boloid. Now the associate surfaces of a sphere are the minimum surfaces,* and the associate surfaces of the equilateral hyperbolic paraboloid are the surfaces of translation whose generating lines are plane and lie in perpendicular planes.† Hence we may state the

THEOREM: *The only non-developable surfaces of translation which may be deformed so that their generating lines remain generating lines throughout the deformation are the minimum surfaces and those surfaces of translation whose two systems of generating lines are plane and lie in perpendicular planes.*

PURDUE UNIVERSITY,
October, 1904.

THE INTERNATIONAL CONGRESS OF MATHEMATICIANS AT HEIDELBERG.

THE International Congress of 1904 was held at Heidelberg, August 8 to 13, under the efficient management of the Deutsche Mathematiker-Vereinigung. The attendance, 336, showed continued advance in comparison with the preceding congresses of Zürich and Paris, the proceedings and papers were of much interest and in some cases of notable importance, and the local arrangements, both for meetings and for hospitable welcome, were generous and admirable. One could have wished only for an increase of the day's normal twenty-four hours, or a distribution of sectional meetings which would render it less difficult to attend more than one section. The programme for the week was in its main outlines as follows:

Monday, August 8.— Informal gathering at the City Hall, with an address of welcome by Professor Moritz Cantor.

Tuesday, August 9.—General session. Addresses of welcome by the son of the Grand Duke of Baden and other official representatives. Address in commemoration of Jacobi by Professor Königsberger. Organization of the six sections. Banquet in the City Hall.

Wednesday, August 10.— Meetings of the sections. Reception at the grand ducal palace at Schwetzingen.

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* See Eisenhart, BULLETIN, March, 1902, p. 243.

† Bianchi-Lukat, p. 338.

Thursday, August 11. — General session. Presentation of the History of the Deutsche Mathematiker-Vereinigung by Professor Gutzmer. Presentation of Volume I of the Encyclopedia by Professor Klein. Addresses by Professors Greenhill and Painlevé. Opening of the exhibitions of models, apparatus, and mathematical literature of the last decade. Neckar excursion.

Friday, August 12. — Meetings of the sections. Demonstrations of models, etc. Reunion and "Kommers" in the castle restaurant.

Saturday, August 13.— Final general session. Resolutions. Addresses by Professors Segre and Wirtinger.

In his preliminary address of Monday evening Professor Cantor, of Heidelberg, as "Deutscher in Deutschland, Badenser in Baden, Heidelberger in Heidelberg," extended to the gathering from many lands a graceful welcome. Zürich, the home of the Bernoullis, of Euler, of Steiner, had offered the first international congress the charms of its renowned lake and of a noble city. The second had met in an exposition year at Paris. With the attractions of these cities Heidelberg might not compete. But a congress consists not merely of sights and beauties of nature, but rather of two quite definite things — sessions and non-sessions, the latter lasting much longer than the former. That these non-sessions might result in the greatest general satisfaction was the most earnest wish of Heidelberg and of himself.

On Tuesday morning Professor Heinrich Weber, of Strassburg, delivered his opening address as president of the congress. He called to mind with brief words of characterization some of the eminent mathematicians who have passed away since the first congress — Weierstrass, Hermite, Sylvester, Salmon, Lie, Brioschi, Cremona, Christoffel, Fuchs — and referred to certain general lines of present mathematical progress.

The son and heir of the Grand Duke of Baden was received with enthusiasm and made an excellent address as honorary president. Official addresses of welcome were also given on behalf of the ministry, the universities of Baden, and the city of Heidelberg.

The commemoration of the hundredth anniversary of the birth of Jacobi in this general session was marked by the delivery of an extended biographical address and the presentation of a stately biography,* both by Professor Leo Königsberger. The presence of the daughters of Jacobi lent an additional element of personal interest.

