

### THE FEBRUARY MEETING OF THE AMERICAN MATHEMATICAL SOCIETY.

A REGULAR meeting of the AMERICAN MATHEMATICAL SOCIETY was held in New York City on Saturday, February 24, 1900, extending, as usual, through a morning and an afternoon session. The first part of the afternoon was devoted to a joint meeting with the American Physical Society at which Professor J. K. Rees presented the paper noted in the list below. The total attendance during the day exceeded fifty persons, and included the following thirty-five members of the Society:

Dr. E. M. Blake, Professor Maxime Bôcher, Professor E. W. Brown, Professor F. N. Cole, Professor E. S. Crawley, Dr. W. S. Dennett, Professor A. M. Ely, Professor T. S. Fiske, Mr. A. S. Gale, Dr. G. B. Germann, Mr. H. E. Hawkes, Dr. G. W. Hill, Dr. J. I. Hutchinson, Professor Harold Jacoby, Mr. S. A. Joffe, Mr. C. J. Keyser, Dr. G. H. Ling, Dr. Emory McClintock, Dr. Alexander Macfarlane, Dr. James Maclay, Dr. Emilie N. Martin, Dr. G. A. Miller, Professor E. H. Moore, Professor F. Morley, Professor James Pierpont, Dr. M. B. Porter, Professor M. I. Pupin, Professor J. K. Rees, Mr. C. H. Rockwell, Professor T. J. J. See, Professor E. B. Van Vleck, Professor J. M. Van Vleck, Miss E. C. Williams, Miss R. G. Wood, Professor R. S. Woodward.

The President of the Society, Professor R. S. Woodward, occupied the chair. The Council announced the election of the following persons to membership in the Society: Professor Anne L. Bosworth, Rhode Island College, Kingston, R. I.; Mr. H. L. Coar, University of Illinois, Urbana, Ill.; Dr. F. R. Moulton, University of Chicago, Chicago, Ill.; Mr. F. G. Radelfinger, U. S. Hydrographic Office, Washington, D. C. Two applications for membership were reported.

A revision of the By-Laws, simplifying their arrangement and removing certain obsolete features, was adopted. Notice was also given of a proposed amendment of the Constitution, recommended by the Council and ordered to be submitted to the Society for action at the April meeting. By this amendment it is provided that the Ex-Presidents of the Society shall be life members of the Council, and that the number of members of the Council, other than officers, elected annually shall be increased from three to four. The presidential term of office is also extended to two years.

The following papers were presented at this meeting :

- (1) Dr. ALEXANDER MACFARLANE : "On the nabla of quaternions."
- (2) Dr. M. B. PORTER : "On the enumeration of the roots of the hypergeometric series between zero and one."
- (3) Mr. H. W. KUHN : "List of the imprimitive groups of degree fifteen."
- (4) Dr. G. A. MILLER : "On the groups of isomorphisms."
- (5) Dr. J. I. HUTCHINSON : "The Hessian of the cubic surface, II."
- (6) Professor MAXIME BÔCHER : "Some theorems concerning linear differential equations of the second order."
- (7) Professor J. K. REES : "The variation of latitude at New York, and a determination of the constant of aberration from observations at the latitude observatory of Columbia University."
- (8) Dr. G. W. HILL : "On the extension of Delaunay's method in the lunar theory to the general problem of planetary motion."
- (9) Professor E. B. VAN VLECK : "On linear criteria for determining the circle of convergence of a power series."
- (10) Professor F. MORLEY : "The metrical geometry of the plane  $n$  line."
- (11) Professor L. E. DICKSON : "Two triply infinite systems of non-isomorphic simple groups of equal order."
- (12) Professor L. E. DICKSON : "Isomorphism between certain systems of simple linear groups."
- (13) Dr. L. W. REID : "A table of class numbers for cubic number bodies, with the method of their calculation."

Mr. Kuhn's paper was presented to the Society through Dr. G. A. Miller. Dr. Reid was introduced by Professor H. B. Fine. In the absence of the authors, Mr. Kuhn's paper was read by Dr. Miller, and Professor Dickson's papers were read by title. The papers of Professor Bôcher and Dr. Porter are published in the present number of the BULLETIN; those of Dr. Miller and Dr. Hutchinson, and the second paper of Professor Dickson will appear in later numbers. Abstracts of the remaining papers are given below.

