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Finite Dehn surgery along A'Campo's divide knots

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Abstract.

We give two geometric methods of constructing plane curves giving cable knots of torus knots via A'Campo's divide knot theory, related to both singularity theory and knot theory. We point out a relationship between "area" of the plane curves and the coefficients of *finite Dehn surgery*, which is Dehn surgery yielding three-dimensional manifolds with finite fundamental group.

§1. Introduction

The *divide* is a relative, generic immersion of a finite number of copies of an arc and a circle in the unit disk D in $\mathbf{R}^2$. N. A'Campo formulated the following definition to associate to each divide P a link $L(P)$ in the 3-dimensional sphere S^3 ([1, 2, 3, 4]):

$$L(P) = \{(u, v) \in D \times T_u D \,|\, u \in P,\ v \in T_u P,\ |u|^2 + |v|^2 = 1\} \subset S^3,$$

where $T_u P$ is the subset consisting of vectors tangent to P in the tangent space $T_u D$ of D at u. The number of components of $L(P)$ is $\sharp$arc + 2$\sharp$circle, where $\sharp$arc (and $\sharp$circle, respectively) is the number of immersed components of arcs (and circles) in P. In this paper, we will study the case where P consists of one immersed arc, thus $L(P)$ is a knot, and we say "a curve P gives a link L" if $L(P) = L$.

The class of links of divides properly contains the class of the links arising from isolated singularities of complex plane curves, for example,

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each torus link $T(a,b)$ of type (a,b) with $a,b>0$ appears as the link of the singularity of the curve $z^a - w^b = 0$ in $\mathbf{C}^2$ at the origin. In particular, if a and b are coprime, then $T(a,b)$ is a torus knot. A *cable knot* of a (non-trivial) knot K is a knot in the boundary T_K of a regular neighborhood of K. A cable knot is called (p,q)-cable of K and denoted by $C(K;p,q)$ if it is homologous to $pl_K + qm_K$ in T_K, where $\{m_K, l_K\}$ is a meridian-longitude system on T_K. For the torus knot $T(a,b)$, if the pair (p,q) of coefficients satisfies the inequarity $q > abp$ then the cable knot $C(T(a,b);p,q)$ also appears as the link of the singularity in $\mathbf{C}^2$, see [11, p.51]. Note that it is well-known that the link of a singularity is a torus knot, a cable knot or a knot obtained by iteration of cablings of them, called an "iterated torus knot".

In this paper, we give two geometric methods of constructing divides that give some cable knots of torus knots. They are different from A'Campo's original method [4]. The first method, in the next section, is a generalization of [14], in which the author and co-authors showed that a billiard curve in a rectangle $a \times b$ gives a torus knot $T(a,b)$ from the view point of knot theory. The second one, in Section 3, is a modification of A'Campo's, but we will use *fold-maps* of rectangles instead of immersions. In Section 4, we point out a relationship between such divide representation of knots and *finite Dehn surgery*, i.e., Dehn surgery yielding a 3-manifold whose fundamental group is finite. The reason why we show such alternative methods is that ours seems more convenient in 3-dimensional topology.

The author would like to thank Professor Tadashi Ashikaga and Professor Masaharu Ishikawa [15, 16] for introducing to him A'Campo's theory. The author would like to thank Professor Mikami Hirasawa, who checked some examples of theorems in this paper by more knot-theoretical and visualized method in [18]. The author would like to express sincere gratitude to the referee for reading the manuscript carefully and giving him valuable advice.

§2. Method 1. Billiard curve

Let X be the infinite $45°$ lattice defined by

$$X := \{(x,y) \in \mathbf{R}^2 |\ \cos \pi x = \cos \pi y\}$$

in the real xy-plane. For a pair (a,b) of positive integers and $(m,n) \in \mathbf{Z}^2$, by $R(a \times b)_{(m,n)}$ we denote the rectangle at (m,n) of size $a \times b$ in the following sense:

$$R(a \times b)_{(m,n)} := \{(x,y) \in \mathbf{R}^2 |\ m \leq x \leq m+a \ \text{ and } \ n \leq y \leq n+b\}.$$

