THE MARCH MEETING OF THE AMERICAN MATHEMATICAL SOCIETY AT CHICAGO.

THE twelfth regular meeting of the American Mathematical Society at Chicago, being also the forty-third regular meeting of the Chicago Section, was held on Friday and Saturday, March 28 and 29, at the University of Chicago. The various sessions were attended by about forty persons, among whom were the following thirty-one members of the Society:

Dr. I. A. Barnett, Professor H. F. Blichfeldt, Professor G. A. Bliss, Dr. Henry Blumberg, Professor W. H. Bussey, Professor R. D. Carmichael, Professor E. W. Chittenden, Professor A. R. Crathorne, Professor D. R. Curtiss, Professor L. E. Dickson, Professor L. W. Dowling, Mr. E. B. Escott, Professor A. M. Kenyon, Professor W. C. Krathwohl, Professor Kurt Laves, Professor A. C. Lunn, Professor G. A. Miller, Professor E. H. Moore, Professor E. J. Moulton, Professor H. L. Rietz, Professor W. H. Roever, Mr. J. B. Rosenbach, Dr. A. R. Schweitzer, Professor J. B. Shaw, Professor E. B. Skinner, Professor H. E. Slaught, Professor E. B. Van Vleck, Professor G. E. Wahlin, Professor E. J. Wilczynski, Professor J. W. A. Young, and Professor Alexander Ziwet.

Twenty-nine persons joined in a dinner at the Quadrangle Club on Friday evening. Following the dinner, a number of short informal speeches were made in response to calls from Professor Bliss who presided. Professor Slaught read interesting extracts from a letter which he had just received from Professor E. R. Hedrick, who is in France helping to organize educational work for members of the American Expeditionary Forces. Professor Ziwet of the University of Michigan extended a most cordial invitation to members of the Section to come to the summer session of the Society next September at Ann Arbor. Several members spoke against the too common practice among mathematicians of giving gratuitously information of a technical nature to persons untrained in mathematics; it was argued that the worth of mathematics would be more widely appreciated and its development correspondingly more rapid if mathematicians demanded adequate compensation for such services.

At a short business session on Saturday morning there was a discussion of the desirability of holding the December meeting of the Section this year in St. Louis in conjunction with the meetings of the American association for the advancement of science. It was voted to leave the matter to the program committee for decision.

In reading his paper on "Differential corrections for anti-aircraft guns," Professor Bliss paid tribute to the efforts of mathematicians who have been engaged in war work of a mathematical nature, mentioning especially the effectiveness of the work headed by Major F. R. Moulton at Washington and by Major Oswald Veblen at the Aberdeen Proving Ground on ballistic problems.

Friday afternoon was devoted to a symposium on the geometry of numbers, relating largely to the work of Minkowski. Formal papers were presented as follows:

I. Professor H. F. BLICHFELDT: "Report on the theory of the geometry of numbers," giving the fundamental theorems with applications to homogeneous and non-homogeneous forms.

II. Professor L. E. DICKSON: "Applications of the geometry of numbers to algebraic numbers."

Synopses of these papers will appear in the July BULLETIN.

Professor G. A. Bliss, chairman of the Section, presided at the various sessions, being relieved by Professors Curtiss, Miller, and Carmichael. At the sessions on Friday and Saturday mornings the following papers were read:

(1) Professor E. J. WILCZYNSKI: "A new geometrical representation for a function of a complex variable."

(2) Dr. A. L. NELSON: "Plane nets conjugate to a given congruence of straight lines" (preliminary report).

(3) Professor A. B. COBLE: "On the ten nodes of a rational plane sextic and of the Cayley symmetroid."

(4) Dr. FLORENCE E. ALLEN: "On a class of sectrix curves."

(5) Professor W. H. BUSSEY: "A note on the problem of the eight queens."

(6) Professors E. W. CHITTENDEN and A. D. PITCHER: "On the theory of developments of an abstract class in relation to the calcul fonctionnel."

