pretation. Then follow chapters on the problem of constructing a conic when certain tangents or points are given; the theory of polarity with reference to a conic; the diameters, axes, center and foci; points common to two conics; the usual higher plane curves; projectivity for forms of two dimensions and projective geometry on a conic.

The book is concluded by eight chapters on solid geometry, —chiefly analytical.

E. B. COWLEY.

Transcendenz von e und π. Ein Beitrag zur höheren Mathematik vom elementaren Standpunkte aus. Von GERHARD HESSENBERG. Leipzig, B. G. Teubner, 1912. x+106 pp.

IT is well known that in approaching the proofs of the transcendence of e and π, either in the original form of Hermite and Lindemann, or in the simplified presentations of Hilbert, Hurwitz, and Gordan, the beginner experiences great difficulty in grasping the significance of such suddenly introduced artifices as the Hermite integral or the Hilbert polynomial.

After some introductory remarks on the " Deus ex machina " appearance of these artifices, the author presents some general reflections, abounding in pedagogical good sense, on " proofs by successive specialization " and " indirect proofs." His point of departure in presenting the proofs—which are in substance those of Hilbert, Hurwitz, and Gordan—is found in the problem of approximating the exponential function by means of the n first partial sums of its power series, each of these sums being weighted in the sense of the method of least squares. In the reviewer's opinion, this mode of presentation should prove natural and plausible to the beginner. All auxiliary propositions (on the rational and exponential functions and on algebraic numbers) are clearly and fully set forth in such a manner as not to obscure with their details the main line of thought in the transcendence proofs, the number-theoretic and analytic features of which are kept well apart.

The preceding developments lead very naturally up to a proof of Lindemann's general theorem on the non-vanishing of a linear aggregate of exponentials with unequal algebraic numbers as exponents and non-vanishing algebraic numbers as coefficients.

This little book is written in a vigorous and pleasing style, as remote from academic dryness as possible without sacrifice of rigor, and its clever handling of a pedagogically difficult subject should recommend it to teachers of mathematics.

T. H. GRONWALL.

Vorlesungen über Variationsrechnung. Von OSKAR BOLZA. Leipzig, B. G. Teubner, 1909. ix+795+10 pp.

THIS book is a revised and considerably enlarged German edition of the same author's "Lectures on the Calculus of Variations," Chicago, University Press, 1904.*

In Chapter I, entitled "The first variation in the simplest class of problems," the author, after some introductory remarks on the scope of the calculus of variations, starts by explaining his system of notations, which is exceedingly precise and consistent, although it would seem to the reviewer that a somewhat less elaborate system would have made the book easier to read without any sacrifice of rigor. The classical results in the theory of the first variation of the integral $\int_{x_1}^{x_2} f(x, y, y')dx$ with fixed and variable end points are set forth, including Euler's differential equation and Du Bois-Reymond's lemma. The proof for the latter given on page 28 is due to Hilbert; it would perhaps have been more appropriate to give the proof of Zermelo (*Mathematische Annalen*, volume 58 (1904), page 558), which is unsurpassed in simplicity, brevity, and elegance.

Chapter II, "The second variation in the simplest class of problems," contains the Legendre and Jacobi criteria; the exposition, excellent already in the English edition, is even better in the present book and stands forth as a model of clearness and precision. Chapter III, "Sufficient conditions in the simplest class of problems," deals with the conditions for a weak minimum, the construction of a field of extremals, Weierstrass's expression for the second variation in terms of the E-function, which is here introduced by means of Hilbert's invariant integral, and various conditions for the existence of a strong minimum.

Chapter IV, "Auxiliary theorems on functions of a real variable," contains various lemmas on implicit functions and existence theorems for differential equations, preparatory to Chapter V, "Weierstrass's theory of the simplest class of problems in parametric representation," which treats anew,

* Reviewed by E. R. Hedrick in BULLETIN, vol. 12 (1906), pp. 80–90.