THE APRIL MEETING OF THE SAN FRANCISCO SECTION.

The thirty-seventh regular meeting of the San Francisco Section of the American Mathematical Society was held at Stanford University, on Saturday, April 9. The chairman of the Section, Professor D. N. Lehmer, presided. The total attendance was twenty-eight, including the following fourteen members of the Society:

Professor R. E. Allardice, Professor B. A. Bernstein, Professor H. F. Blichfeldt, Professor Florian Cajori, Professor M. W. Haskell, Professor L. M. Hoskins, Professor D. N. Lehmer, Professor W. A. Manning, Professor H. C. Moreno, Dr. F. R. Morris, Professor C. A. Noble, Professor T. M. Putnam, Dr. S. E. Urner, Dr. A. R. Williams.

After the regular programme, interesting papers were read by Professor D. L. Webster, of Stanford University, on the quantum theory, and by Dr. T. L. Kelly, also of Stanford University, on the new theory of dispersion.

The dates of the next two regular meetings of the Section were fixed as October 22, 1921, and April 8, 1922.

Titles and abstracts of the papers read at this meeting follow below. Professor Bell's papers were read by title.

1. Professor Florian Cajori: *Euclid of Alexandria and the bust of Euclid of Megara.*

Professor Cajori proves that the portrait issued by the Open Court Publishing Company as representing Euclid of Alexandria, the geometrician, is in fact a portrait of Euclid of Megara. This article appeared in SCIENCE for April 29, 1921.

2. Professor Florian Cajori: *The spread of the Newtonian and the Leibnizian notations of the calculus.*

This paper appears in the present number of this BULLETIN.

3. Professor H. F. Blichfeldt: *The approximate solution in integers of a set of linear equations.*

Consider the n functions $f_1 = |x_1 + \alpha_1 z + a_1|$, $\cdots$, $f_n = |x_n + \alpha_n z + a_n|$, involving $n + 1$ variables x_1, $\cdots$, x_n, z and $2n$ given constants $\alpha_1, \cdots, \alpha_n, a_1, \cdots, a_n$, the numbers $\alpha_1, \cdots, \alpha_n$ being irrationals which satisfy no equation of the form $k_1\alpha_1 + \cdots + k_n\alpha_n + k = 0$, where $k, k_1, \cdots, k_n$ are integers not all zero. By a theorem of Kronecker it follows that, if a positive quantity ϵ as small as we please is given, then a set of integers $x_1, \cdots, x_n, z$ exist for which the functions f_i are all $< \epsilon$. It is proved in this paper that, given an arbitrary function $\varphi(z)$ of a positive integer z, provided that it represents a positive number approaching infinity with z and that $\varphi(z + 1) \geqq \varphi(z)$, a set of irrationals $\alpha_1, \cdots, \alpha_n$ may be defined, restricted as above, such that the condition $F < 1/\varphi(z)$ is satisfied by only a finite number of sets of integers $x_1, \cdots, x_n, z$; here F represents any one of the elementary symmetric functions of degree not higher than $n - 1$ of the n functions f_i. On the other hand, whenever the set of irrationals $\alpha_1, \cdots, \alpha_n$ is given, restricted as above, a set of integers $x_1, \cdots, x_n, z$ exists for which every $f_i < \epsilon$, and at the same time $\Pi_{i=1}^n f_i < N/z$, where N depends on n only.

4. Professor M. W. Haskell: *Autopolar curves and surfaces.*

In this paper, the author gives the following method for deriving all autopolar curves and surfaces. There is a (1, 1) correspondence between the line-elements of a given curve and the line-elements of its polar reciprocal with respect to a given conic. Every pair of corresponding line-elements determines a pair of line-elements belonging to the given conic. The locus of the foci of the involution defined by these two pairs of line-elements is a curve which is autopolar with respect to the given conic. Every autopolar curve with respect to the given conic can be derived in this way. Curves and surfaces autopolar with respect to a given quadric surface can be derived in the same way, substituting surface-elements for line elements.