(7) Professor E. J. WILCZYNSKI: "The scientific work of Gabriel Marcus Green."

(8) Professor G. A. BLISS: "Differential corrections for anti-aircraft guns."

(9) Professor H. L. RIETZ: "Functional relations for which the coefficient of correlation is zero."

(10) Professor L. L. DINES: "Systems of linear inequalities."

(11) Professor E. J. MOULTON: "A note on a proof of the law of the mean."

(12) Dr. A. R. SCHWEITZER: "On pseudo-groups with an application to the algebra of logic."

(13) Professor G. A. MILLER: "Groups containing a relatively large number of operators of order two."

(14) Dr. HENRY BLUMBERG: "On a theorem of W. H. Young and G. C. Young."

(15) Professor E. B. VAN VLECK: "On Green's lemma."

In the absence of the authors, the papers of Dr. Nelson, Professor Coble, Dr. Allen and Professor Dines were read by title.

Abstracts of the papers follow below. The abstracts are numbered to correspond to the titles in the list above.

1. In this paper, Professor Wilczynski interprets the complex variables z and w as points upon the same Neumann sphere, and joins by straight lines the points which correspond to each other by means of a functional relation $w = F(z)$. The resulting congruence has a large number of interesting properties. Its focal surfaces are real surfaces of positive curvature, and the foci on each line of the congruence divide harmonically its two points of intersection with the sphere. The congruence is a W-congruence, and all of the asymptotic lines of the focal surface belong to linear complexes. The Neuman sphere is the directrix quadric of the focal surface. The asymptotic lines of the focal surface correspond to the minimal lines of the z-plane and can be obtained with ease. The developables of the congruence, which are real, are also obtained and interesting relations are shown to exist between the directrices, axes, and rays of the two focal sheets, as well as between the directrix, axis, and ray curves.

2. Dr. Nelson has made use of Wilczynski's methods in the study of the projective properties of a congruence of straight lines as related to those of the plane nets which are conjugate to it, in the sense of Guichard. Following are some of the results.

(1) If the plane nets have straight lines for constant values of one parameter, one of the nappes of the focal surface of the congruence is developable, and conversely.

(2) If the first Laplace transforms of the plane nets have straight lines corresponding to the constant values of one parameter, the new focal nappe of the minus first derived congruence becomes developable, and conversely.

(3) If the plane nets are periodic under the Laplace transformation, of period three, the following properties hold for the congruence:

(*a*) The lines which join the first and second, and the minus first and minus second, Laplace transforms of the surface point of either of the focal nappes will intersect on the axis of that surface point.

(*b*) This point of intersection will be the harmonic conjugate of the surface point with respect to the foci of its axis. It will also be one of the foci of the joint axis which corresponds to the surface point.

(*c*) The joint axis curves and joint ray curves coincide and become parametric.

(*d*) The parametric tangents at a particular surface point of either focal nappe will be the double rays of the involution determined by the axis tangents and ray tangents at the same point.

3. The notion as developed by Professor Coble of the congruence of n points in a projective space under Cremona transformation leads to an infinite discontinuous group which can be exhibited as a group of linear transformations with integer coefficients. Thus the group has a modular theory and in an earlier paper the types of finite groups obtained by reduction modulo 2 were determined. It now appears that these modular groups have not merely an arithmetic but also a geometric existence when the n points are properly specialized. Such cases for example are the ten nodes of a rational plane sextic, and in space the ten nodes of a Cayley symmetroid. Some of the theorems obtained are as follows. A general rational plane sextic can be transformed into precisely $2^{13} \cdot 31 \cdot 51$ projectively distinct sextics and under such transformation these types are permuted by a group isomorphic with a theta modular group of genus 5. Again if there exists a quartic curve with a triple point at one node

and on the nine others (one condition on the sextic) then there exists a similar quartic with a triple point at *any* one of the nodes. Similar theorems apply to the symmetroid. In this case the genus of the group is 4. There is thus foreshadowed a uniform parametric representation of the ten nodes in terms of abelian modular functions of genus four.