The six sections organized on Tuesday afternoon, with brief addresses by the respective "Einführende," and the general introductory events culminated in the evening with the grand banquet in the city hall, attended by some 600 members and guests. The speakers—alternating with the courses—included, besides the two presidents of the congress, Professor Klein, of Göttingen, who expressed the thanks of the Deutsche Mathematiker-Vereinigung to the authorities who had coöperated with it; Professor Krazer, of Karlsruhe, in honor of the foreign guests and Professor Geiser, of Zürich, in response; Professor Noether, of Erlangen, in honor of Heidelberg, and Professor Heffter, of Bonn, in honor of the ladies.

The visit to the grand ducal palace at Schwetzingen was the social event of Wednesday and indeed of the entire week. The visitors were met at the station by assembled fire companies, veterans, musical societies and school children, and had the distinction — unusual for mathematicians — of marching to the castle gates between these ranks of honor. The grand duke himself was in the distant mountains, but his son and heir represented him as host not less admirably than on preceding days as honorary president of the congress. Strolls in the pleasant old park and a collation in the orangery brought this Nicht-Sitzung to a successful end.

Thursday morning the congress held its second general session. Professor Gutzmer, of Jena, secretary of the Deutsche Mathematiker-Vereinigung, presented to the congress a copy of his admirable new history of the Vereinigung. He referred particularly in his brief presentation address to the questions which can only be properly settled by coöperation of mathematicians, for example, questions of mathematical education, of the applications of mathematics, of the preparation of reports, encyclopedias, etc. To these questions the Vereini-

* By arrangement with the publishing house of Teubner, copies of this biography were supplied to members of the congress at the nominal price of five marks. The addresses of Professors Königsberger, Greenhill, Painlevé, and Wirtinger, as well as Professor Gutzmer's History, were distributed gratis, a great convenience to those who did not readily understand the spoken language. Here should be mentioned also the very useful Tageblatt, which published the daily programme, lists of those in attendance and their local addresses, excursions, notices, etc.

gung had devoted particular attention and it was therefore fitting to review its activity and progress in Heidelberg, where fifteen years earlier (September, 1889) Georg Cantor gave a new impulse toward the foundation of a society of mathematicians. The history is dedicated to the participants of the Heidelberg congress, each of whom received a copy.

In the absence of Professor von Dyck, chairman of the academic commission, Professor Klein, of Göttingen, presented to the congress the now completed Volume I of the Encyclopedia of the Mathematical Sciences. The final installment of the volume contains a general introductory report by the chairman, a special preface by the editor, Professor Meyer, and, among other matters, a detailed author and subject index. Thanks were extended to all whose generous coöperation has rendered the success of the work possible; to Professor Meyer, who gave the first impulse to the plan and who now has the satisfaction of seeing its partial accomplishment; finally to the publishing house which has placed its great resources without limitation at the service of the undertaking. Volumes II–VI are progressing at such a rate that their completion can be foreseen in a reasonable period; but at the same time the scope of the work becomes ever wider. Professor Molk would present the first part of a French edition prepared by the coöperation of the most eminent French authors. This may be regarded as a second revised edition of the original, and it may be hoped that later a new German edition will follow which shall be still more complete.

Professor Molk, of Nancy, followed in explanation of the French edition. It is not a mere translation, although the general character of the German edition is preserved. French mathematicians and engineers have edited the articles of the German edition with due reference to French usage, additions being marked by asterisks. In each case there has been exchange of views between the German and the French writer — offering thus the first example of collaboration between numerous mathematicians belonging to different countries and speaking different languages. The first part, now presented to the international congress, contains:

1. An article on the foundations of arithmetic, based on the German of H. Schubert, by J. Tannery and J. Molk.

2. An article on combinatory analysis and on determinants, based on the German of E. Netto, by H. Vogt.

3. The first part of an article on irrational numbers, based on the German of A. Pringsheim, by J. Molk.

The French edition is published by Teubner and by Gauthier-Villars.

Professor Painlevé of Paris, delivered an address on "The modern problem of the integration of differential equations."