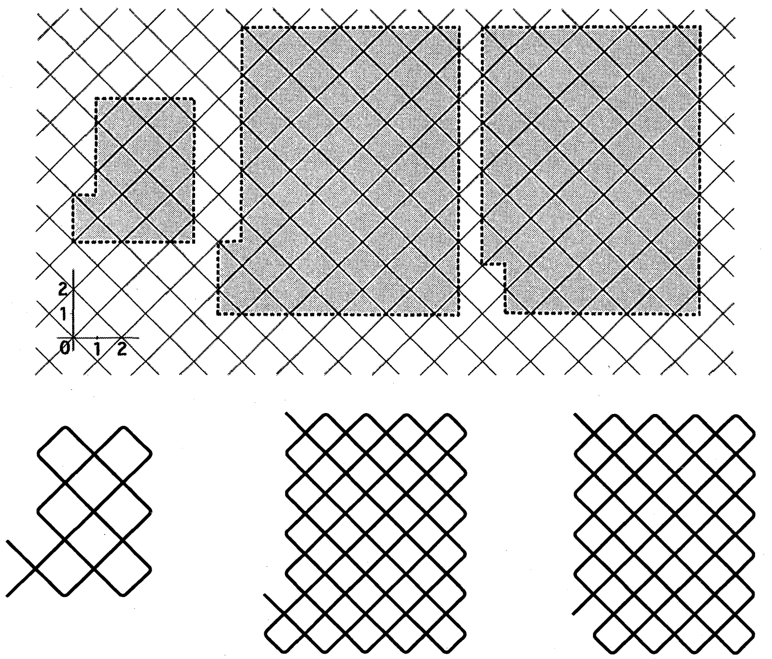

Fig. 1. $P(2,3;2,13)$, $P(3,4;3,37)$ and $P(3,4;3,35)$

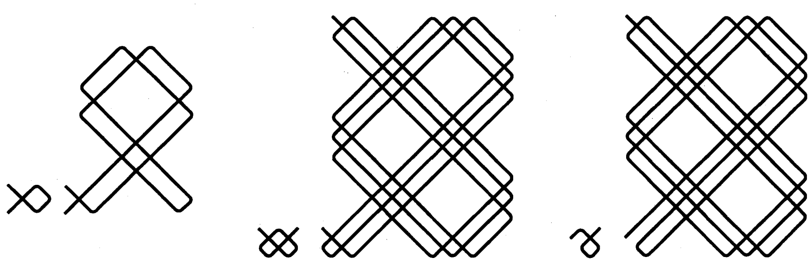

Fig. 2. $B(2,3)\langle B_0^+(2)\rangle$ and $B(3,4)\langle B_0^\pm(3)\rangle$

For such a rectangle or a *union* $\mathcal{R}$ of such rectangles, we regard $X \cap \mathcal{R}$ as a piecewise linear curve (shortly, a PL curve), where we regard each point in $X \cap \partial\mathcal{R}$ as a break point if it is on the edges of $\mathcal{R}$, or a endpoint if it is on a corner of $\partial\mathcal{R}$. From such a PL curve, we get a divide by rounding the break points smoothly and setting it in the unit disk in $\mathbf{R}^2$

by an isotopy. By $X \cap \mathcal{R}$, we also denote such a smooth divide derived from the PL curve.

In [14], the author and co-authors proved the following proposition and found a nice diagram of $T(a,b)$ and Murasugi-sum structure of their fiber surfaces from the view point of knot theory. (Proposition 2.1 itself has been shown in [17], [5], or can be shown by more recent works [10], [18].)

Proposition 2.1. ([14]) *Let (a,b) be a pair of positive integers and (m,n) any pair of integers. Then a divide $X \cap R(a \times b)_{(m,n)}$ gives a torus link $T(a,b)$.*