In a paper on "The principles of differentiation in space analysis," read before the Society, January 26, 1895, Dr. Macfarlane pointed out that in space analysis there are two essentially distinct ways of differentiating a product: one result is obtained when the factors are first differentiated

and the product then formed ; a different result is obtained when the product is first formed and then differentiated. The main features of the second kind of differentiation as regards scalars were there developed. In the present paper Dr. Macfarlane applies the second kind of differentiation to the space operator denoted by Nabla,  $\nabla$ . The paper shows how to form perfectly general expressions for  $\nabla$  and for  $\nabla^2$ , of which latter Laplace's operator is a special case.

Mr. Kuhn's paper is in abstract as follows: In constructing the imprimitive groups of any degree, the first problem is to find all the intransitive groups that may be used as heads. The systems of intransitivity of these heads are transformed according to a transitive substitution group whose degree is equal to the number of systems of intransitivity of the given heads. For degree fifteen the number of these systems is either three or five. Hence every imprimitive group of degree fifteen is isomorphic with at least one of the seven transitive groups of degree three or five. Where the imprimitive group contains no substitution besides identity that transforms each one of its systems of imprimitivity into itself, this isomorphism must be simple. There are just two such groups. They are simply isomorphic respectively to the alternating and the symmetric group of degree five.

There are 21 groups that may be used as heads for the groups which contain three systems of imprimitivity ; for those with five systems there are 9 such groups (including identity). The heads having been found, the next problem is to find the remaining substitutions in the possible imprimitive groups. There are 55 distinct imprimitive groups of degree fifteen that contain three systems of imprimitivity, and 56 that contain five such systems. Of the latter, 13 also contain three systems. Hence the total number of imprimitive groups of degree fifteen is 98. Of this number, 64 are solvable and 34 are unsolvable.

The latitude observatory under Professor Rees's charge was first placed on the present site of Columbia University near the corner of 118th Street and Amsterdam Avenue, New York. Observations were made at this station from April 24, 1893, to December 5, 1895. When the new buildings were begun, the latitude observatory was moved to the corner of 120th Street and Broadway. Here the observations were made from January 9, 1895, to the present time. The two stations were connected by a care-

ful survey which showed the second position to be north of the first 7".337. All observations made at the second point were reduced to the first. The zenith telescope made by Wanschaff of Berlin was employed throughout. Its aperture is 80 millimeters, and its focal length one meter. The observers were Professors Rees and Jacoby and Dr. H. S. Davis. Four groups of stars were used, having mean right ascensions approximately as follows :

Group I, 6 hours ;      Group II, 14 hours ;  
Group III, 18 hours ;      Group IV, 22 hours.

Each group contained seven pairs of stars, and the groups were observed both morning and evening whenever the weather permitted.

SERIES A.

April 24, 1893, to July, 1894 :	818 pairs by Rees,
	302 " " Jacoby,
	654 " " Davis.

SERIES B.

July, 1894, to January, 1896 :	771 " " Rees,
	310 " " Davis.

SERIES C.

January, 1896, to January, 1898 :	1065 " " Rees,
	774 " " Davis.

SERIES D.

January, 1898, to December, 1899 :	951 " " Rees,
	873 " " Davis.

Total pairs : 6,518

This table shows the individual observers to have measured : Rees, 3,605 pairs ; Jacoby, 302 pairs ; Davis, 2,611 pairs ; total, 6,518 pairs.