He referred to the origin of the calculus in connection with the scientific study of natural phenomena, and to the direction of the first applications of this study, differentiation seeking always to analyze the phenomenon, integration always to reconstitute it. The successors of Newton and Leibniz, limiting themselves to the simplest facts in each class, achieved brilliant results. Every advance in analysis had its immediate echo in physics, and conversely. It was the most glorious and fruitful epoch in the history of mathematics, which seemed to be in truth the key of the universe.

The following period — of Cauchy and others — was occupied with efforts to enlarge the bounds of integrability and appears at first view barren in comparison. The apparent deflection of mathematics from the real to the imaginary seems arbitrary and unfruitful, but only to a superficial critic, as is illustrated by the significance of the theory of functions of the complex variable for the Taylor expansion of a function of a real variable.

Complex variables once introduced, the integration of a differential equation could be attempted in two different ways: (1) one could seek to reduce it to simple equations (quadratures, linear equations, etc.), that is to aim — in accordance with the traditions of the older analysis — at the formal integration of the equation; (2) failing a reduction of this sort, one could seek to study the general integral by approximations, determining its properties, singularities, etc., in the entire complex plane.

In the end the integration of a differential equation will consist in two successive operations: (1) reduction of the equation to irreducible equations; (2) direct study of the integrals of these irreducible equations in the entire complex plane.

In order to define reducibility, let us consider an algebraic differential equation, for example,

$$\frac{dy}{dx} = f(x, y). \tag{1}$$

The general integral $y(x)$ of this equation depends on an arbitrary constant u; in other words, the equation defines a function y of two variables x and u, or rather an infinite number of such functions (which may be derived from any one by replacing u by an arbitrary function of u). It is admissible also to consider u as a function of x and y; this function satisfies the equation

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} f = 0, \tag{2}$$

which is equivalent to the preceding equation. The general integral of equation (1) will be called reducible if one can adjoin to equation (2) other (algebraic) equations in x, y, u, $\partial u/\partial x$, $\partial u/\partial y$, $\partial^2 u/\partial x^2$, $\cdots$, which are consistent with (2) without being derived from it. The definition may be extended to an equation — or a differential system — of any order.

On this definition can be based a proposition which dominates the whole theory of reducibility, namely, in the case of the first order:

When a differential equation is reducible, among the reduced systems there is always one, of lowest differential order, which has the following properties: (1) It is automorphic, that is, its general solution $u(x, y)$ is deduced from any particular solution $u_1(x, y)$ by the transformations $u = \psi(u_1)$ of a group; (2) all the other reduced systems are derived from this by replacing u by a function $U(u)$ which satisfies an algebraic differential equation (in u and U) chosen arbitrarily.

This theorem, which may be extended immediately to a differential equation of any order, shows the fundamental reason for the great and almost exclusive share which the theory of groups has in the reduction of differential equations.

The simple enumeration of continuous groups for one variable leads to the theorem:

When an equation of the first order is reducible, four cases only can present themselves: (1) The equation is integrable algebraically; (2) it admits an algebraic integrating factor; (3) it admits an integrating factor whose logarithm has its two first derivatives algebraic; (4) a first integral $u(x, y)$ is given by a differential system of which the general solution is of the form $u = (au_1 + b)/(cu_1 + d)$ where a, b, c and d are arbitrary constants, a system leading back, as is well known, to an equation of Riccati.

In general an equation known to be irreducible must be considered as integrated if one can by any process of indefinite approximation — series, continued fractions, etc. — represent the general integral in its entire domain (real and complex) with an error as small as one pleases and of known limit, such representation exhibiting the fundamental properties of the integral, accounting for initial conditions, etc.

