and important problems in probability itself, and then on to statistics in general and to the statistical problems of theoretical physics or mechanics. Some section headings in the latter topics are the following: Linked events, Lexis theory and the laws of large numbers, Mendel's theory of heredity, Industrial statistics, Galton's board, The second law of thermodynamics, Machines dependent on chance, Small causes and large consequences, Kinetic theory of gases, Brownian motion, Entropy theory and Markoff chains, Radio-active radiations, Quantum theory, The renunciation of causality, Heisenberg's uncertainty principle.

EDWARD L. DODD

La Notion de Point Irrégulier dans le Problème de Dirichlet. By Florin Vasilesco. (Actualités Scientifiques et Industrielles, no. 660.) Paris, Hermann, 1938, 61 pp.

This is the twelfth of the booklets containing "Exposés sur la Théorie des Fonctions" published under the direction of Paul Montel as a part of the series named above. This number contains an interesting compilation of the results of some recent researches on the Dirichlet problem. The discussion makes frequent use of material treated in another booklet by the same author: *La Notion de Capacité* (Actualités Scientifiques et Industrielles, no. 571, 1937).

The first chapter is devoted to a brief discussion of artificial (that is, removable) singularities of harmonic functions of three variables. The author takes as his starting point a theorem to the effect that a function continuous at a point P and bounded and harmonic elsewhere in the neighborhood of P is harmonic at P. As this theorem is attributed to Picard (1923) the author is evidently unfamiliar with an earlier paper by Bôcher in which the same result is established (this Bulletin, vol. 9 (1903), p. 455 ff.; the priority of Bôcher's theorem has already been noted by Raynor, ibid., vol. 32 (1926), p. 537 ff. and by Kellogg, ibid., vol. 32 (1926), p. 664 ff.).

The second chapter, which is the longest of the pamphlet, is devoted to the study of the solution of the generalized Dirichlet problem. The author discusses such topics as conditions for regularity and irregularity of boundary points (especially those expressed in terms of capacity or the conductor potential) barriers, the generalized Green's function, Lebesgue's example of an irregular point, and Kellogg's lemma and some of its corollaries.

Chapter III is devoted mainly to a discussion of the results concerning balayage published by Frostman in 1935.

Chapter IV contains a brief summary of the contents of a booklet by de la Vallée Poussin, *Les Nouvelles Méthodes de la Théorie du Potentiel et le Problème de Dirichlet Généralisé* (Actualités Scientifiques et Industrielles, no. 578, Paris, 1937). One of the topics discussed is the lightening of the requirement of continuity at a multiple boundary point of a spatial domain. Apparently both de la Vallée Poussin and Vasilesco have overlooked an earlier discussion of the Dirichlet problem for three dimensional domains with multiple boundary points (Perkins, Transactions of this Society, vol. 38 (1935), p. 106 ff.).

The remaining chapters of the booklet contain brief sketches of some of the most recent work on topics connected with the Dirichlet problem. Much of this exposition is based on researches published in 1938. Chapter V is concerned principally with Marcel Riesz's notion of generalized potentials of order α (an extension of earlier work by Frostman); Chapter VI is devoted primarily to an account of recent work on balayage by Brelot. Chapter VII is unique in that it makes use of logarithmic capacity; it contains some results (due mainly to Frostman) concerning functions of a complex variable.

An extensive bibliography is given at the end of the booklet.

A few typographical errors were noted. The footnote on page 19 is clearly intended to refer to the first paragraph after the italicized statement of the theorem concerning continuity. In the second formula on page 31, for γ read λ. In the first theorem of section 28, page 46, for (*d'ordre a*, $0<\alpha\leqq 2$) read (*d'ordre* α, $0<\alpha\leqq 2$).

The author assumes that the reader has some knowledge of modern potential theory, especially the theory of capacity. The reviewer feels that even though the pamphlet may not be intended for novices, the discussion of Kellogg's lemma (pages 20–23) could be made clearer by a few prefatory remarks recalling the distinction between Wiener's and de la Vallée Poussin's definitions of capacity.

Despite some imperfections of detail, the pamphlet is unquestionably a notable contribution to the literature on modern potential theory, and will undoubtedly prove highly valuable to many students of this subject.

FRED W. PERKINS

Les Fonctions de Matrices. I. *Les Fonctions Univalentes*. By Hans Schwerdtfeger. (Actualités Scientifiques et Industrielles, no. 649.) Paris, Hermann, 1938. 58 pp.

This monograph should be of particular interest to the student of analysis; the introductory algebraic treatment, in fact, is suggested and delimited by problems in the field of differential equations.

The opening chapter is an exposition of those properties of matrices that are standard equipment for every geometer and algebraist. It is in Chapters II and III that one finds the author's own contributions.

Chapter II deals mainly with the following problem: Given a set of single-valued functions $f(\xi)$, $g(\xi)$, $\cdots$ of the real or complex variable ξ, and an n-rowed square matrix A with fixed coordinates in the complex field, it is required to define n-rowed square matrices $f(A)$, $g(A)$, $\cdots$ so that the following requirements are satisfied:

1. If $f(\xi)$ is a constant α for every ξ in the set S, then $f(A)=\alpha E$ (E the identity matrix) for every A whose characteristic roots lie in the set S.

2. To the functions $f(\xi)\pm g(\xi)$ and $f(\xi)g(\xi)$ correspond respectively the matrices $f(A)\pm g(A)$ and $f(A)g(A)$.

3. To the function $g(f(\xi))$ corresponds the matrix $g(f(A))$, assuming that the variation of ξ is so restricted that the values η of $f(\xi)$ lie in the domain of definition of $g(\eta)$.

When each function is a polynomial $\sum\alpha_i\xi^i$, the problem is solved by choosing for $f(A)$ the polynomial $\sum\alpha_i A^i$. For a large class of functions the author shows that the definition of $f(A)$ can be obtained from the "polynomial definition" by using an interpolation formula of Hermite. In particular, if $f(\xi)$ is a single-valued function such that $f(\lambda_i)$, $f'(\lambda_i)$, $\cdots$, $f^{(n-1)}(\lambda_i)$ exist, where λ_i is a root, of multiplicity n_i, of the characteristic equation of A, then the author obtains for $f(A)$ the formula $f(A)=\sum_{i=1}^{m}A_i\sum_{r=0}^{n_i-1}(1/r!)f^{(r)}(\lambda_i)(A-\lambda_iE)^r$, where A_i and m are respectively the Frobenius covariants and the number of distinct characteristic roots of the matrix A. In the final section of Chapter II the author shows that from this formula one obtains a definition for $f(At)$, t a real or complex variable, if one replaces $f^{(r)}(\lambda_i)$ by $f^{(r)}(\lambda_it)$. For $f(\xi)=e^\xi$, in particular, the author's formula leads to $e^{At}=\sum_{i=1}^{m}e^{\lambda_it}A_i\sum_{r=1}^{n_i-1}(t^r/r!)(A-\lambda_iE)^r$.

The function e^{At} occupies a central position in Chapter III. Here the author applies the results of Chapter II to the theory of systems of differential equations. As one would expect, the matrix notation enables him to exhibit solutions in a compact and elegant form.

CHARLES HOPKINS