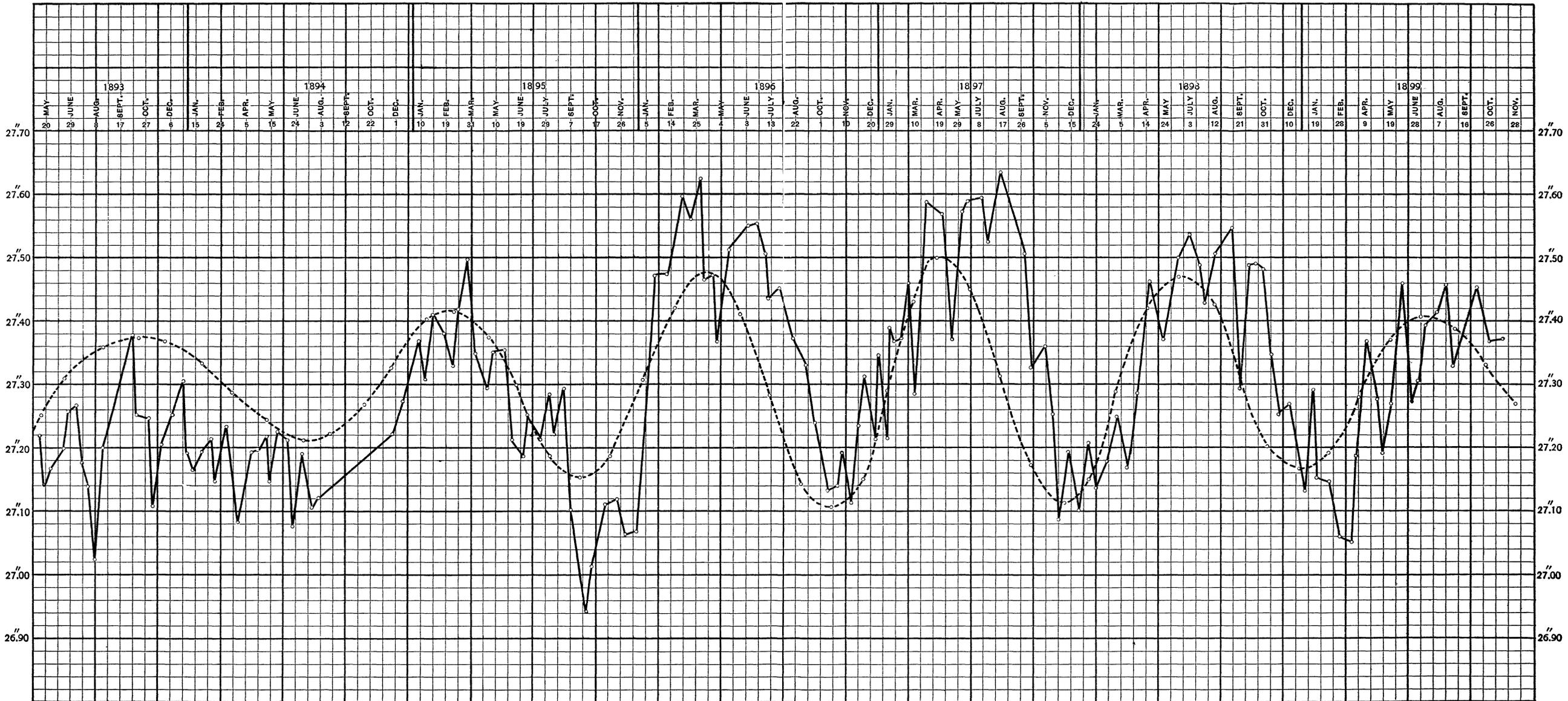
The record shows that observations were taken on 758 nights. The computations give the accompanying table and curve which show the variation of latitude. The curve required by Dr. S. C. Chandler's formula (*Astronomical Journal*, No. 446) is shown in the dotted line. From 1896 the observed epochs of maxima and minima follow the computed in time.

The four series of observations gave the following values of the aberration constant :

Series A.	20"4566	weight 18	Series C.	20"4695	weight 26
" B.	20 4525	" 16	" D.	20 4704	" 27

Taking the probable error of a single latitude observation as 0".16, the constant of aberration is  $20.464 \pm 0".006$ .

COLUMBIA UNIVERSITY OBSERVATORY, NEW YORK CITY. VARIATION OF LATITUDE.



The dotted curve is constructed from Chandler's formula, *Astronomical Journal* No. 446.