Now, for a pair (a,b) of positive coprime integers and a positive integer p, we define a region $\mathcal{R}(a,b;p,pab+1)$ in $\mathbf{R}^2$ and a divide $P(a,b;p,pab+1)$ as:

$$\begin{aligned} (+) \quad \mathcal{R}(a,b;p,pab+1) &:= R(pa \times pb)_{(0,0)} \cup R(1 \times p)_{(-1,0)}, \\ P(a,b;p,pab+1) &:= X \cap (\mathcal{R}(a,b;p,pab+1) + \vec{\delta}), \end{aligned}$$

and also define a region $\mathcal{R}(a,b;p,pab-1)$ in $\mathbf{R}^2$ and a divide $P(a,b;p,pab-1)$ as:

$$\begin{aligned} (-) \quad \mathcal{R}(a,b;p,pab-1) &:= cl\left(R(pa \times pb)_{(0,0)} \backslash R(1 \times (p-1))_{(0,0)}\right), \\ P(a,b;p,pab-1) &:= X \cap (\mathcal{R}(a,b;p,pab-1) + \vec{\delta}), \end{aligned}$$

where cl means the closure of the region, $\vec{\delta} = (\delta_1, \delta_2) \in \mathbf{Z}^2$ with $\delta_1 + \delta_2 \equiv p+1 \mod 2$ and $+\vec{\delta}$ means the parallel transformation by $\vec{\delta}$ in $\mathbf{R}^2$, see Figure 1.

Theorem 2.2. *For a pair (a,b) of positive coprime integers and a positive integer p, the divide $P(a,b;p,pab \pm 1)$ gives a $(p, pab \pm 1)$-cable of the torus knot $T(a,b)$, i.e.,*

$$L(P(a,b;p,pab \pm 1)) = C(T(a,b);p,pab \pm 1).$$

Proof. First, from the pair $(a_0,b_0) := (a,b)$, we construct a word $w_1 w_2 \cdots w_n$ of two letters L (left) and R (right) by Euclidean algorithm, see Figure 3:

If $a_i > b_i$, then $w_{i+1} := L$ and $(a_{i+1}, b_{i+1}) := (a_i - b_i, b_i)$.

If $a_i < b_i$, then $w_{i+1} := R$ and $(a_{i+1}, b_{i+1}) := (a_i, b_i - a_i)$.

By coprime-ness of (a,b), after some n steps, the pair (a_n, b_n) becomes $(1,1)$. Then this step is over.

Second, we regard the word as a rule of constructing the $a \times b$ rectangle. In fact, $R(a,b)_{(0,0)}$ is obtained from $R(1 \times 1)_{(0,0)}$ ruled by the word in inversed order as follows $(j = 1, 2, ..., n)$ (see also [19]):

$$(3,5) \xrightarrow[R]{} (3,2) \xrightarrow[L]{} (1,2) \xrightarrow[R]{} (1,1)$$

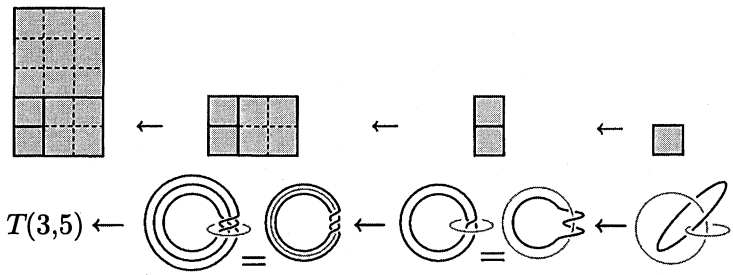

Fig. 3. Euclidean Algorithm

If $w_{n+1-j} = L$, then we add a square from the right.

If $w_{n+1-j} = R$, then we add a square from the top, see Figure 3. This process corresponds to a blowing-up sequence of the singularity $z^a - w^b = 0$, thus also to a twisting sequence of torus knots. According to growing up of the rectangle from $R(1 \times 1)$ to $R(a \times b)$, the corresponding knot K changes from the unknot $K_0 := T(1,1)$ to $K_n := T(a,b)$ by Proposition 2.1.

Finally, we starting with the region $R((p+1) \times p)_{(-1,0)} + \vec{\delta}$ or $cl\left(R(p \times p)_{(0,0)} \backslash R(1 \times (p-1))_{(0,0)}\right) + \vec{\delta}$ according to the sign at $\pm$, which gives $C(T(1,1); p, p \pm 1) = T(p \pm 1, p)$ regarded as a curve in T_{K_0}. We add $p \times p$ extended squares to the starting rectangle from the right or the top according to the word w_{n+1-j} is L or R ($j = 1, 2, ..., n$) as same as in the last step. Then we have $\mathcal{R}(a, b; p, pab \pm 1)$ and the divide $P(a, b; p, pab \pm 1)$.