It is natural to seek in the first place the differential equations whose general integrals can be obtained by equating uniform functions to zero, and especially the differential equations which possess uniform first integrals. It must not be overlooked that the search for such first integrals is a problem of extreme difficulty. But in a particular (yet somewhat extended) case the problem can now be attacked with success, viz., when the general integral is itself a uniform function in its entire domain of existence. The problem thus particularized is a direct generalization of the discoveries of Abel and Jacobi. If we limit ourselves to the first order, the problem leads only to known results. But it is quite otherwise with equations of higher order. These may define new transcendentals not expressible by any combinations of the classical transcendentals.

Again by generalizing the last problem a little, we may study differential equations of which the integral is a function with n branches or has fixed critical points. But whatever progress may be achieved in these difficult problems, and in other vaster problems of the same nature, it is evident on the one hand that we shall never exhaust all the perfectly integrable equations, and on the other hand that an equation taken at random will not come under these special types. The interest of these remarkable classes of equations, which is evident from a purely mathematical point of view, might then appear smaller from the point of view of the applications, if we did not know what benefit general methods always derive from precise and difficult problems, exhaustively studied.

When a differential equation which occurs in any problem admits neither formal nor analytic integration, the sole resource is to attack it directly by the aid of processes of approximation, guiding one's self as much as possible by the real problem represented. It should not be forgotten that the methods of approximation applicable today to any differential equation, if still theoretically imperfect, prove in many cases amply efficacious. Thus the quantitative problem of n bodies is not yet rigorously solved, but the ephemerides of our solar system are nevertheless calculated for three centuries with surprising precision.

Formal integration, analytic integration as perfect as possible in the complex field, approximate integration in the real domain, these are the three directions of development thus far. At the center of all these stands the theory of functions in a directing and commanding position. It need not be inferred that this dominance will always belong to it. It may one day be judged as is now, for example, the arithmetical work of Gauss, one of the most harmonious and best constructed parts of the mathematical edifice, but a monument of the past. We shall then, perhaps, possess methods more profound and powerful, which will permit us to attack differential equations boldly, concerning ourselves only with the real problem which they represent. But these fruitful and living methods will have sprung from the theory of functions.

Professor Greenhill, of Woolwich, next made an address on "The mathematical theory of the top considered historically."

Beginning with a citation of Klein-Sommerfeld: Theorie des Kreisels, he traced the subject backward through the work of Routh, Hess, Lottner, Jacobi, Poisson, Lagrange, to Euler and Segner, of Halle, taking especially the latter's Specimen theoriæ turbinum (1755) as the effective starting point. Segner was inspired originally by Serson's artificial horizon, a spinning top to be employed in navigation.

An apparatus was exhibited to show gyroscopic motion by a bicycle wheel suspended from a beam by a brass rod and a bicycle hub. Attention was called to the simplicity of the apparatus, which can easily be duplicated. The wheel is spun by a stick and then so deflected as to have a gyroscopic motion, undulating, cusped, or looped, as may be desired.

The fundamental equations of motion were derived by simple geometric consideration of a diagram, on which the conditions of steady motion and slight nutation were also interpreted.

Jacobi's theorem that top motion is compounded of two associated Poinsot movements was analyzed, and the Poinsot rolling cone, as well as the Kirchhoff kinetic analogue and Darboux's deformable hyperboloid, were shown by simple models easily constructed.

A challenge was thrown down to any writers on the theta function who might be present, questioning its dynamical utility, and suggesting as an alternative the function which is the quotient of two theta functions having a constant phase difference in the argument.

The fine large hall of the university museum used for the opening general session on Tuesday was afterwards devoted to an extensive exhibition of mathematical literature, apparatus, and models of the last decade. This exhibition was formally opened with addresses on Thursday afternoon, and thereafter proved a constant center of active interest.

In opening the exhibition of mathematical literature, Professor Gutzmer referred to the fact that this exhibit is the first of its kind. It is confined to scientific mathematical literature, but even here completeness is not attainable. After presenting statistics of the literary production of the world, both in general and in mathematics, Professor Gutzmer recommended restriction of the scope of future exhibitions of literature, for example to a complete exhibit of the mathematical literature of the country in which the congress is meeting. He referred also to the value for the higher mathematical instruction of a special exhibit of English and American text-books.