4. A system of curves discussed in a paper presented to the Society by Dr. James H. Weaver in April, 1917, is shown by Miss Allen to be a special case of the Schoute sectrix curves and of the araneïd or "spider" curves. It is shown that the system for successive values of n may be generated by a birational transformation, consisting of an inversion followed by a perspectivity. The generation of the entire system of araneïds by alternate applications of these two operations is also proved.

5. The problem of the eight queens is the determination of the ways in which eight queens can be placed on a chess board—or more generally, in which n queens can be placed on a square board of n^2 cells—so that no queen can take any other queen. Professor Bussey's note shows that in the special case in which n is a prime number p, there are $p^2 - 3p$ solutions furnished by the $p^2 - 3p$ straight lines of slopes 2, 3, 4, $\cdots$, $(p - 2)$ of the finite euclidean geometry of order p^2. The solutions furnished by this method are not the only solutions, however.

6. In a paper, which has not been published, entitled "Classes which admit a development" (presented to the Society, March 21, 1913), Professors Chittenden and Pitcher discussed the theory of developments Δ of an abstract class in relation to the calcul fonctionnel of Fréchet. [A development Δ of a class P is a system $((P^{ml}))$ of subclasses of P $(m = 1, 2, 3 \cdots; l^m = l_1^m, l_2^m, \cdots)$. Cf. E. H. Moore, Introduction to a Form of General Analysis, Yale University Press, New Haven, 1910.] In the present paper the above mentioned theory is refounded and based upon a set of five properties of a development Δ which together imply that the class P is a compact metric space (cf. Hausdorff, Grundzüge der Mengenlehre, Leipzig, 1914). The five properties are proved to be completely independent in the sense of E. H. Moore (loc. cit.).

The results of the general theory are applied to obtain the necessary and sufficient conditions that a topological space (cf. Hausdorff, loc. cit.) be equivalent to a compact metric space. A further application is made to spaces of non-metrical analysis situs. Every limited domain in a space δ satisfying the axiom system Σ_1 (Σ_2) of R. L. Moore (*Transactions of the American Mathematical Society,* volume 17, 1916) is a compact metric space. This result was obtained independently by R. L. Moore by another method.

The paper will appear in the *Transactions.*

7. Professor Wilczynski's paper, intended for publication in the BULLETIN, is devoted to a brief description of the mathematical work of Gabriel Marcus Green, whose premature death inflicts a heavy loss on American mathematics.

8. In the applications of ballistics it is customary to compute a first approximation to the path of a projectile on the assumption that there is no wind, and on the further assumptions that the density of the air and the weights of the projectile and powder charge are normal. These are ideal conditions rarely if ever realized in actual firing. The range table must therefore contain differential corrections which can be applied in the field, either mechanically or personally, to account for the actual conditions at the time when the firings are made. The so-called differential corrections appear as the solutions of a system of linear differential equations which are solved by mechanical quadratures. In the paper of Professor Bliss an auxiliary system of linear differential equations is used by means of which the number of solutions to be computed by mechanical quadratures is reduced to three. The remainder of the computation is of the simplest sort, requiring no especial skill, and relatively rapid.

9. In this paper, Professor Rietz treats some simple and striking cases in which the correlation coefficient of two variables x and y is zero although there is a simple functional relation between x and y. The result $r = 0$ may be accounted for by a consideration of the lines of regression. This brings out the importance of testing the curve of regression, and the danger of making applications of the correlation coefficient without knowledge of the underlying theory. In the cases

cited, for a single valued function $y = f(x)$, the correlation ratio η_y can be used appropriately instead of r to give a summary notion of correlation.

