THE MECHANICS OF THE ATMOSPHERE.

The Mechanics of the Earth's Atmosphere. A collection of Translations. By CLEVELAND ABBE. City of Washington: published by the Smithsonian Institution, 1891. 8vo, pp. 324.

THE student who in these days sets out to explore any field of mathematical physics, with a view to extending its boundaries, assumes an arduous task. He must, in the first place, become familiar with a certain number of facts, or phenomena, which constitute the basis of his science. He must then learn how these facts are expressed in mathematical symbols, and along what lines the processes of deduction may be applied. This introduction to the mere elements of the science will often find him deficient in some branch of analysis, to which reference must be had before intelligent progress can begin. When this deficiency is supplied, new difficulties are almost certain to arise in the novelty of the conceptions required and in the complexity of the specifications needed for generality. Sooner or later in the course of his studies he learns that the beginnings of the science, as well as the latest conclusions concerning it, are to be found in certain original memoirs. The mere number and volume of these are generally appalling enough. But additional difficulties are commonly met in the fact that they are published in several languages and bound up in the bulky transactions of learned societies. To reach and appreciate them, the student must have access to one or more of the great libraries, and possess at least a reading knowledge of two or three foreign languages. But the obstacles he must encounter and overcome do not end here. He will find that the field of his special science joins on or laps over the boundaries of other sciences, so that he must trace out and define numerous analogies and finally end with a pretty thorough knowledge of several sciences before he is competent to advance any one. The time and energy required to attain this preliminary equipment are very great, and those who do not possess special facilities, along with exceptional talents and unflagging industry, can hardly hope to rise above the plane of mediocrity.

In view of these obstacles in the way of all but the most favored students, we must welcome every effort which seeks the unification and simplification of allied sciences or renders the sources of knowledge concerning them more accessible. Most noteworthy and commendable in the latter regard are the pains recently taken by the French government to republish in collected form the works of Lagrange, Laplace, Fourier, Cauchy, etc., thus enabling the student to place himself in intimate contact with the methods and appliances of these masters in mathematical and physical research. Private and public enterprise and the zeal of learned societies have also done much of late in the same direction in Germany and England, as shown in the collected works of Bessel, Kirchhoff, Sir George Stokes, Lord Kelvin, Maxwell, and others. The readiness with which such works can now be obtained by the majority of students is one of the most hopeful signs for future progress. Not less noteworthy and commendable are those collections wherein a series of memoirs on the same or allied subjects are brought together in one language. Taylor's Scientific Memoirs of a half century ago are familiar examples of such important aids to readers of English; and the more recent Collection de Mémoirs relatifs à la Physique, publiés par la Société Française de Physique, are similar worthy examples to readers of French.

The collection of translations before us falls in line with the class of works just mentioned, and it is a pleasant duty to thank Professor Abbe and the administration of the Smithsonian Institution for what amounts, almost, to a systematic treatise on the mechanics of the earth's atmosphere, as presented by some of its most recent expounders.

The volume contains twenty papers, all but two of which were published originally in the German language. The subjects embraced in these papers are so inviting to mathematicians generally as well as to those who would bring meteorology within the domain of mathematical physics, that we think it not out of place here to give a full list of the titles and authors. The list is then as follows:

I. HAGEN, 1874. The measurement of the resistances experienced by plane plates when they are moved through the air in a direction normal to their planes.

II. HELMHOLTZ, 1858. On the integrals of the hydrodynamic equations that represent vortex motions.

III. HELMHOLTZ, 1868. On discontinuous motions in liquids.

IV. HELMHOLTZ, 1873. On a theorem relative to movements that are geometrically similar, together with an application to the problem of steering balloons.

V. HELMHOLTZ, 1888. On atmospheric motions. First paper.

VI. HELMHOLTZ, 1889. " " " Second paper. On the theory of wind and waves.

VII. HELMHOLTZ, 1890. The energy of the billows and the wind.

VIII. KIRCHHOFF, 1869. The theory of free liquid jets.

IX. OBERBECK, 1877. On discontinuous motions in liquids.

X. OBERBECK, 1882. The movements of the atmosphere on the earth's surface.

XI. OBERBECK, 1882. On the Guldberg-Mohn theory of horizontal atmospheric currents.

XII. OBERBECK, 1888. On the phenomena of motion in the atmosphere. First paper.

XIII. OBERBECK, 1888. On the phenomena of motion in the atmosphere. Second paper.

XIV. HERTZ, 1884. A graphic method of determining the adiabatic changes in the condition of moist air.
XV. BEZOLD, 1888. On the thermodynamics of the atmosphere. First paper.
XVI. BEZOLD, 1888. On the thermodynamics of the atmosphere. Second paper.
XVII. BEZOLD, 1889. On the thermodynamics of the atmosphere. Third paper.
XVIII. RAYLEIGH, 1890. On the vibrations of an atmosphere.
XIX MARGULES, 1890. On the vibrations of an atmosphere periodically heated.
XX. FERREL, 1890. Laplace's solution of the tidal equations.

The first paper, by Professor G. H. L. Hagen, deals chiefly with experimental data. It was published in 1874, and in addition to its intrinsic merits it is noteworthy for what we would now pronounce a decided demerit in its use of antiquated units.