Besides periodicals the exhibit included about 900 publications, Teubner contributing about 250, Gauthier-Villars 175. The number in English was small.

The collection of models numbered about 300, including not only the usual styles in plaster, paper, thread and metal, but also integrators, harmonic analyzers, calculating machines, instruments for drawing curves and solving equations, tops, gyroscopes, kinematical models, optical instruments, etc. There were about 25 exhibitors, but the most important collections were shown by Professor Wiener of Darmstadt, Schilling of Halle, the technical schools in Darmstadt and Karlsruhe, Coradi of Zürich, the University of Göttingen, Zeiss of Jena, and Chateau Frères of Paris.

Leibniz's calculating machine was exhibited and its manner of operation explained by Professor Runge of Hannover. It was pointed out that, while the machine probably never worked with entire accuracy, this was due to faulty mechanical construction alone, the principles underlying its construction being the same as in some successful modern machines.

The Wiener exhibit included many models of space curves and developable surfaces, movable wire models of surfaces of the second order, thread models of intersecting surfaces, etc. In the great variety of Schilling's models the kinematical were notable. Coradi presented beautifully constructed instruments, including an apparatus for drawing parabolas, a harmonic analyzer and an integrator. The Göttingen exhibit included interesting tops.

Zeiss showed a great variety of optical apparatus and in particular a new form of projecting lantern, in which, if the object placed in the apparatus is transparent or translucent, projection is effected by transmitted light in the ordinary manner, while if opaque objects are to be projected, their images are thrown on the screen by reflected light.

Other objects of interest were Christian Wiener's origina model of a surface of the third order with twenty-seven straight lines, Brückner's large collection of polyhedrons having congruent faces and congruent or symmetric angles, and Finsterwalder's movable models of minimum surfaces. Several lectures were held in connection with the exhibit.

Thursday evening was devoted to a delightful Neckarfahrt. After an invasion of a Gartenwirthschaft in numbers for whom the landlord unfortunately could not duplicate the miracle of the loaves and fishes, decorated stone-boats were taken in the evening for the romantic descent of the Neckar. Below the arches of the Carlsbrücke, each pier poured a fountain of fire into the river, while high above the town the castle burst into dazzling light with red fires, burning steadily for some fifteen minutes. In the midst of a beautiful display of fireworks from a boat on the river, the pythagorean diagram stood out brilliantly against the sky, an appropriate symbol of the nature of the meeting. The whole effect was finely spectacular.

After the section meetings of Friday there was again a social evening, and again an illumination of the castle, viewed this time from the fine Scheffel terrace. The large open hall of the castle restaurant was crowded for the Kommers which fittingly ended the general festivities of the week. Professor Schubert, of Hamburg, with a gallant and dashing young corps officer on each side, presided with the skill of experience and the dignified enthusiasm of renewed youth. Oratory and singing vied in their contrast with the more serious pursuits of the week.

At the final general session on Saturday the following resolution was adopted, as presented by P. Tannery, A. von Braunmühl, E. Lampe, G. Loria, M. Simon, D. E. Smith, P. Stäckel, E. Wölffing :

"Whereas, the history of mathematics forms to-day a subject of indisputable importance, its value as well from the purely mathematical standpoint as from the pedagogical becomes continually more conspicuous, and it is therefore indispensable to assign it a fitting place in public education,

"Resolved, that the Third International Congress of Mathematicians at Heidelberg (August, 1904), referring to the recommendations of Section 5 of the International Congress for Comparative Historical Investigation (Paris, July, 1900) and of Section 8 of the International Congress of Historians (Rome, 1903), adopts as its own the following recommendation, international in character, as expressed by the congress at Rome :

"(1) That the history of the exact sciences should be taught by the universities, with provision for lectures in the four divisions, mathematics and astronomy, physics and chemistry, natural sciences, medicine.