The author then gives illustrations of multiply auto-polar curves: a special quintic with five cusps autopolar with respect to six conics, and a special quintic with three cusps and three conjugate points autopolar with respect to four conics.

5. Mr. P. H. Daus (introduced by Professor Lehmer): *Normal ternary continued fractions.*

The author discusses an extension of Jacobi's ternary continued fraction algorithm, and points out certain similarities between it and the ordinary continued fraction expansion.

6. Mr. D. V. Steed (introduced by Professor Lehmer): *The hyperspace generalization of the lines on the cubic surface.*

In this paper it is shown that a necessary condition for the existence of a finite number of linear spaces of dimensions d on the general hypersurface of order r in space of n dimensions is that r, n, and d satisfy the equation

$$\binom{r+d}{d} = (n-d)(d+1).$$

In particular the case where $d = 1$ is considered, and a method for the enumeration of the lines on the general hypersurface of order $2n - 3$ in space of n dimensions is developed. The numbers found for space of dimensions four, five, six, and seven are 2875; 698,005; 306,142,821; and 211,039,426,895, respectively, but by means of the recursion formulas given it is possible to make the corresponding enumeration for space of any dimension.

7. Professor D. N. Lehmer: *On the computation of interest on certain kinds of investments.*

The equations of high degree which are often involved in the computation of the rate of interest on annuities, when treated by Newton's method, yield certain correction formulas which avail to compute the rate to as high a degree of accuracy as may be desired and which require of the computer no more knowledge of the theory of equations or of algebra than is necessary to substitute quantities in a formula. The author exhibits these correction formulas for the rate of interest on a bond, bought at a certain price, and on an annuity whose present value, or whose amount, is given.

The paper is to appear in the AMERICAN JOURNAL OF ACCOUNTANCY.

8. Professor E. T. Bell: *The Bernoullian functions occurring in the arithmetical applications of elliptic functions.*

In applying elliptic functions to theorems on divisors, Glaisher, Halphen and others have given formulas involving Bernoullian functions.

In this paper it is shown that sixteen distinct Bernoullian functions, and no more, can arise in the arithmetic applications of elliptic theta functions and their quotients. The set is considered in detail with applications.

9. Professor E. T. Bell: *Anharmonic polynomial generalizations of the numbers of Bernoulli and Euler.*

In this paper the author discusses two cyclic sets and one anharmonic set which for every degree n degenerate when $z = 1$ either to Bernoulli or Euler numbers of rank n or to others naturally dependent upon these. The entire subject is developed by means of an extension of the symbolic calculus of Blissard and Lucas, and is shown to be simply isomorphic to the algebra of the twelve Jacobian elliptic functions of Glaisher. This analysis leads to several novel features, among which may be mentioned: the definition and solution of purely symbolic linear difference equations; symbolic addition theorems whereby the polynomials of degree $m + n$ can be easily calculated from those of degrees m, n; and a wholly new interpretation of Kronecker's symmetric functions which he took as the point of departure for his discussion of Bernoulli numbers. It is shown that the polynomials cannot be obtained by linear recurrences, and the appropriate recurrences are derived. Finally congruences to a prime modulus are discussed in detail. The algebraic relations and congruences between the polynomials degenerate for $z = 1$ to recurrences and congruences for Bernoulli and Euler numbers. Some of the degenerate cases were given by Lucas and others, thus affording checks.

10. Professor E. T. Bell: *Note on the prime divisors of the numerators of Bernoulli's numbers.*

This note, which will appear shortly in the AMERICAN MATHEMATICAL MONTHLY, contains a generalization of a result due to John Couch Adams, viz. if p is an odd prime which divides neither $2^r + 1$ nor $2^r - 1$, then the numerator of B_{2pr} (in Lucas' notation) is divisible by p. For $r = 1$ we get Adams' theorem. The proof follows from the series for sn u.

11. Professor E. T. Bell: *Proof of an arithmetic theorem due to Liouville.*

This paper appeared in the March number of this BULLETIN.

12. Professor E. T. Bell: *On a general arithmetic formula of Liouville.*

This paper appeared in the April number of this BULLETIN.

B. A. BERNSTEIN,
Secretary of the Section.