## OBSERVATORY OF COLUMBIA UNIVERSITY.

Date.	40° 48'	$\phi - \phi_0$	No. pairs.
1893. May 9	27".222	-.082	77
19	.144	-.160	44
27	.162	-.142	42
June 8	.178	-.126	59
17	.198	-.106	45
28	.259	-.045	32
July 8	.262	-.042	30
19	.177	-.127	38
26	.141	-.163	46
Aug. 6	.029	-.275	46
Oct. 6	.377	+.073	31
15	.244	-.060	43
30	.241	-.063	44
Nov. 9	.111	-.193	46
20	.204	-.100	48
Dec 3	.232	-.072	18
11	.255	-.049	28
23	.302	-.002	43
1894. Jan. 2	.192	-.112	36
11	.168	-.136	65
23	.198	-.106	61
Feb. 2	.214	-.090	56
12	.148	-.156	59
24	.193	-.111	86
March 5	.234	-.070	68
21	.083	-.221	51
Apr. 1	.192	-.112	33
11	.199	-.105	65
23	.218	-.086	83
May 2	.153	-.151	89
12	.225	-.079	62
29	.216	-.088	50
June 8	.079	-.225	68
17	.193	-.111	64
July 1	.114	-.190	34
16	.120	-.184	38
Nov. 15	.226	-.078	39
29	.274	-.030	43
Dec. 18	.367	+.063	25
1895. Jan. 2	.305	+.001	20
15	.412	+.108	53
Feb. 2	.380	+.076	39
14	.332	+.028	40
26	.425	+.121	35
March 10	.500	+.196	18
21	.346	+.042	24
Apr. 8	.298	-.006	30
20	.353	+.049	48
May 7	.354	+.050	44
18	.213	-.091	47
June 4	.184	-.120	28
13	.257	-.047	51
July 5	.217	-.087	45
16	.286	-.018	23
28	.222	-.082	46

Date.	40° 48	$\phi - \phi_0$	No. Pairs.
1895. Aug. 10	27".295	-.009	19
23	.103	-.201	46
Sept. 15	26 .943	-.361	26
Oct. 1	.996	-.308	38
18	27 .111	-.193	55
Nov. 5	.119	-.185	50
18	.063	-.241	36
Dec. 2	.070	-.234	41
1896. Jan. 14	.474	+.170	37
28	.476	+.172	54
Feb. 11	.542	+.238	45
22	.600	+.296	41
March 5	.560	+.256	42
17	.627	+.323	37
28	.464	+.160	21
April 9	.477	+.173	37
19	.367	+.063	33
May 5	.513	+.209	56
18	.532	+.228	27
June 4	.551	+.247	47
20	.554	+.250	30
July 1	.501	+.197	48
12	.437	+.133	31
31	.458	+.154	47
Aug. 19	.379	+.075	57
Sept. 9	.331	+.027	23
26	.240	-.064	27
Oct 13	.135	-.169	35
25	.142	-.162	53
Nov. 10	.193	-.111	59
23	.118	-.186	23
Dec. 5	.232	-.072	37
15	.314	+.010	15
28	.219	-.085	27
1897. Jan. 8	.345	+.041	23
25	.214	-.090	41
Feb. 5	.387	+.083	28
18	.362	+.058	53
28	.378	+.074	55
March 10	.452	+.148	38
23	.283	-.021	10
April 25	.583	+.279	41
May 21	.565	+.261	73
June 1	.376	+.072	16
17	.573	+.269	48
July 2	.595	+.291	44
20	.597	+.293	26
Aug. 4	.522	+.218	70
22	.639	+.335	44
Sept. 30	.503	+.199	97
Oct. 16	.324	+.020	13
Nov. 4	.361	+.057	38
16	.254	-.050	20
26	.089	-.215	37
Dec. 10	.193	-.111	13

Date.	40° 48'	$\phi - \phi_0$	No. Pairs.
1897. Dec. 26	27".100	— .204	22
1898. Jan. 10	.205	— .099	33
26	.139	— .165	43
Feb. 10	.177	— .127	53
March 1	.246	— .058	88
17	.169	— .135	25
April 4	.285	— .019	33
19	.461	+ .157	22
May 12	.365	+ .061	69
26	.429	+ .125	28
June 13	.501	+ .197	64
26	.539	+ .235	59
July 12	.488	+ .184	75
26	.430	+ .126	23
Aug. 12	.506	+ .202	32
Sept. 6	.549	+ .245	47
17	.295	— .009	28
Oct. 2	.488	+ .184	45
15	.490	+ .186	42
Nov. 1	.485	+ .181	59
15	.354	+ .050	40
Dec. 1	.251	— .053	48
14	.264	— .040	28
1899. Jan. 14	.138	— .166	36
23	.291	— .013	44
Feb. 3	.151	— .153	28
23	.146	— .158	35
March 13	.060	— .244	24
26	.055	— .249	16
April 6	.189	— .115	28
26	.366	+ .062	48
May 9	.275	— .029	29
14	.194	— .110	39
29	.267	— .037	61
June 16	.459	+ .155	11
30	.275	— .029	46
July 14	.309	+ .005	48
25	.391	+ .087	21
Aug. 12	.418	+ .114	51
22	.458	+ .154	42
Sept. 4	.322	+ .018	63
30	.397	+ .093	31
Oct. 17	.451	+ .147	35
Nov. 5	.361	+ .057	51
24	.369	+ .065	53
Mean	27".304		