Generally, if a knot K' is obtained from K by a positive twisting along a disk d, the homology class m_K in T_K becomes to $m_{K'}$ in $T_{K'}$ and the class l_K in T_K becomes to $l_{K'} + lk(K, \partial d)^2 m_{K'}$ in $T_{K'}$, where $lk(K, \partial d)$ is the linking number of K and the boundary of d, see [20, p.11]. In the divide theory, the intersection number between two divides equals to the linking number of the corresponding components of the link.

In our j-th process in the case of $w_{n+1-j} = R$, the boundary of the disk d_j corresponds to the right edge whose length is b_{n+1-j}, which equals to the linking number $lk(K_{j-1}, \partial d_j)$.

Thus $pl_{K_{j-1}} + (pa_{n+1-j}b_{n+1-j} \pm 1)m_{K_{j-1}}$ becomes to

$$\begin{aligned} &p(l_{K_j} + b_{n+1-j}^2 m_{K_j}) + (pa_{n+1-j}b_{n+1-j} \pm 1)m_{K_j} \\ &\quad = pl_{K_j} + \{p(a_{n+1-j} + b_{n+1-j})b_{n+1-j} \pm 1\}m_{K_j} \\ &\quad = pl_{K_j} + (pa_{n-j}b_{n-j} \pm 1)m_{K_j}. \end{aligned}$$

The case of $w_{n+1-j} = L$ is similar (a and b are changed). After the final n-th step, we have the cable knot $C(T(a,b); p, pab \pm 1)$. The proof is completed. Q.E.D.

§3. Method 2. Fold-immersion

Let P be a PL curve obtained by cutting out $X \cap \mathcal{R}$ from the lattice X as in the last section.

Definition 3.1. For such a PL curve P, by b we denote the number of break points of P on the edges of $\mathcal{R}$. We say that a map $f : [0,1]^2 \to \mathbf{R}^2$ is a *fold-immersion of* $[0,1]^2$ *along* P if it satisfies the following condition:

(1) $f([0,1] \times \{\frac{1}{2}\}) = P$,

(2) There exists a sequence $0 < t_1 < t_2 < \cdots < t_b < 1$ such that

 (i) f is an immersion over $([0,1] \backslash \{t_1, t_2, \cdots, t_b\}) \times [0,1]$ and

 (ii) Near each $\{t_i\} \times [0,1]$, f is locally given as shown in Figure 4. For example, in the case of Figure 4, f near $\{t_i\} \times [0,1]$ is determined by the map

$$\begin{array}{rccc} \varphi : & (t_i - \epsilon, t_i + \epsilon) \times [0,1] & \to & \mathbf{R}^2 \\ & (t,s) & \mapsto & (t + s, |t|). \end{array}$$

 If the break point is not on the bottom edge, then f is locally given by the $\pi/2, \pi$ or $3\pi/2$ rotation of Figure 4 or its reflection.

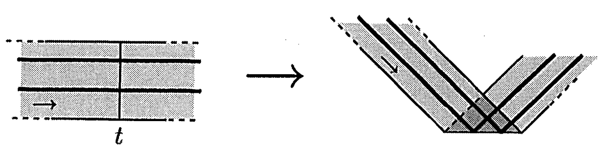

Fig. 4. Fold-map

We remark that triangle moves on divides shown in Figure 5 do not change the ambient isotopy type of the links of the divides. Let $B_0^{\pm}(p)$

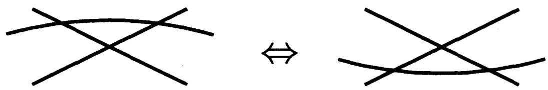

Fig. 5. Triangle move

and $B(a,b)$ be the billiard curves defined by

$$\begin{aligned} B_0^+(p) &:= X \cap R((p+1) \times p)_{(0,-p)}, \\ B_0^-(p) &:= X \cap cl\left(R(p \times p)_{(0,-p)} \backslash R(1 \times (p-1))_{(0,-p)}\right) \text{ and} \\ B(a,b) &:= X \cap R(a \times b)_{(0,0)}. \end{aligned}$$

By scaling smaller, we regard $B_0^{\pm}(p)$ in the rectangle as a curve in $[0,1]^2$. Then, the image of $B_0^{\pm}(p)$ under a (generic) fold-immersion along $B(a,b)$ is well-defined up to triangle moves. We denote such a curve by $B(a,b)\langle B_0^{\pm}(p)\rangle$, see Figure 2, placed near Figure 1 for convenience.

