lished, as well as the standard theorems on intersection and linking numbers. A final chapter deals with the Brouwer degree of a mapping, and the Lefschetz fixed point formula treated by the methods of Hopf.

The book contains a very complete bibliography, it is well indexed, and as a further help to the reader most chapters are prefaced by a summary of their contents. At many points in the text the reader is referred to supplementary notes collected at the end of the book, which indicate extensions of the theory and links with other work. Little attempt is made, outside these notes, to attribute ideas to their originators; this is particularly glaring when the ideas are heavily exploited. The authors apologize for their failure to treat the Alexander duality theorem and the theory of compact metric spaces (so beautifully rounded out by the recent work of Pontrjagin), but they promise to write a second volume on these matters if someone else does not do so first.

A. W. TUCKER

WEDDERBURN ON MATRICES

Lectures on Matrices. By J. H. M. Wedderburn. American Mathematical Society Colloquium Publications, volume 17. New York, American Mathematical Society, 1934. vii+200 pp.

For the past seventy-five years matric theory has been growing in stature and in favor among men. Many branches of mathematics have been more promising infants, but few have shown such sustained growth and ever widening field of application. The concept of matrix, like that of group, extends its roots under algebra, number theory, geometry, differential and integral equations, wave mechanics—a fairly representative cross-section of modern mathematics. This fundamental nature of matric theory has never been so generally appreciated by mathematicians as at present. Thus Wedderburn's book is timely.

An evident fact in the history of matric theory is that the important theorems are not due to any small group of men. A few names stand out prominently, of course, but it has taken close to a thousand distinct contributions to bring the theory to its present state. Why this was so is not evident, but it must be true that the theorems which now seem so clear to us were not intuitive to mathematicians at the time of their discovery. It has been the common history of the important theorems that they were discovered first in special cases, then generalized and laboriously proved, and finally furnished with neat direct proofs. In the book under review most of the theorems have reached the last stage of development, and the reader is apt to be unaware of the amount of publication which it renders obsolete.

In recent years Wedderburn has been one of the most important contributors to matric theory. His discoveries have been published as they came, and have taken their place in matric lore. It would be absurd, therefore, to expect a large proportion of the results in this book to be new. But theorems have been extended, sharpened, and clarified to a remarkable degree.

It is the organization and presentation of the material, however, which

make the peculiar appeal of the book. This is no mere compendium of results—the subject has been completely reworked and the proofs recast with the skill and elegance which come only from years of devotion.

The treatment is primarily that of the British school. The matrix is introduced through its representation as a linear vector function, and treated alternatively from this point of view and abstractly as an element of a matric algebra, according to convenience. Vector analysis, matric theory, and linear associative algebra are all integrated in this treatment, and each is used to advantage in proving results in the others. The reviewer's chief regret is that the theory was not brought into closer harmony with the modern German school of algebra; in many cases only a rewording is necessary to bring this about.

Chapter 1 is concerned with vectors, linear vector functions, basis, adjoint, and rank. Chapter 2 treats matric polynomials, the characteristic equation, and the square root of a matrix. In Chapter 3 we find the all-important theory of invariant factors and elementary divisors. Chapter 4 is devoted to the theory of vector polynomials and the equivalence of pencils of the type $A\lambda+B\mu$. The author gives his own presentation of this rather difficult topic, and while there is no great gain in brevity, the treatment seems to be rigorous and complete.

In Chapter 5 we have a treatment of compound matrices, direct products, power matrices, and a neat treatment of Schur's associated matrices. Chapter 6 is on hermitian, unitary, and orthogonal matrices with several nice proofs by the author. Chapter 7 is on commutative matrices, and includes a sharpening of the famous theorem of Frobenius concerning the characteristic roots of $\psi(x, y)$, where x and y are commutative.

Chapter 8, under the title of functions of matrices, contains many interesting morsels. The determinations of a multiform function of a matrix are well treated, with examples. We also find an amplification of the author's theory of the absolute value of a matrix, generalizing Hadamard's theorem on the maximum value of a determinant. Then come the derivative and integral of a matrix, matric functions of a scalar variable, then functions of a variable vector, and functions of a variable matrix. These lead to the covariant derivative of a tensor and to quantum matrices of finite order.

Chapter 9, on the automorphic transformation of a bilinear form, was first published by the author in 1922 in but slightly different form.

The last chapter is an elegant treatment of linear associative algebras, the subject which was started on its modern development by the publication of the author's dissertation in 1907. The linear algebra is a natural generalization of the matric algebra as the author has presented it, so the inclusion is a natural one. The principal decomposition theorems have here been materially simplified.

In brief, this book consolidates the gains of the last seventy-five years, filling in gaps and simplifying proofs, and laying a firm foundation on which the matric theory of the next century will arise.

C. C. MacDUFFEE