Probable error of one observation of unit weight  $\pm 0''.16$

In Dr. Hill's paper, which will be published in the *Transactions*, Delaunay's method is explained as it would be applied to a planetary system, like the solar system. The attribution of greater generality to the method has the effect of rendering its establishment much easier. The limits of the domain in which the method is applicable are considered and are shown to be quite simple in their character. In the Delaunay transformation the special form of the equation of the conservation of energy constitutes a relation between the principal linear and angular variable. This relation is graphically exhibited by a plane curve. The general properties of this curve are noted. These curves are divisible into three classes. Delaunay's formulas of transformation are established by applying the theorem of Poisson to the deduction of the value of the Jacobian of the old variables with reference to the new. Some simplifications have been found for the expression of the partial derivatives of the general linear variable with reference to those conjugate to the angular variables employed. The subject is illustrated by making an application to the asteroid of the Hecuba type acted on by Jupiter; two cases are separately considered, viz., first, where the motion of the principal angular variable is continuous through the circumference, and second, where it executes a libration.

The criteria given by Professor E. B. Van Vleck for determining the radius of convergence of a power series  $a_0 + a_1x + a_2x^2 + \dots$  have a close connection with one of the two well known criteria which are based upon Cauchy's theorems concerning the convergence of an infinite series. According to one of these two criteria the radius of convergence is the limit of  $1/|a_n^{1/n}|$  or, in case no limit exists, it is the greatest affix of a point of condensation of the sequence  $1/|a_n^{1/n}|$ , ( $n = 1, 2, 3, \dots$ ). The other criterion gives the radius of convergence only when there is a limit for  $|a_n/a_{n-1}|$ . The reciprocal of this ratio is then the desired radius. But, though the former criterion is perfectly general and of the highest value for theoretical purposes, it nevertheless has little or no superiority as a practical test for determining the radius of convergence of a given series. For in most cases in which  $|a_n/a_{n-1}|$  does not approach a limit, the determination of the limit of the sequence  $1/|a_n^{1/n}|$  or of its points of condensation is extremely difficult. The object of Professor Van Vleck's paper is to establish criteria which apply in certain cases where the ratio test fails; namely, in

the cases in which a number of coefficients of the power series are connected together by a linear relation which tends to take a limiting form as  $n$  increases.

The first of four theorems obtained in the paper is as follows: Given a series  $a_0 + a_1x + a_2x^2 + \dots$  in which, after some fixed term,  $p + 1$  consecutive coefficients are connected by a linear relation

$$a_n = (b_1 + \varepsilon_1^{(n)})a_{n-1} + (b_2 + \varepsilon_2^{(n)})a_{n-2} + \dots + (b_p + \varepsilon_p^{(n)})a_{n-p};$$

if, for an arbitrarily assigned positive value  $\varepsilon$ , a positive integer  $m$  can be found such that for  $n \geq m$  every  $|\varepsilon_i^{(n)}|$  will be less than  $\varepsilon$ , the series will converge within a circle whose center is the origin and whose radius is the distance from the origin to the nearest root (or roots) of the polynomial  $1 - b_1x - b_2x^2 - \dots - b_px^p$ . A special case of this theorem is that in which  $p = 1$ . The root of the polynomial  $1/b_1$  is then also the limit of  $a_{n-1}/a_n$ , and we have Cauchy's ratio test for the determination of the radius of convergence.

The existence of a limiting relation between  $p + 1$  coefficients of the series necessitates also the existence of other limiting linear relations containing a greater number of coefficients. Among all these linear relations there is, however, one and only one containing the minimum number of coefficients. If in the above theorem this relation be used, the circle will be, in general, the circle of convergence.

