the following theorem concerning functions defined by sequences of continuous functions. We assume that each of the sequence of functions $\varphi_n(x)$ is continuous in the interval (0, 1), and that $\lim_{n=\infty} \varphi_n(x) = \varphi(x)$ exists. A necessary and sufficient condition for the continuity of $\varphi(x)$ in the interval (0, 1) is that, corresponding to any positive number ϵ and any integer m, the condition $| \varphi(x) - \varphi_n(x) | < \epsilon$ is satisfied for every value of x in (0, 1), where n has one of a finite number of values all greater than m, the value to be given to n depending on the value assigned to x.

CORNELL UNIVERSITY,
March 2, 1912.

ON THE V_3^3 WITH FIVE NODES OF THE SECOND SPECIES IN S_4.

BY DR. S. LEFSCHETZ.

(Read before the American Mathematical Society, April 27, 1912.)

CUBIC varieties in four-space were first investigated by Segre, in two memoirs* which are still classic, and in which he gave a generation of those having more than six nodes, especially the one with ten nodes, while he also considered varieties containing a plane, and gave some of their properties. Castelnuovo† investigated also the V_3^3 with ten nodes, and a good account of the theory of the latter is to be found in Bertini.‡ So far as we know however, varieties having nodes for which the hypercone tangent degenerates into one cut by any V_3^1 in a cone—points which we define as nodes of the second species—have been but little considered. In a previous paper § the writer has given the maximum of these nodes for surfaces, or rather a method for obtaining it. This method admitted of an evident extension to n-space, and in particular gives for V_3^3 in four-space, a maximum of these nodes equal to half the number of absolute invariants of the most

* "Sulle varietà cubiche," *Memorie dell' Academia di Torino*, ser. 2, vol. 39 (1888). "Sulla varietà cubicha con 10 punti doppi," *Atti di Torino*, vol. 22 (1887).

† "Sulle congruenze dell 3° ordine," *Atti dell' Ist. Veneto*, ser 6, vol. 6 (1888).

‡ Geometria proiettiva degli iperspazi, p. 176.

§ "On the existence of loci with given singularities." Read before the Poughkeepsie meeting of the Society, Sept. 12, 1911.

general V_3^3, that is five. We propose, in the present paper, to show briefly the existence of this variety having five such nodes.

The equation of a V_3^3 having 5 nodes, one at each of the vertices of the pentahedron of reference, is

$$\sum_{i,\,k=1}^{i,\,k=5} \frac{a_{ik}}{x_i x_k} = 0 \qquad (a_{ik} = a_{ki}, \quad a_{ii} = 0).$$

The hypercone tangent to the V_3^3 at the point (0, 0, 0, 0, 1) has for its equation

$$\sum_{i,\,k=1}^{i,\,k=4} \frac{a_{ik}}{x_i x_k} = 0,$$

and its section by a V_3^1 will be a cone if we have

$$\begin{vmatrix} 0 & a_{12} & a_{13} & a_{14} \\ a_{12} & 0 & a_{23} & a_{24} \\ a_{13} & a_{23} & 0 & a_{34} \\ a_{14} & a_{24} & a_{34} & 0 \end{vmatrix} = 0.$$

If we apply now the transformation

$$X_1 x_1 = X_2 x_2 = X_3 x_3 = X_4 x_4 = X_5 x_5,$$

our V_3^3 becomes

$$\sum_{i,\,k=1}^{i,\,k=5} a_{ik} X_i X_k = 0,$$

that is, a V_3^2, which will pass through the five vertices of the pentahedron of reference, and will be cut by $X_5 = 0$ in a quadricone on account of the vanishing of the above determinant. It will therefore be tangent to $X_5 = 0$. It follows that to show the existence of a V_3^3 with five nodes of the second species, we need only construct a V_3^2 both inscribed and circumscribed to a pentahedron formed by five V_3^1. For this purpose let us consider an arbitrary V_3^2, and three arbitrary V_3^1 tangent to it, which we will denote by A, B, C. We want to find two more V_3^1, say D and E, such that together with the three given ones they satisfy the required condition with respect to V_3^2. Let L be the right line intersection of A, B, C; P_a, P_b, P_c the planes (BC), (CA), (AB); and C_a, C_b, C_c the conic intersections of these planes with V_3^2. These three conics have in common the points α, β where L cuts V_3^2. With the conditions imposed it is clear that D must go through α (or β) and E through β (or α). . . . Suppose then that D goes through α and E through β. D and E are to be determined by the conditions that they shall be tangent to V_3^2 and such that the plane (DE) shall meet C_a, C_b, C_c respectively in one point. Consider now a fixed point a on C_a, and a point b on C_b. There are two V_3^1 going through a, b, α and tangent to V_3^2, and if c is the point other than α where one of them meets C_c, there are two V_3^1 tangent to V_3^2 and going through a, c, β. If b' is the point where one of them cuts C_b, it is seen at once that (b, b') are in (4, 4) correspondence, and for any of the 8 coincidences it is evident that we have two hyperplanes D, E which together with (A, B, C) form a system of the kind required.

It may be remarked in passing that

$$\prod_{i=1}^{i=4} x_i \sum_{k=1}^{k=4} \frac{a_k}{x_k} + x_5^3 = 0$$

represents a V_3^3 with four nodes of the second species, and is a mere generalization of the cubic surface with three such nodes represented by

$$x_1x_2x_3 + x_4^3 = 0.$$

We reserve for a later occasion the consideration of the special cases that may arise in the construction given above.

LINCOLN, NEB.,
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WHAT IS MATHEMATICS?

Principia Mathematica. By ALFRED NORTH WHITEHEAD and BERTRAND RUSSELL. Vol. I. Cambridge, 1910. Royal 8vo. xvi + 666 pp. $8.00.

THE game of chess has always fascinated mathematicians, and there is reason to suppose that the possession of great powers of playing that game is in many features very much like the possession of great mathematical ability. There are the different pieces to learn, the pawns, the knights, the bishops, the castles, and the queen and king. The board possesses certain possible combinations of squares, as in rows,