NOTES. 109

locum habere queant, in se complectatur. Neque vero idcirco problema principale resolvere liceret, quo pro statu initiali quocunque ejus motus secuturus requiritur; ad hoc enim necesse esset, infinitas illas constantes arbitrarias pro statu initiali dato debite determinare, quod certe opus omnes vires

analyseos longe esset superaturum."

In order to see how remarkable this passage is it is necessary to recall the celebrated triangular contest between D'Alembert, Euler and Daniel Bernoulli which began at about the middle of the eighteenth century.* We have here to deal with the question under discussion between Bernoulli and Euler, the former maintaining that the most general vibration of a stretched string could be expressed as a trigonometric series, the latter that this could give only a particular class of solutions, since (a fact no one doubted at that time) a function which would be graphically represented by a broken line could not possibly be expressed as a trigonometric series, or in fact by any analytic formula.

Now the above quotation refers to an entirely analogous problem, the principal difference being that the trigonometric series is replaced by a series involving Bessel's functions, and here we find Euler taking Bernoulli's position and asserting that without any doubt this series expresses the most general motion possible! I am at a loss to explain this passage, for we must apparently either assume that Euler had forgotten the previous discussion which had lasted so long and formed such an important episode in his life and in which, considering the mathematical knowledge of the time, his position had been unassailable; or that he had anticipated Fourier's wonderful idea that any function can be developed in a trigonometric series, or a series involving Bessel's functions. That he should have arrived at this point without the method of determining the coefficients seems incredible.

HARVARD UNIVERSITY, January, 1893.

NOTES.

THE annual meeting of the NEW YORK MATHEMATICAL SOCIETY was held Thursday afternoon, December 29, at four o'clock, the president, Dr. McClintock, in the chair. The council announced that Professor Henry B. Fine had been appointed to act as delegate of the Society to the meetings of the American Philosophical Society on the occasion of the

^{*} See RIEMANN, Works, pp. 214-218 (sec. ed. pp. 227-231).

110 NOTES.

150th anniversary of its founding. Reports were presented by the secretary, treasurer and librarian. The secretary stated that the membership of the Society was 234, of whom 44 resided in New York city and the immediate vicinity. The average attendance of members at the meetings had been 15 or 16, and the average total attendance 18. The librarian stated that in the library there were 36 volumes and 54 pamphlets, and that 15 journals had been placed on the exchange list of the Society. The treasurer's report having been read, an auditing committee was appointed to examine his accounts.

The committee on nominations elected at the preceding meeting, reported a ticket for the officers and other members of the council for the ensuing year, and, a ballot being taken, this ticket was unanimously elected. The officers and council for 1893, are:—President, Dr. Emory McClintock; Vice-President, Professor Henry B. Fine; Secretary, Dr. Thomas S. Fiske; Treasurer, Mr. Harold Jacoby; Librarian, Professor D. A. Murray; Committee of Publication, Dr. Thomas S. Fiske and Mr. Harold Jacoby; other members of the Council, Professor Thomas Craig, Professor W. Woolsey Johnson, Professor J. E. Oliver, Professor J. K. Rees, Professor J. H. Van Amringe.

Professor M. I. Pupin communicated an informal note on a proof of the principal property of two perspective triangles, viz., that the points in which the corresponding sides intersect are all on the same straight line. If the triangles are plane sections of a triangular pyramid, it is sufficient to point out that these points must lie on the line of intersection of their planes. To prove the property for two perspective triangles in the same plane, it is necessary only to show that they are projections of plane sections of a triangular pyramid. The converse proposition also follows at once by

the method of projections.

The same method of demonstration may be applied to show that the corresponding sides of two perspective polygons intersect in points which are all on the same straight line provided that these polygons represent the projections of plane sections of some pyramid. If the corresponding sides of two polygons intersect on the same straight line without making the two polygons perspective to each other, they may be taken as representing the projections of plane sections of wedge-shaped bodies. To this class of figures belong any two closed funicular polygons corresponding to the same system of forces. (Compare Minchin's Statics, p. 105.) The proper qualification of the results mentioned in the latter portion of the note, gave rise to some discussion in which Professors Webb and Merriman took part.

NOTES. 111

REPORT OF THE TREASURER FOR THE YEAR 1892.

$oldsymbol{Receipts}.$	Expenditures.
Balance from 1891 \$271.00 Net receipts from members, subscribers, and advertising	Printing list of members \$98.00 "Bulletin
\$1450.70	\$1450.70

HAROLD JACOBY, Treasurer.

We have examined the treasurer's accounts, and found the same correct.

A REGULAR meeting of the New York Mathematical Society was held Saturday afternoon, January 7, at half-past three o'clock, the president, Dr. McClintock, in the chair. A paper by Professor W. H. Echols, "On a general formula for the expansion of functions in series," was read by the secretary. This paper may be regarded as an exposition of a general theorem which is the basis of a series of papers in course of publication in the *Annals of Mathematics* under the title "On certain determinant forms and their applications."

THE council of the German mathematical association (Deutsche Mathematiker-Vereinigung) has issued a preliminary report, from which it appears that the membership is at present 215. The annual meeting will be held at Munich, in the month of September, during the week preceding the Naturforscher-Versammlung; the exact date will be announced later.