The second theorem of the paper is a restricted extension of the first to the case in which the number of terms in the linear relation increases without limit with increasing  $n$ . In place of the above polynomial an infinite series  $1 - b_1x - b_2x^2 - \dots$  is then to be introduced. The theory can also be extended to cover power series containing two or more variables. This extension is made for the case of two variables in theorems III. and IV., which may possibly be of special interest, since criteria for the convergence of series in two or more variables seem as yet to be very rare.

Professor Van Vleck's paper is to be published in the *Transactions*. The results of the paper will be applied in a subsequent article to the theory of linear differential equations.

Professor Morley's paper, which will be published in the April number of the *Transactions*, first establishes for  $n$  lines of a plane the existence of a circle, previously detected for 4 lines by Steiner and for 5 lines by Kantor; and shows that for all  $n - 1$  lines included in the  $n$  lines, such circles pass

through a point. The method used is a general analytic method of handling  $n$  lines of a plane, by means of certain characteristic constants. The circle and point form the beginning of a series of figures, of which Clifford's circle or point is the end; accordingly Clifford's theorem is proved by the method, and the coordinates of his chain found.

The curves considered are included in  $x = \text{rat. alg. } t$ , where  $t$  is a point on the unit circle; and it proves to be desirable to consider in connection with the curves those obtained by polarizing the expression in  $t$ , somewhat as is done in the theory of osculants. The curves so obtained are examined in the case of a limaçon, which first presents itself; and in the case of an ellipse, where it appears that, given  $2p$  lines of an ellipse, the Clifford circle of any  $2q - 1$  lines and that of the remaining lines are partners in a complex involution  $I_1^2$ .

The fact that the theory of 5 lines takes a peculiarly simple form when the lines touch a hypocycloid of class 3 suggests special examination of hypocycloids. The theory as given by Kantor for 5 lines is thus generalized at once, and further it is indicated how the theorems of the memoir itself, which concern merely the intersections of  $n$  lines, can be replaced by theorems in which the lines are taken 3, 4, or more at a time.

In a résumé of the known finite simple groups published in the BULLETIN, July, 1899, p. 470, Professor Dickson noted that for  $p > 2$  the simple abelian group  $A(2m, p^n)$  had the same order as the simple orthogonal group  $O(2mH, p^n)$  and that the two were isomorphic for  $m = 1$  and 2. [For the case  $m = 1$ ,  $p^n = 3$ , the groups are not simple.] The present note announces\* the truth of the conjecture there made, that the two groups were not isomorphic for  $m > 2$ . Besides these two triply infinite systems, there has been noted† but a single, isolated, case of non-isomorphic simple groups of equal orders, viz, for the order  $\frac{1}{2} 8!$ . By an elaborate investigation, Professor Dickson establishes the following theorems:

The general abelian group on  $2m$  indices in the  $GF[p^n]$ ,  $p > 2$ , contains exactly  $m$  sets of conjugate substitutions of period two, those of the  $r$ th set being conjugate with the product  $T_{1,-1} T_{2,-1} \dots T_{r,-1}$ , where  $T_{i,-1}$  merely changes the signs of  $\xi_i$  and  $\eta_i$ . It contains a single conjugate set of

\* The complete investigation was offered January 25th to the *Quar. Jour. of Math.*

† Cf. BULLETIN, 1899, p. 473. An immediate proof is given by the writer in an article to appear in the *Amer. Jour. of Math.* for July, 1900.

substitutions whose squares give  $T \equiv T_{1,-1} T_{2,-1} \cdots T_{m,-1}$ , the only self-conjugate substitution of the abelian group, the set being represented by  $M_1 M_2 \cdots M_m$ .

From these theorems the following result is derived concerning the quotient group of the Abelian group by its maximal self-conjugate subgroup  $\{1, T\}$ , the case  $m = 1$ ,  $p^n = 3$  being excluded:

According as  $m$  is even or odd, the simple group  $A(2m, p^n)$ ,  $p > 2$ , has exactly  $\frac{1}{2}(m+2)$  or  $\frac{1}{2}(m+1)$  distinct sets of conjugate operators of period two.

