

## THE NINETEENTH REGULAR MEETING OF THE SOUTHWESTERN SECTION

The nineteenth regular meeting of the Southwestern Section of the Society was held at the University of Nebraska on Saturday, November 27, 1926. Visiting members were entertained at the University Club on the Friday evening before the meeting. The morning and afternoon sessions on Saturday were presided over by Professor W. C. Brenke.

The total attendance at the meetings was twenty-five, among whom were the following eighteen members of the society.

Ashton, Brenke, Candy, Collins, J. T. Colpitts, Congdon, Doole, G. C. Evans, Gaba, Gouwens, F. S. Harper, Holl, Louis Ingold, J. V. McKelvey, Pierce, Runge, Sherer, J. S. Turner.

The morning session closed with an extended paper, *A survey of discontinuous boundary value problems for Laplace's equation in two dimensions*, by Professor Griffith C. Evans of Rice Institute. This paper was presented by invitation of the program committee.

At the close of the afternoon session, which was devoted to the reading of further papers, a resolution was passed thanking the Department of Mathematics of the University of Nebraska for their hospitality. The section also extended a vote of thanks to Professor Evans for his address.

Abstracts of the papers presented at this meeting, except the extended paper mentioned above, are given below. The papers by Anderson, Ettlinger, and Wahlin were read by title. Mr. Atanasoff was introduced by Professor E. S. Allen.

1. Professor D. L. Holl: *A solution of  $\nabla^4 = 0$  occurring in the problem of viscous fluid motion.*

The problem of viscous incompressible fluid motion is the problem of determining a function  $\psi$  satisfying  $\nabla^4\psi = 0$  in the region considered and subject to a given set of boundary conditions. In this paper a solution

is found by the use of conjugate cylindrical coordinates characterized by the transformation  $\alpha + i\beta = \log(z+k) - \log(z-k)$  for a region between two eccentrically placed cylinders. The solution contains the special case of concentric cylinders, for which certain minimizing properties are evident immediately. Subsequent computations for all the relevant physical quantities concerned are found by which the stability of an apparatus, designed for experimental determinations, is predicted.

2. Professor G. E. Wahlin: *On a quadratic algebra over a field.*

This report contains the definition of an algebra of order  $2^r$  over a general field. The multiplication of the basal elements as defined is not associative. Every element of the algebra is a root of a quadratic equation with coefficients in the field.

3. Professor J. S. Turner: *Tchebycheff determinants.*

Tchebycheff gave forms  $\pm(x^2 - Dy^2)$  with small values of  $D$ , and explained their use in finding the factors of large integers (Journal de Mathématiques, vol. 16(1851), pp. 257-282). In the present paper 40 determinants are given which have the following properties: (a)  $D$  contains no square factor, and  $T^2 - DU^2 = 1$ ,  $U \leq 4$ , (b) each genus of  $D$  contains only one class of reduced forms. The method of using reduced forms  $ax^2 - by^2$  to find the factors of large integers is explained. In certain cases this method is superior to that in which Euler determinants are used.

4. Miss Nola L. Anderson: *An extension of Maschke's symbolic method.*

Maschke represented the quadratic differential form  $ds^2 = \sum g_{ij} dx^i dx^j$  in the symbolic form  $(df)^2 = (\sum f_i dx^i)^2$  where the  $f_i$  were treated as partial derivatives of a symbolic function. In the present paper the coefficients of this differential form are represented symbolically by the equations  $f_i f_j = g_{ij}$ , but it is not assumed that  $\partial f_i / \partial x^j$  is equal to  $\partial f_j / \partial x^i$ . It is found that differential parameters of the first order by the generalized method are represented in precisely the same manner as in Maschke's theory. Invariant expressions of higher order, however, are best regarded as invariants of the set of quantities  $g_{ij}$  and a certain other set analogous to Christoffel's triple-index symbols. A geometric interpretation is given for the two-dimensional case, and certain invariant vectors related to a two-dimensional surface are studied.

5. Professor M. G. Gaba: *A set of postulates for euclidean geometry in terms of point and inversion.*

The basis consists of a class of elements called points and a class of permutations among the points. Sufficient postulates or restrictions are placed upon the permutations to make them the circular inversions. Circles are defined and, by specializing a point, lines are defined. The usual

axioms of plane euclidean geometry are then derived as theorems or stated without using any other undefined notion.

6. Professor H. J. Ettlinger: *Note on Riemann-Stieltjes integrals.*

By means of the Duhamel-Moore theorem, a direct and simple proof is given of the following theorem established by J. M. Whittaker (*On integrals with a nucleus*, Proceedings of the London Society, vol. 25(1926), pp. 213-218): If  $f(x)$  is bounded on  $(a, b)$  and  $g(x)$  is bounded and  $R$ -integrable on  $(a, b)$ , and  $h(x)$  is the indefinite integral of  $g(x)$  from  $a$  to  $x$ , then a necessary and sufficient condition that the Stieltjes integral  $\int_a^b f(x)dh(x)$  exist is that the Riemann integral  $\int_a^b f(x)g(x)dx$  exist.

7. Mr. J. V. Atanasoff: *On the dynamics of a new type of molecular model.*

A molecular model is chosen in which the distribution of charge approximates that of modern theories of atomic structure. Force distance relations are developed on the assumption of classical electrodynamics. An investigation is made of the law of state of a gas composed of such molecules.

8. Professor Louis Ingold: *Quadratic first integrals in affine and related geometries.*

In a previous paper the author has indicated a method of transforming a given affine geometry by means of an arbitrary non-singular matrix  $\|a_i^j\|$  whose elements are functions of the coordinates. The present paper discusses conditions for the existence of quadratic first integrals for the differential equations defining the paths in the transformed geometries.

9. Professor Julia T. Colpitts: *The real zeros and other properties of a certain entire function of genus unity.*

The function considered is  $f_a(x) = \sum_{n=0}^{\infty} x^n / (n+a)^n$ ,  $a > -1$ . There is an even number of negative zeros. There is not more than one greater than  $-2e$ . When  $a \geq 2$ , there is no real zero. Approximations of the greatest zero for small values of  $a$  are obtained.

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*Secretary of the Section.*