

briefly; the various types of broadening and shift which are due to inter-molecular forces and which have been of interest in the terrestrial laboratory are not discussed, presumably because of their absence in stellar spectra.

Several chapters are then devoted to the chemical constitution, to the dissociative equilibrium of stellar compounds, and to the temperatures of the stars, with special reference to the sun. After this the discourse again turns toward spectroscopy, the subject being that of forbidden transitions. Near the end of the book we find a discussion of evidence regarding the state of nebulae, their degree of ionization, their temperature, their chemical composition.

Every reader will close the book with an acute realization of the power of analysis which has shed considerable light on these most elusive problems, and of the vastness of the field which still waits for investigation. A book by Roseland on astrophysics needs no recommendation. It is highly suitable that it should have appeared in the International Series of Monographs on Physics, thus aligning itself with some of the most outstanding treatises on modern physics.

HENRY MARGENAU

Éléments de géométrie infinitésimale. By G. Julia. Deuxième édition. Paris, Gauthier-Villars, 1936. 7+262 pp.

The second edition of Julia's differential geometry has entirely kept its original character. As such, it takes a very special position among the textbooks on the subject. In no other book are questions concerning contact and envelopes treated with so much elaboration and so much care. Not only does their discussion take more than a hundred pages of the first of the three chapters, but they also permeate many topics in later parts of the book. Great care is therefore taken in the application of the theory of implicit functions. In the new edition the author has here and there changed the order of the subjects, and added a few remarks, partly in footnotes, which have added to the clarity of an already very well written book.

D. J. STRUIK

Actualités Scientifiques et Industrielles. Exposés mathématiques publiés à la mémoire de Jacques Herbrand. Paris, Hermann.

1. *Sur quelques propriétés de polynomes.* By J. Dieudonné, 1934. 24 pp.
2. *Sur les suites stationnaires.* By N. Lusin, 1934. 19 pp.
3. *Étude des fonctions sousharmoniques au voisinage d'un point.* By Marcel Brelot, 1934. 55 pp.
4. *Charakterisierung des Spektrums eines Integral Operators.* By J. von Neumann, 1935. 20 pp.

1. The author develops an elementary method for determination of the radii of "starshapedness" and of "convexity" of some special classes of analytic functions and applies this theory to the case $z[P(z)]^{\alpha/n}$, where $P(z)$ is a polynomial of degree n and α is any real number.

2. Let (*) $E_0, E_1, \dots, E_\alpha, \dots, \alpha < \Omega$ be a transfinite sequence of Borel measurable sets, which is such that if α is a limit ordinal, then $E_\alpha = \lim_{\xi < \alpha} E_\xi$. The sequence (*) is said to be stationary if there exists a $\gamma < \Omega$ such that

$E_\gamma = E_{\gamma+1} = \dots = E_\alpha = \dots$, $\gamma \leq \alpha < \Omega$. In generalizing some classical results of Baire the author proves that (*) is stationary if it is decreasing and if, in addition, each E_α is F_σ . The problem in the general case is unsolved.

3. This useful pamphlet consists of an Introduction followed by three chapters. In Chapter I the author gives a rapid but clear and rigorous survey of properties of convex and subharmonic functions, in some places giving new proofs or supplying missing details in older proofs existing in the literature. Chapter II contains the author's own investigation concerning the behavior of a subharmonic function in a neighborhood of an isolated singular point. The last Chapter, III, contains applications to the theory of harmonic functions and of solutions of the equation $\Delta u = \phi(P)$. The bibliographical references are numerous.

4. The operator $Af(x)$ defined over the space $L_2(-\infty, \infty)$ is said to be an integral operator if it can be represented in the form $\int_{-\infty}^{\infty} a(x, y)f(y)dy$ where $\int_{-\infty}^{\infty} |a(x, y)|^2 dy < \infty$ for almost all x . The main purpose of the author is to find necessary and sufficient conditions which have to be satisfied by a spectrum of a self-adjoint Hermitian operator in order that it be the spectrum of an integral operator. By a very elegant argument it is shown that necessary and sufficient conditions in question consist merely in the requirement that $\lambda=0$ should be a limit point of the spectrum. An important place in the proof is occupied by the following theorem: *Being given any self-adjoint Hermitian operator H , it is always possible to find a completely continuous operator X of E . Schmidt's type of arbitrarily small norm, so that the spectrum of $H+X$ would reduce to a pure point-spectrum.* This theorem was proved by H. Weyl in the case of a limited operator H .

J. D. TAMARKIN

Introduction Mathématique aux Théories Quantiques. By Gaston Julia. Paris, Gauthier-Villars, 1936. vi+220 pp.

This book is the sixteenth of the well known series, *Cahiers Scientifiques*, and is the first of a series which proposes to give the mathematical foundation of quantum mechanics. In this first volume the essential difficulties of quantum mechanics (some of which concern the fact that Hilbert space is not finite dimensional) are merely foreshadowed, the attention being directed in the main to vector analysis in a space of finite dimensions. However, the treatment is sophisticated and designed, as far as possible, to carry over to the infinite dimensional case. Thus the treatment of linear equations is given in which the concept of rank is developed without the introduction of determinants. The derivation of the Jordan normal form for matrices is particularly clear, as is also the chapter on metric spaces. We can heartily recommend the book as an easy introduction to the well-known works of von Neumann (*Quantum Mechanics*), Stone (*Hilbert Space*), and Wintner (*Infinite Matrices*). There are only two parts of the book which seem to the reviewer other than highly commendable. The first of these contains the statement that unitary and Hermitian operators "se correspondent biunivoquement par les relations" (Cayley) $U = (H+i)/(H-i)$; $H = i(U+1)/(U-1)$. This is misleading since unitary matrices with a characteristic root unity are not cared for (except by a some-