THE BOSTON COLLOQUIUM.

Lectures on Mathematics. By E. B. VAN VLECK, H. S. WHITE, F. S. WOODS. New York, The Macmillan Company, 1905. xii + 187 pp.

THIS volume contains the lectures delivered at the Boston Colloquium of the AMERICAN MATHEMATICAL SOCIETY, September 2–5, 1903; it is not only a convenient form in which to preserve these valuable lectures, but is a most welcome addition to the meagre catalogue of English works on the higher mathematics. In the brief space of 166 pages is given a very clear and readable summary of three important fields of recent development. The topics treated have the following titles:—

Linear systems of curves on algebraic surfaces, by Professor White, pages 1–30;

Forms of non-euclidean space, by Professor Woods; pages 31–74;

Selected topics in the theory of divergent series and of continued fractions, by Professor Van Vleck, pages 76–166, followed by a bibliography of memoirs relating to algebraic continued fractions, pages 167–187.

The authors have displayed admirable judgment in selecting from the abundance of material at their disposal those theorems and results most suited to give a distinct and vivid picture of the fields with which it is intended to make the reader acquainted. A judicious and carefully considered reserve is exercised in the presentation of details, some of the theorems being demonstrated in full, while for others only an outline is given sufficient to present a clear idea of the argument and the essential nature of the method, and still other results are merely stated when their character and bearing can be made evident without the details of proof. The value and suggestiveness of the lectures are augmented by the various critical remarks scattered throughout the work, which serve not only to elucidate and emphasize the relationships of the various parts, but also to indicate their deeper and less obvious import. Then, too, the pointing out of unsolved problems and incomplete features, whether of detail or theory, will at least be of service in stimulating the constructive imagination of the reader, even if it does not lead to additional contributions in these particular lines.

The first group of lectures deals with the theory of systems of curves on an algebraic surface, with especial reference to properties that are invariant under birational transformations and the kinds of surfaces that admit given systems. This is followed by an account of Picard's theory of linear exact differentials of the first kind on an algebraic surface, a subject of great interest not only from the geometric point of view but also on account of its connection with the slowly developing theory of functions of two variables.

The second set of lectures begins with a clear and simple account of the Riemann hypotheses relative to a restricted portion of space. By the introduction of additional hypotheses the geometry is extended to an unlimited space. With restriction to space of constant curvature three essentially distinct types of geometry are considered, the spherical and elliptic, the lobachevskian, and the euclidean, — these corresponding to positive, negative, and zero curvature respectively. In the first two cases the non-euclidean spaces investigated by Clifford, Klein, and Killing are considered in detail.

The last group of lectures occupies considerably the greater half of the book and thus is afforded a correspondingly larger scope for breadth and variety of treatment, which has been utilized by the author in a most thorough and scholarly manner. The subject, too, is of peculiar interest, since until recent years the divergent series has been looked upon by mathematicians as a monstrosity to be carefully excluded from the well-regulated society of exact science. As, however, the solutions of many differential equations present themselves in the form of divergent series, this subject has during the past twenty years made good its claims to serious recognition, and, thanks to the labors of Poincaré, Stieltjes, Borel, and others, a substantial foundation has been laid for a theory of divergent series. To determine the conditions under which a divergent series may be manipulated as the analytic representative of an unknown function, to develop the properties of the function, and to formulate methods for deriving a function uniquely from the series constitute the chief aim of the theory and form the subject matter of the lectures. The greater share of discussion is given to the last named object. The attention is here directed towards convergent series of rational functions, and the valuable instrumentality of algebraic continued fractions leading to a somewhat full and independent discussion of the theory of such fractions.

A few typographical errors have been observed, none of which would be confusing to the reader. The formulas in x, pages 33–34, should, of course, be expressed in terms of z. The word *sixth*, page 29, line 20, is apparently incorrect, as Humbert states explicitly (*Liouville*, 1893, page 436) that the minimum degree of hyperelliptic surfaces is not yet determined, but he believes it to be *eight*, and considers a number of cases of surfaces of that degree (pages 436–449).

The book is unfortunately printed on very thick paper; while not bulky, it could have been made into a more tasty and compact volume of less than half the thickness.

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CORRECTION.

The following correction should be made in the paper by Mr. Lennes in the October BULLETIN: Page 14, lines 14–16, *for* where M is the difference $\cdots$ of $f(x)$ on ab *read* where M is twice the least upper bound of the absolute value of $f(x)$ on ab.

NOTES.

BY arrangement with The Macmillan Company a limited number of copies of the Chicago Congress Mathematical Papers are offered to members of the AMERICAN MATHEMATICAL SOCIETY at $1.50 per volume, one half the former price. Orders should be sent direct to the Macmillan Company or to the Secretary of the Society, no commission being allowed to agents. Details concerning the book will be found in the advertising pages of the BULLETIN.

THE seventy-sixth annual meeting of the British association for the advancement of science was held at York, England, August 3 to 8. Professor R. LANKESTER was president of the association, and Dr. E. H. GRIFFITHS president of section A,