"(2) That the elements of the history of the sciences should be included in the secondary school program of the several branches of science."

The following resolution was presented by F. Morley and A. Vassilief:

"The Third International Congress of Mathematicians, considering that the publication of the complete works of Euler has great scientific importance, supports the recommendation made to the Carnegie Institution by the mathematical committee of which Professor Moore was chairman, and expresses the hope of its early realization. Considering, on the other hand, that the success of this understanding requires the coöperation of scholars from all lands, whose assemblage for the elaboration of the plan and the discussion of other related questions will be possible during the next congress,

"Resolved, that the Third Congress requests the committee of organization of the next congress to present to it a report on the status of the question as well as on the steps which the congress could take to contribute to the success of this important scientific undertaking."

The congress thereupon expressed its recognition of the great importance of the projected publication, observed that steps in the same direction had already been taken by the academies of St. Petersburg and Berlin, and expressed the hope that a report of progress could be presented at the next congress.

The following resolutions, presented respectively by Sections 5 and 6, were adopted :

"Resolved, that a closer union of the historians of the mathematical sciences should be brought about. While such an association, like its objects, should be international, the coöperation of similar and related national societies, journals, museums, etc., should be sought. It is requested that this resolution be placed on the programme of the next congress.

"Resolved, that the Congress greets with warmest sympathy the efforts of mathematicians to secure everywhere the facilities now indispensable for the prosecution of mathematical studies — sufficiently numerous professorial chairs, adequate libraries, drawing-rooms, work-rooms, collections of models, stereopticon apparatus, etc. The Congress expresses the urgent wish that the state and other controlling authorities afford the necessary support for this purpose."

In choosing the place of the next congress the eloquent invitation presented by Professor Volterra on behalf of the Italians for a meeting at Rome in the spring of 1908 was accepted with enthusiasm. A preliminary invitation for 1912 was extended on behalf of the English mathematicians.

Professor Segre, of Turin, delivered an address on "The geometry of to-day and its relations to analysis."

The relations of geometry to analysis depend principally on the fact that in many respects the objects with which the two sciences are occupied — even if under different names — are the same, at least in an abstract sense. The difference between geometry and analysis consists in the problems which they present and still more in the methods by which the problems are treated.

One of the dominant characteristics of the geometry of to-day is the great generality and abstractness which exists in its concepts. This characteristic appears in the higher theory, as also in recent researches on the foundations of geometry. The evolution of this science has relegated to a secondary rank the space intuition which at first constituted one essential element. Reason has become instead the sole essential instrument. Hence it comes that modern geometry can be as rigorous as analysis, and in fact aspires to be so. Examples of this occur in the determination of the multiplicity of the solutions of geometric problems, and in other questions of enumerative geometry. The latter must resort to algebra to establish its principles, and in return can give algebra many results. The algebraic researches of Kronecker, Hilbert, etc., have great importance for the geometry of algebraic manifolds.

The abstractness already mentioned as characterizing the entities or objects studied in modern geometry extends to the group of transformations which appear as fundamental in such study. Today the dominant geometric tendency, inherited from Riemann, concerns itself with properties invariant under birational transformation. To its triumph much has been contributed by the celebrated memoir of Brill and Noether, as also by the Italian school of geometers. Thus have originated various researches on the algebraic curves, and particularly the more recent ones of Castelnuovo and Enriques on the algebraic surfaces. For birational correspondence and its groups important results have also been reached.

In the direction of increasing generality there may be considered, in place of the algebraic manifold, a more general one to which Segre has devoted a series of papers (about 1890) under the name hyperalgebraic. These arise by assuming algebraic relations, not between the complex coördinates of the geometric elements, but between the real components of those coördinates. For example the so-called Hermite forms, with conjugate complex variables, represent some of the geometric entities considered by Segre, and are studied very easily by these geometric means.

