed of by a simple argument.

Ex. 9(2) of p. 164 is incorrect. The word "is" in 1.30 should be "contains." Consider, for example, the subspace $(0, 1/2, 1/3, 1/4, 1/5, \cdots)$ of the real number space.

The book is generally accurately and clearly written. A beginner who has had the prerequisites mentioned previously should find little difficulty in reading it. Every chapter of the book contains a large collection of exercises. These examples can serve well to illustrate the concepts and methods which occur; the more difficult ones among them might, in addition, help the reader to delve more deeply into the subject matter. Solutions of more difficult exercises are given or indicated in the text.

HING TONG

Integration in finite terms. By J. F. Ritt. New York, Columbia University Press, 1948. 7+100 pp. $2.50.

Advances in mathematical analysis have been intimately bound to the development of the notion of function. It was only in the last century that, both in the real and complex domain, the function concept was explicitly and completely elaborated. With this achievement it was possible to place on a sound foundation the calculus of earlier times. Having progressed so far, certain questions concerning the properties of functions of classical analysis lost their logical import. But their historical significance remains untarnished as generation after generation of young mathematicians is trained through the medium of the calculus.

The functions of classical analysis are the elementary functions: that is, those which can be constructed from the variables $x, y, \cdots$ by a finite number of algebraic operations and the taking of logarithms and exponentials. For example, $\cos y^{1/2}+\log [x^3+\arctan (x \log y)]$ is elementary. The young student quickly discovers that the closure of this set of functions under the operations of analysis is not an obvious fact, if it be a fact at all. Indeed, his instructor assures him that certain functions cannot be integrated in finite terms, that is, their integrals cannot be given an elementary representation. One hazards the guess that in a substantial number of the good courses in calculus offered in this and other countries, this is the one subject about which the instructor may not have first-hand knowledge. Rather, he imparts to his young charges, frequently with embarrassment, information which is based on hearsay.

Professor Ritt has now written a short book in which the reader will find all the material on this subject which should be the property of the complete mathematician. The theory exposed is one of considerable charm and of classical importance. A method is developed which may be used in attempting the solution of arbitrary problems on the representation of functions in an elementary manner. This method is then applied to certain specific questions where it yields complete results. It does not provide easy answers to the great variety of questions which one could propose. In spite of this fact, it possesses a certain degree of finality. It is truly extraordinary that no book has been written previously on this subject except in Russian.[1]

[1] The book by G. H. Hardy, *The integration of functions of a single variable*, Cambridge Tracts, 1905, is not one in which the Liouville theory is expounded. The author