one given by the author in a previous paper (*Projective differential geometry of a pair of plane curves*, to appear in Duke Math. J.). Here two projective invariants determined by the fourth order terms in the Taylor expansions of the functions representing the two curves, at the point of intersection, are obtained and characterized geometrically. Further, on the basis of the vanishing or nonvanishing of these two invariants, the author arrives at four different types of canonical representation for the two curves at the point of intersection; the absolute invariants in the expansions of each type are interpreted geometrically in terms of certain double ratios. (Received June 11, 1943.)

234. Edward Kasner: *Motion in a resisting medium.*

Consider the motion of a particle moving in the plane under a generalized field of force and influenced by a resisting medium, the resistance R acting in the direction of the motion and varying as some function of the position (x, y), the direction y', and the speed v of the particle. Through a given lineal element E, there pass ∞^1 trajectories. If the osculating parabolas are constructed to these trajectories at E, the locus of the foci varies in shape with the nature of the resistance R. If the focal locus is a circle through the point of E, it is found that R must be of the form $A(x, y, y')v^2 + B(x, y, y')$. In the final part of the paper, this result is extended to space. Construct the osculating spheres at E to the ∞^1 trajectories passing through the lineal element E. If the locus of the centers of these spheres is a straight line, the resistance R is of the form $A(x, y, z, y', z')v^2+B(x, y, z, y', z')$. Finally it is shown that if a single trajectory is known in the field of gravity with a resisting medium $R(v)$, then the law of resistance $R(v)$ can be completely determined. (Received July 27, 1943.)

235. Edward Kasner and John DeCicco: *A generalized theory of contact transformations.*

The authors present a generalized theory of contact transformations in the plane. Any union-preserving transformation of differential elements of order n into lineal elements is obtained by considering the osculating (up to and including contact of order n) of a given parameterized family of ∞^{n+1} curves to an arbitrary curve. If a union-preserving transformation T is such that any two unions which possess $n \geqq 2$ as the order of contact are converted by T into two unions which have at least second order contact, then T must be a contact transformation between lineal elements. As a consequence of this work, the authors find a general theory of evolutes and involutes which contains the osculating circle theory of Huygens and Bernoulli as a special case. (Received July 27, 1943.)

Logic and Foundations

236. Theodore Hailperin: *A set of axioms for logic.*

Two well known logical systems claiming adequacy for mathematics are currently studied. These are appellatively described as "Principia Mathematica" and "set-theory." A third and stronger system, called "New Foundations," has been proposed by W. V. Quine. (This system is not to be confused with his *Mathematical logic*, 1940.) Quine's system uses the usual logical primitives for the propositional calculus and the theory of quantifiers, and class membership, but makes no restrictions on the ranges of the variables. To avoid the contradictions, Quine has a metalogical axiom to the effect that only "stratified" formulas determine classes. The purpose of the present paper is the presentation of nine formal axioms and a demonstration of their equivalence with Quine's metalogical axiom in the presence of the restricted predicate calculus and the axiom for identity. (Received July 29, 1943.)

Numerical Computation

237. H. E. Salzer: *Coefficients for numerical integration with central differences.*

The numbers $M_{2s} \equiv B_{2s}^{(2s)}(s)/(2s)!$, where $B_{2s}^{(2s)}(s)$ denotes the $(2s)$th Bernoulli polynomial of order $2s$ for argument equal to s, were computed for $s=1, 2, \cdots, 10$, the result being exact values in lowest terms. Then these quantities were all checked by a cumulative recursion formula. (Previous calculations went only as far as $s=4$.) The numbers M_{2s} are coefficients of central differences of order $2s-1$ in the well known formula for numerical integration, sometimes called the Gauss-Encke formula or the second Gaussian summation formula. All important formulas for and references to these coefficients are included. This calculation was performed in the course of work done for the Mathematical Tables Project, National Bureau of Standards. (Received July 7, 1943.)

Statistics and Probability

238. E. J. Gumbel: *On the plotting of statistical observations.*

It is well known that there exist two stepfunctions corresponding to a continuous variate. We may attribute to the mth observation the ranks m or $m-1$. To obtain one and only one serial number m, which will, in general, not be integer, we attribute to x_m an adjusted frequency $m-\Delta$, namely the probability of the most probable mth value. The correction Δ for the rank is unlimited and possesses a mode, Δ increases for increasing value of the variate from zero up to unity. The correction is important for small numbers of observations. For large numbers of observations and for the ogive it is sufficient to choose $\Delta=1/2$. The calculation of Δ allows a correct plotting of all observations (including the first and last) on probability paper (equiprobability test). For the return periods, the ranks m and $m-1$ correspond to the observed exceedance and recurrence intervals. Generally the corrected return periods pass for increasing values of the variate from the exceedance to the recurrence intervals, provided the variate is unlimited and possesses a single mode. The asymptotic standard error of the partition values may be used to construct confidence bands for the ogive, the equiprobability test, and the return periods. This control for the fit between theory and observation may be applied to all observations which are not extreme. (Received July 30, 1943.)

239. Henry Scheffé: *On a measure-theoretic problem arising in the theory of non-parametric tests.*

Let μ be any measure on the real line, such that the measure of the whole line is unity, and form the "power" measure μ^k in Euclidean k-space—that is, the product