As last year's meeting had to be abandoned on account of the cholera, the exhibition of mathematical models and instruments planned for that meeting will be held this year at Munich, in the rooms of the Polytechnic School; it is to be open during the whole month of September and promises to be of unusual interest. Prof. W. Dyck, in connection with many others, had undertaken the preparation of a descriptive and illustrative catalogue for the proposed exhibition of 1892. This catalogue, a volume of about 450 pages, with numerous illustrations, is now ready, and can be obtained directly from Prof. W. Dyck (München, Hildegardstrasse 1½), at the price of mk. 9.80 (including postage). While this volume can of course not be regarded as a complete catalogue of the coming

exhibition, it will serve to show the range and importance

of the work to be accomplished.

The objects to be exhibited are classified under three main heads which we may briefly describe as I. analysis, II. geometry, III. mechanics. Class I. is subdivided as follows: (1) arithmetic, comprising calculating machines and apparatus used in the theory of probability; (2) algebra and theory of functions, where, in addition to models and drawings, we find mechanical devices for solving equations and constructing the dependence of functions on their variables; (3) integral calculus, with instruments for rectifying curves, measuring areas. In the class devoted to geometry we have, besides drawing instruments, a large variety of models and graphical illustrations exhibiting the properties of geometrical solids, surfaces, and curves. Class III. has three principal divisions. In the first, mechanics proper, are comprised (a) models and instruments for demonstrating dynamical propositions, and (b) kinematical models and instruments, with applications to machinery (a very extensive division). In the second division, mathematical physics, we find (a) apparatus illustrative of the wave-theory; (b) and (c) models illustrating the structure and physical properties of crystals; (d) thermodynamical models and drawings; (e) apparatus for the illustration of electrodynamic phenomena. The third division, technical applications, contains geodetic, nautical and meteorological instruments.

This description of instruments is preceded by a series of monographs and notes covering about 135 pages. The authors and subjects are as follows: F. Klein, Geometrical illustration of the enumeration of the real roots of algebraic equations: A. Voss, On systems of equidistant curves on curved surfaces; A. Brill, On the reduction of the higher singularities of an algebraic curve to elementary singularities; G. Hauck, On the geometric postulates of solid geometry in their relation to the methods of descriptive geometry; A. von Braunmühl, Historical sketch of the organic generation of plane curves from the oldest times to the end of the eighteenth century; L. Boltzmann, On the methods of theoretical physics; A. Amsler, On mechanical integrations; O. Henrici, On instruments for harmonic analysis.

R. S. WOODWARD, C.E., Ph.D. (University of Michigan), of the U.S. Coast Survey, Washington, D. C., has been appointed Professor of Mechanics at Columbia College, the appointment to take effect on July 1.

THE nine conferences on the curriculum of the secondary schools, appointed by the National Educational Association to be held at various places throughout the United States, met on December 28, 1892. That on Mathematics met at Cambridge, Mass. The members of the committee were: Professor W. E. Byerly, Harvard University; Professor Florian Cajori, Colorado College; Chancellor W. S. Chaplin, Washington University; Mr. Arthur H. Cutler, New York; Professor H. B. Fine, Princeton College; Mr. W. A. Greeson, Grand Rapids, Mich.; Professor Simon Newcomb, Johns Hopkins University; Mr. J. L. Patterson, Lawrenceville, N. J.; Professor G. D. Olds, Amherst College; Professor T. H. Safford, Williams College. It was the unanimous opinion of the committee that important changes are needed. Its proceedings, however, can not be made public until laid before the general committee of ten in April or May.

NEW PUBLICATIONS.

1. HIGHER MATHEMATICS.

- ECKHARDT (E.). Ein Rotationsproblem. Die Dreitheilung des Winkels. Die Darstellung der Wurzeln der Gleichungen dritten Grades durch Zeichnung. Marburg 1892. 8vo. 56 pp. 2 plates. Mk. 1.80
- HAYWARD (R. B.). The algebra of coplanar vectors and trigonometry. London, Macmillan, 1892. Svo. 876 pp. 8s. 6d.
- JAHRESBERICHT der deutschen Mathematiker-Vereinigung. Band I: 1890-91. Enthaltend Chronik für 1890-91, Bericht über die in Halle 1891 gehaltenen Vorträge und ausführlichen Bericht über die Fortschritte der projectiven Invariantentheorie im letzten Vierteljahrhundert, von Dr. W. F. Meyer. Herausgegeben von G. Cantor, W. Dyck, E. Lampe. Berlin, Reimer, 1892. 8vo. 292 pp. Mk. 7.60
- LAISANT (C. A.). Recueil de problèmes de mathématiques, classés par divisions scientifiques, contenant les énoncés avec renvoi aux solutions de tous les problèmes posés, depuis l'origine, dans divers journaux : Nouvelles annales de math., Journal de math. élém. et de math. spéc., Mathésis, Nouvelle correspondance math. (In 7 vols.) Paris. Gauthier-Villars. 8vo.
 - Vol. IV. Géométrie analytique à deux dimensions (et géométrie supérieure). 1893. Fr. 6.00
 - Vol. V. Géométrie analytique à trois dimensions (et géométrie supérieure). 1893. Fr. 2.50
- MOLIEN (T.). Über Systeme höherer complexer Zahlen. (Dissertation.)
 Dorpat, Karow, 1892. 8vo. 74 pp. Mk. 2.00
- Müller (M.). Über algebraisch-logarithmische Integrale von Systemen algebraischer Differentialgleichungen. Heidelberg, 1892. 8vo. 64 pp. Mk. 1.50