Within the hyperalgebraic class are included as particular cases those composed of the real elements of a given algebraic class. The study of the questions of reality or form is not sufficiently cultivated today; nevertheless important works exist, among which those of Hilbert, Klein, Juel and others may be cited. With this are connected the analysis situs and the researches on the form of the integral curves of differential equations.

There have been considered recently in geometry new species of complex points — the bicomplex, etc., and there have also been employed with advantage new species of complex numbers, for example the numbers $a + b\epsilon$ where $\epsilon^2 = 0$, as used by Study in his Geometrie der Dynamen. On the other hand, geometric researches have also been made restricting the field of points for example to the points with integral coördinates (Minkowski, Geometrie der Zahlen, etc.), or to those with rational coördinates (Poincaré, *Journal de Mathématiques*, 1901). Thus in this, as in so many other examples, are revealed the bonds uniting geometry more and more closely with analysis — bonds which can give to the one science as to the other immense benefits.

Professor Wirtinger, of Vienna, presented an address on "Riemann's lectures on the hypergeometric series and their significance."

The speaker gave a review of the sources, the plan and the ideas of Riemann's lectures of 1859 on the hypergeometric series. He showed how these ideas came to fruition in subsequent mathematical literature. He recalled first those problems already enunciated by Riemann but today still unsolved. In this connection he formulated other problems leading to new classes of functions.

He dealt in particular with the application of the methods of the principle of Dirichlet and the equation of Fredholm to Riemann's problem in the theory of differential equations, and with the analogy of the relations between the solutions of a non-homogeneous linear differential equation to the solutions of the homogeneous equation on the one hand and to the elliptic integrals and their periods on the other. The classical figure of p parallelograms with $(2p-2)$ branch points he wished treated from points of view similar to those employed for the figure of the single parallelogram and its reduced form. To previous generalizations of the hypergeometric series he added a new one to which he was led through a new and simple representation of the hypergeometric function by means of the elliptic modular functions. The new transcendental functions are definite integrals on the elliptic base extended between $n$ths of periods, and for example for $n = 5$ the group of the icosahedron substitutions has the same significance for them, as the well-known group of the substitutions x, $1/x$, $1-x$, etc., for the ordinary hypergeometric function.

The speaker extended his method also to general algebraic bases and pointed out the relation of his functions to the moduli of the theta functions.

He concluded with the general observation that it is exactly the working out of what seem at first only interesting particular cases that enables us to make real progress in the general theory. In spite of the deductive form which we give our results, we can as little dispense with the inductive method in mathematics as in any other branch of human learning.

The week ended with further attractive excursions for those whose departure was not too urgent.

The writer acknowledges with cordial appreciation the courteous response of participants in the Congress to his request for abstracts of their papers. By the kind coöperation of Dr. E. B. Wilson, of Yale University, the abstracts of the papers read before the several sections will appear later in the BULLETIN.

H. W. TYLER.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY,
November, 1904.

THE HEIDELBERG CONGRESS: SECTIONAL MEETINGS.

The several sections of the Congress at Heidelberg met on Tuesday afternoon, August ninth, to organize and arrange programs. For each section there had been previously appointed two or three presidents, of whom one delivered an address, usually merely a few words of welcome and of pleasurable anticipation of the scientific value of the work of the section. In the cases of von Brill and Klein, however, the address widened out into as truly a scientific communication as any presented at the subsequent meetings. Instead of expecting the presidents to take the chair at all the meetings of their sections, the excellent custom of selecting honorary chairmen from among those present was adopted.

The meetings were held Wednesday and Friday mornings. Professor Tyler, representing the AMERICAN MATHEMATICAL SOCIETY, inserted in the *Tageblatt* a statement that the Society would be glad to receive for publication in the BULLETIN abstracts of the various papers, and sent to each speaker whose address could be obtained a circular letter repeating the request. The response was general and cordial, and we have here especially to thank the members of the congress for their