The second paper, by Helmholtz, is his classic memoir published originally in *Crelle's Journal*. Readers of English have hitherto had access to this memoir through a translation of Professor P. G. Tait published in the *Philosophical Magazine* for June, 1867. But no one can doubt the propriety of republishing it in the present list; for besides marking a distinct advance in hydromechanics, it is at once the most natural and most readable exposition extant of the profound questions considered.

The following five papers, also by Helmholtz, afford a good illustration of the fact alluded to above, namely, that one's foundation for research in any science must be broad as well as deep. They have to deal primarily with the atmosphere, but they present numerous analogies with and allusions to the sciences of thermodynamics, electricity, magnetism, attraction, etc.

The paper by Kirchhoff, No. VIII. of the collection, finds its proper place as an extension to No. III. by Helmholtz. Both Nos. III. and VIII. are of interest in pure mathematics as affording illustrations of the use of conjugate and discontinuous functions for the expression of natural phenomena.

The first paper by Professor Oberbeck gives a very interesting account of his experiments in forcing jets of colored liquid into quiescent colorless liquids. The observed effects, illustrated by numerous meridian sections of the jets, are specially fruitful in suggestions to those who would extend the theoretical investigations of Helmholtz and Kirchhoff. The following papers, X. to XIII., by Oberbeck, deal particularly with the phenomena of the motions of our atmosphere. They are admirable specimens of the application of the principles of hydromechanics to the interpretation of such phenomena, by reason especially of the clear specification of the assumptions made. We may question the adequacy of these assumptions, but we are not left in doubt as to the limitations they impose on the problems presented by nature. These papers are also of interest and value from the independent confirmation they afford of many results reached previously by Ferrel.

The paper by Dr. Hertz, No. XIV., and those by Bezold, Nos. XV. to XVII., as their titles indicate, deal almost exclusively with the thermodynamics of the atmosphere. While the paper by Hertz is limited to problems requiring the assumption of adiabatic changes only, those of Bezold seek to outline processes which do not rest on so narrow a basis, and which will therefore enable us to understand more clearly the actual phenomena. Both authors are evidently admirers of the graphical method of illustrating mathematical relations, so that those who think best by the aid of diagrams, as well as those who can do without them, will find these investigations attractive. The observer in meteorology, as distinguished from the theorist, will also recognize in Bezold's papers, especially, some fine opportunities for the application of good observational data.

Those who might be deterred from entering the rather difficult field of mathematical meteorology will find encouragement in the words of Lord Rayleigh in the opening paragraph of his paper, No. XVIII., on vibrations of an atmosphere. "In order," he says, "to introduce greater precision into our ideas respecting the behavior of the earth's atmosphere, it seems advisable to solve any problems that may present themselves, even though the search for simplicity may lead us to stray rather far from the actual question. It is proposed here to consider the case of an atmosphere composed of gas which obeys Boyle's law, viz., such that the pressure is always proportional to the density. And in the first instance we shall neglect the curvature and rotation of the earth, supposing the strata of equal density are parallel planes perpendicular to the direction in which gravity acts." He also assumes that the acceleration of gravity is constant through the height considered.

These would seem to be quite simple assumptions, yet they lead to some very interesting results, with which one may well become familiar before passing to the considerations required by the actual atmosphere. The most suggestive of the results referred to are the natural vibration periods of Rayleigh's ideal atmosphere. These are, for the first two harmonics, 23.8 hours and 13.7 hours, respectively, and they may be important in throwing light on the relative amplitudes of the observed diurnal and semi-diurnal changes of atmospheric pressure.

The paper by Rayleigh is fitly followed by that of Margules, who conceives the periodic variations of pressure in the atmosphere to be due to periodic changes of its temperature. His investigation rests on a foundation of assumptions, which seem nearly, if not quite, adequate. The clearness of presentation and the comprehensiveness of statement render this one of the most important of recent contributions to dynamical meteorology.

Margules' discussion of the case presented by a rotating sphere leads to an application of the differential equation of the second order first treated by Laplace in his theory of tides. The determination of the constants of integration of this equation is a matter of controversy, the principal contestants being Laplace and Lord Kelvin on the one side and Airy and Ferrel on the other. This fact accounts for the unexpected appearance at the end of the collection of the paper by Ferrel on Laplace's solution of the tidal equations. May we not hope that some readers of the BULLETIN will clear up the difficulties and obscurities surrounding this important question?

In a work of this kind one would hardly expect the translator to vary much from a literal rendition. We might wish that he had in the present case taken larger liberties and pruned off some of the verbose indirectness of the originals; but, on the other hand, the literalness serves to keep the reader close to the thought of the author, and this advantage may properly offset the desire for directness in translations so free as these from ambiguity.

The typography of the book is in general good, and we must commend Professor Abbe's effort to secure conformity with the best models of mathematical notation. His use of the "round *d*" to indicate partial differentiation is especially gratifying in view of the perplexing and aimless diversity in this respect of most works in the English language. We have noticed a considerable number of such typographical errors as it is difficult to keep out of a first edition of a book containing a large number of formulas. We may confidently expect that the need of a new edition of the work will afford an opportunity for correcting these inevitable blemishes.

In closing this hasty and quite inadequate sketch of these papers, we would wish to join in the hope expressed by Professor Abbe, "that a coming generation of American meteorologists may prosecute to further conquests the mathematical studies begun by Ferrel and perfected by our European colleagues."

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