This can be proved by combining the two formulas in Elliott's paper in the *Messenger of Mathematics*, volume 7 (1878), page 151 and in Minkowski's paper in the *Mathematische Annalen*, volume 57 (1903), page 463.

But any similar formula for the relative volume of two convex ovoid bodies cannot be established.

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CONCERNING THE DEFINITION OF A SIMPLE CONTINUOUS ARC.

BY DR. GEORGE H. HALLETT, JR.

(Read before the American Mathematical Society October 26, 1918.)

IN a paper entitled "Curves in non-metrical analysis situs with an application in the calculus of variations," *American Journal of Mathematics*, volume 33 (1911), pages 285–326, N. J. Lennes gives the following definition of a simple continuous arc.*

"A continuous simple arc connecting two points A and B, $A \neq B$, is a bounded, closed, connected set of points $[A]$ containing A and B such that no connected proper subset of $[A]$ contains A and B."

I shall show that the word "bounded" in this definition is superfluous.†

Lennes proves the simpler properties of formal order on an arc without any use of the assumption that it is bounded. He also proves (§§ 4, 8) that "if A_0 is any point of an arc AB, and t_1 any triangle containing A_0 as an interior point, then (in case $A_0 \neq A$) there is a point A_1 on the arc AA_0 and (in case $A_0 \neq B$) a similar point B_1 on the arc BA_0 such that every point of the arc A_1B_1 lies within t_1."

The following theorem also follows readily without use of the assumption that an arc is bounded:

If a point A_0 of an arc AB is a limit point of a set of points $[S]$ of the arc AB, and C is A or (if $A_0 \neq A$) any point of the

* Loc. cit., p. 308.

† Since I wrote this paper it has been pointed out to me by Professor R. L. Moore that a modification of the argument used in the proof of Theorem 49 on p. 159 of his paper "On the foundations of plane analysis situs," *Transactions Amer. Math. Society*, vol. 17 (1916), pp. 131–164, would accomplish the same result.

subarc AA_0 of AB, and D is B or (if $A_0 \neq B$) any point of the subarc A_0B of AB, then the subarc CD of AB contains at least one point of $[S]$.

This theorem is tacitly assumed by Lennes in his proof of Theorem 7 (§ 4).

All the above mentioned theorems thus hold if the term simple continuous arc is defined without the use of the word "bounded." Using this definition of an arc, I now prove the

THEOREM. *A simple continuous arc is bounded.*

Proof. Suppose AB is an arc which is not bounded. Let $[S_1]$ consist of A and all points S_1 of AB such that the arc AS_1 is bounded. Let $[S_2]$ consist of all other points of AB. By hypothesis both $[S_1]$ and $[S_2]$ exist. No point of S_2 is between A and a point of $[S_1]$. Since AB is connected, $[S_1]$ contains a limit point P_1 of $[S_2]$ or $[S_2]$ contains a limit point P_2 of $[S_1]$. In the first case any triangle t_1 containing P_1 contains an arc a_1 of AB containing P_1. The arc a_1 contains a point Q_2 of $[S_2]$. The arc AP_1 of AB is contained in a polygon p_1. Therefore the subarc $AQ_2 = AP_1 + P_1Q_2$ lies entirely within a polygon (Lennes, Theorem 15, § 2), and is bounded, contrary to hypothesis. In the second case any triangle t_2 about P_2 contains an arc a_2 containing P_2, a_2 contains a point Q_1 of $[S_1]$, AQ_1 is contained in a polygon, and therefore $AP_2 = AQ_1 + Q_1P_2$ is contained in a polygon and is bounded, contrary to hypothesis. Thus in either case the supposition that AB is not bounded leads to a contradiction.

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THE TRANSFORMATION OF A REGULAR GROUP INTO ITS CONJOINT.

BY DR. J. E. MCATEE,

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1. CONSIDER a regular substitution group G of order g. All the substitutions on the same letters that are commutative with every substitution of G form a group G', of order g, called the conjoint of G. These groups are conjugate.* If G is abelian, $G = G'$. In the contrary case the statement that a

* Finite Groups, Miller, Blichfeldt and Dickson, 1916, p. 35.