10. In this paper, Professor Dines constructs a theory relative to systems of linear inequalities, which to a considerable extent parallels the classic theory of systems of linear equations. The paper will appear in the *Annals of Mathematics.*

11. A proof of the law of the mean for derivatives has been based on the law of the mean for integrals (Goursat-Hedrick, Mathematical Analysis, volume I, page 155). Professor Moulton calls attention to an error in the proof, and shows how it can be corrected. The hypotheses of the theorem proved in this way are more restrictive than when proved in another common way. It is pointed out that if Denjoy's generalization of integration is used the derivative $f'(x)$ need not be continuous on the interval (a, b), nor integrable in the sense of Riemann or Lebesgue. The restrictions are that $f'(x)$ must exist and be finite at every point of the interval.

12. A set E of elements forms a pseudo-group if E satisfies a set of properties $\Sigma[\rho_1, \rho_2, \cdots]$ necessary for an abstract group (finite or infinite) and including closure with regard to the undefined relations $\rho_1, \rho_2, \cdots$ generating Σ. Every group is a pseudo-group as are also the semi-groups of Dickson and Hilton. In a previous paper Dr. Schweitzer exhibited a pseudo-group $\Sigma(\theta)$ satisfied by $\theta(x, y) = x^{-1}y^{-1}$ of an abstract group. In the present paper Dr. Schweitzer constructs further pseudo-groups, in particular with reference to distributive properties. Among the latter class there exists a pseudo-group satisfied by $\zeta(x, y) = x^{-1}yx$ and $\tau(x, y) = xyx^{-1}$ of an abstract group and possessing duality with reference to the generating relations $\zeta(x, y)$ and $\tau(x, y)$. This pseudo-group $\Sigma[\zeta, \tau]$ has the property that it is satisfied by any set of elements obeying the "additive" and "multiplicative" laws of the algebra of logic. Incidentally, a new set of postulates for the latter discipline is obtained merely by assuming that for the pseudo-group $\Sigma[\zeta, \tau]$ the "principle of absorption" $\zeta[x, \tau(x, y)] = x$ is valid. Fundamental for the discussion is

the fact that the properties $\zeta(x, x) = x$, $\zeta[x, \tau(x, y)] = x$ or y, $\zeta[x, \tau(y, z)] = \tau[\zeta(x, y), \zeta(x, z)]$ and their duals are valid both for the abstract group $[\zeta(x, y) = x^{-1}yx, \tau(xy) = xyx^{-1}]$ and the algebra of logic $[\zeta(x, y) = x \cdot y, \tau(x, y) = x + y]$.

13. Professor Miller's paper appears in full in the present number of the BULLETIN.

14. A given planar set S is said to have a triangular void at a point P, if P is the vertex of a triangle containing no points of S in its interior. According to a recent result of W. H. and G. C. Young (*Proceedings of the London Mathematical Society*, 1918), a planar set having a triangular void at every one of its points is of measure zero. Dr. Blumberg gives a simple proof of this theorem, and a generalization based on the following definition: Let C_r be the circle of center P and radius r and $m_e(S, C_r)$ the exterior measure of the portion of S in C_r; then we define the "upper exterior measure of S at P" as

$$\frac{\limsup}{r \to 0} \frac{m_2(S, C_r)}{\pi r^2}.$$

The generalization is as follows: A planar set having its upper exterior measure < 1 at every one of its points is of measure zero. This theorem is utilized to obtain certain general properties of functions of two variables. Extension to higher spaces is immediate.

15. In the proofs which have been given for Green's fundamental lemma

$$\int\int \frac{\partial P(x, y)}{\partial x} dxdy = \int_c P(x, y) dy$$

notable restrictions have been imposed upon the boundary. The usual condition is that the contour C shall be cut by a parallel to the axis of X in a finite number of points or consist of a finite number of "regular" pieces. The paper of Professor Van Vleck gave a proof for the general case in which C is merely conditioned to be a simple closed rectifiable curve. Over its closed interior $P(x, y)$ is supposed to be continuous, while the requirement for $\partial P/\partial x$ is that it shall be properly integrable.

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