## SHORTER NOTICES

*Über einige neuere Fortschritte der additiven Zahlentheorie.* By Edmund Landau. (Cambridge Tracts in Mathematics and Mathematical Physics, no. 35.) London, Cambridge University Press; New York, Macmillan, 1937. 94 pp.

This tract presents the last published book of the great mathematician, and reflects in crystal clear fashion many characteristic traits of his mathematical work: an exceptional ability to refine the arguments of workers in the same field, anatomical exposure of the pattern of proof, elimination of detours, a tendency toward elementary methods (not at all synonymous with simple), the reduction of the use of words to a very minimum. The result is a completely self-contained booklet (with the exception of the *Anhang* on Siegel's theorem on quadratic forms of negative discriminant) which could (theoretically) be read by any graduate student after a course in elementary number theory and advanced calculus. In reality, the tract represents very difficult reading, on account of its extreme conciseness.

It is interesting that Landau has in this work completely eliminated the use of his $O(\ )$, $o(\ )$, using instead a set of a hundred thirty-five $c_\gamma$'s.

After a very condensed introduction of nine pages, in which the aims of the various chapters are outlined, follow four chapters and an appendix: 1. Der Winogradoffsche Satz, 30 pp. 2. Der Schnirelmannsche Satz, 20 pp. 3. Die Sätze von Erdös und Romanoff, 11 pp. 4. Sätze von Khintchine, Besicovitch und I. Chowla, 11 pp. Anhang. Der Siegelsche Satz, 13 pp.

The object of Chapter 1 is to prove Winogradoff's theorem: $\lim_{x\to\infty} G(k)/k \log k \leqq 6$, where $G(k)$ is the smallest number sufficient to represent all sufficiently large positive integers as the sum of $G(k)$ or fewer $k$th powers of positive integers.

This theorem of 1934 is a landmark in the history of Waring's problem, which was first proposed about a century and a half ago, and to which elementary methods (in the ordinary sense of the word) were applied until Hilbert, in 1909, first proved the finiteness of $G(k)$. However, his analytic method, which involved the use of 5-fold Dirichlet integrals (first, 25-fold integrals), yielded a pure existence proof, and his methods, much as they excited the admiration of mathematicians, have not proved capable of such extension as was at the time hoped. Then followed the incredible applications of the theory of analytic functions by Hardy and Littlewood which first led to a definite large bound for lim $G(k)$. From Winogradoff's theorem it is known that every integer $x$, sufficiently large, can be represented by $G(k)$ $k$th powers, where $k < G(k) < 6k \log k$.

The contributions of American mathematicians to Waring's problem, particularly those of Dickson and his students, are well known.

The proof of Winogradoff's theorem is broken down, in a manner familiar to readers of Landau's former books, into a sequence of sixty-eight theorems, of which the last is the theorem itself. The theorem is quoted as (1) in the Introduction, and it is a typical Landau touch to end this chapter after thirty pages with the two lines:

Satz 68. (1).

Beweis. Klar.

The two theorems of Schnirelmann which comprise the second chapter are as follows:

THEOREM 91. *Let $M(x)$ be the number of positive integers $y \leqq x$ for which $y = p + p'$ ($p$, $p'$ primes) has solutions. Then, for $x \geqq 4$, $M(x) \geqq x/c_8$, ($c_8$ a constant).*

THEOREM 95. *Every positive integer $x > 1$ can be represented as a sum of at most $c_6$ primes, ($c_6$ a constant).*

Hardy and Littlewood had earlier derived the result that every sufficiently large odd number is the sum of three odd primes, but under the assumption of an unproved theorem concerning the distribution of zeros of a certain transcendental function. Since Schnirelmann, Winogradoff has proved the Hardy-Littlewood theorem without any unproved assumption, thus including both the Hardy-Littlewood theorem and Schnirelmann's result. This is the present status of Goldbach's conjecture that every even number greater than 4 is the sum of two odd primes.

The theorems of Chapters 3 and 4 are of a newer and less well known type than those of the first two chapters. An illustration is the following theorem:

THEOREM 96 (Khintchine-Erdös). *Let* $\mathfrak{A}$ *be a set of positive integers a, and let* $\mathfrak{B}$ *be a set of positive integers b enjoying the property that a certain positive integer l exists such that every positive integer is a sum of l or fewer numbers b. Let* $\mathfrak{S}$ *be the set of all numbers of the form a or* $a+b$, $A(x)$ *the number of a's* $\leqq x$, $S(x)$ *the number of s's* $\leqq x$, *and, for some* $\alpha$, $(0<\alpha<1)$, $A(x)\geqq\alpha$ *for all x greater than zero. Then, for all* $x>0$, $S(x)\geqq\alpha(1+(1-\alpha)/2l)x$.

The proof of this theorem is extremely short and involves a minimum of mathematical prerequisites. It gives almost the impression of an exercise in formal logic.

By specializing the set $\mathfrak{B}$, many important results are obtained, such as:

*For a given positive integer* $a>1$ *and for a positive integer* $x>2$, *the number of solutions of* $p+i^a\leqq x$, *p a prime, for any integer* $i>0$, *is greater than* $x/c(a)$, *where* $c(a)$ *is a constant dependent only on a; and a similar theorem for* $p+a^i\leqq x$.

To another theorem by Khintchine, Theorem 110, which belongs to the same general range as Theorem 96, Landau pays the compliment: "Der Kintchinesche Beweis ist elementar und doch ein sehr kompliziertes grosses Kunstwerk."

This book is a fitting memorial to a mathematician who was at the same time a leader in research and unsurpassed in his ability and eagerness to make the most advanced work done by others available to a larger group.

A. J. KEMPNER

*Zur Differentialgeometrie der Kurven und Flächen fester Breite.* By Hans Bückner. (Schriften der Königsberger gelehrten Gesellschaft, vol. 14, no. 1.) Halle, Niemeyer, 1937. 22 pp.

This pamphlet gives a readable and very brief account of this subject, based on the support function of the curve or surface. New results include lower bounds for the area of a surface of constant breadth and for length of a space curve of constant breadth; characterization of a curve of constant breadth as an oval for which any two of the support normals intersect within the closed region bounded by the curve; an analogous result for surfaces; and an equation connecting lengths and areas of the two parts into which a support normal divides a curve of constant breadth and the included area.

E. H. CUTLER

*The Theory and Construction of Nondifferentiable Functions.* By A. N. Singh. (Lucknow University Studies, no. 1, Faculty of Science, Session 1934–1935.) Lucknow, Newul Kishore Press, 1935. 7+110 pp.

The author of this tract has published numerous papers on nondifferentiability and curves without tangents since 1924, mostly in Indian periodicals, though a couple of papers have found their way to this country. In the present volume, which is based