We readily see that the simple subgroup  $O(2m+1, p^n)$  of the orthogonal group contains at least  $m$  substitutions of period two which are not conjugate, for example,

$$C_1 C_2, \quad C_1 C_2 C_3 C_4, \quad \dots, \quad C_1 C_2 C_3 C_4 \cdots C_{2m-1} C_{2m},$$

where  $C_i$  denotes the orthogonal substitution altering  $\xi_i$  alone whose sign it changes. It follows that for  $m > 2$ , the orthogonal group contains a greater number of distinct sets of conjugate operators of period two than the simple group  $A(2m, p^n)$  of equal order.

Dr. Reid's table gives the class number  $h$ , the discriminant  $\Delta$ , and a basis for each of 161 cubic number fields, and the factorization of certain rational primes into their prime ideal factors. When  $h = 1$ , the prime number factors of these primes are given. Units are also given for a majority of the fields. The method employed in the calculation of  $h$  is based upon the following theorem of Minkowski's: In every ideal class of an algebraic number field,  $k$ , there exists an ideal, whose norm is

$$< \left( \frac{4}{\pi} \right)^r \frac{|m|}{m^m} \left| \sqrt{\Delta} \right|,$$

where  $m$  is the degree and  $\Delta$  the discriminant of the field, and  $r$  the number of pairs of imaginary fields which are found among the  $m$  conjugate fields  $k, k^{(1)}, \dots, k^{(m-1)}$ . The process consists: 1° in obtaining all ideals satisfying the above condition; 2° in determining the number of ideal classes into which they fall. All prime ideals which satisfy the above condition are obtained by factoring the rational primes

$$< \left( \frac{4}{\pi} \right)^r \frac{|m|}{m^m} \left| \sqrt{\Delta} \right|$$

This is effected in the case  $\frac{d(\theta)}{\Delta} \not\equiv 0 \pmod{p}$  by means of

the theorem : If  $p$  satisfy the condition  $\frac{d(\theta)}{\Delta} \not\equiv 0 \pmod{p}$ , and if  $f(x)$  be resolved into its prime factors with respect to the modulus  $p$  :

$$f(x) \equiv \{P(x)\}^e \{P'(x)\}^{e'} \dots \pmod{p},$$

where  $P(x), P'(x), \dots$  are different prime functions with respect to  $p$ , of degrees  $f, f', \dots$  respectively, then

$$(p) = (p, P(\theta))^e (p, P'(\theta))^{e'} \dots$$

is the required factorization of  $(p)$ , where  $(p, P(\theta)), (p, P'(\theta)), \dots$  are different prime ideals of degrees  $f, f', \dots$  respectively.  $\theta$  is an integer defining the field,  $f'(\theta) = 0$  the equation of lowest degree satisfied by  $\theta$ , and  $d(\theta)$  the discriminant of this equation. When  $\frac{d(\theta)}{\Delta} \equiv 0 \pmod{p}$  the

factorization of  $(p)$  can be effected in the case of cubic fields by a method due to Woronoj. The determination of the number of classes into which these ideals fall is made to depend upon the determination of the lowest power of a given ideal which is a principal ideal, and this upon the determination whether a given principal ideal  $(a)$  is the  $p$ th power of a principal ideal. Attention is called to the saving in reckoning effected by observing that, when we have  $N - n + k < 2K$ , then  $h = K$ , where  $N$  is the number of ideals to be investigated,  $n$  the number of these ideals whose classes have been determined,  $k$  the number of the known classes which have found representatives, and  $K$  the number of known classes. If the  $s$ th and  $t$ th are the lowest powers of  $j$  and  $a$  respectively, which are principal ideals, then a necessary condition that  $j$  belong to one of the classes represented by a power of  $a$  is  $t \equiv 0 \pmod{s}$ , in which case there are  $\varphi(s)$  of these classes to one of which it is possible for  $j$  to belong. They have as representatives  $a^{i'}$ , where  $t = t's$ , and  $i$  is prime to  $s$ . The question whether  $j$  belongs to a given class is decided by multiplying  $j$  by an ideal  $b_j$  belonging to the reciprocal class and determining whether  $jb_j$  is a principal ideal by the method indicated. See Hilbert: "Bericht über die Theorie der algebraischen Zahlkörper," *Jahresbericht der deutschen Mathematiker-Vereinigung*, Vol. 4 (1894-5), §§ 11, 24; Minkowski: *Geometrie der Zahlen*; Woronoj: "The algebraic integers, which are functions of a root of an equation of the 3d degree," (translation of the Russian title).

F. N. COLE.