Theorem 3.2. *For a pair (a,b) of positive coprime integers and a positive integer p, the divide $B(a,b)\langle B_0^{\pm}(p)\rangle$ gives a $(p, pab \pm 1)$-cable of $T(a,b)$, i.e.,*

$$L(B(a,b)\langle B_0^{\pm}(p)\rangle) = C(T(a,b); p, pab \pm 1).$$

Proof. It is easy to see that, for a nice choice of fold-immersion, or in other words, by some triangle moves, the curve $B(a,b)\langle B_0^{\pm}(p)\rangle$ is isotopic to $P(a,b;p,pab \pm 1)$. Q.E.D.

We remark that A'Campo constructed in [3] the divide $B(2,3)\langle B_0^{+}(2)\rangle$ in our notation as the image of $B(2,9)$ under <u>an immersion</u> along $B(2,3)$ and denoted by $P_{2,9} * P_{2,3}$, see Figure 6.

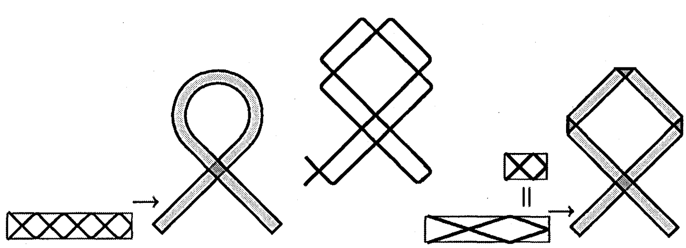

Fig. 6. Comparison

§4. Area of divide and Dehn surgery

Let K be a knot in S^3 and n an integer. By $M(K,n)$ we denote the 3-manifold obtained by Dehn surgery along K with coefficient n, i.e., removing a solid torus V_K along K and regluing it back such that the meridian comes to a curve homologous to $l_K + n\, m_K$, where $\{m_K, l_K\}$ is a meridian-longitude system on the boundary of V_K. Here we are concerned with $M(K,n)$ whose fundamental group $\pi_1(M(K,n))$ is finite. Such research is called "finite Dehn surgery" ([6, 9, 13]). Note that $H_1(M(K,n);\mathbf{Z}) \cong \mathbf{Z}/n\mathbf{Z}$.

What we would like to point out in this section is that in some examples of integral finite surgery $M(K,n)$, the knot K is given by a plane curve as $X \cap \mathcal{R}$ via A'Campo's divide theory, and that in such a case the coefficient n is near to the area $A(\mathcal{R})$ of the region $\mathcal{R}$ in the plane.

The links which can be obtained from the billiard curves by the method in section 2 and 3 are only cable knots of torus knots. In [13] and [6], it is proved that an iterated torus knot other than a torus knot or a cable knot of a torus knot has no finite surgery. The cable knots of torus knots with finite surgery, completely listed in [6], can be obtained as the knots of billiard curves except for the following four cases: $C(T(2,3);p,q)$ with $(p,q) = (2,9), (2,15), (3,16)$ and $(3,20)$. We can state the following:

Theorem 4.1.

(i) *Let $\mathcal{R}$ be a rectangle $R(a \times b)_{(m,n)}$ with a pair (a,b) of any coprime integers, K the knot of the billiard curve obtained from $\mathcal{R}$, and n a coefficient of finite surgery of S^3 along K. Then the inequality $|n - A(\mathcal{R})| \le 1$ holds.*

(ii) *Let $\mathcal{R}$ be a region defined by $(+)$, K the knot of the billiard curve obtained from $\mathcal{R}$, and n a coefficient of finite surgery of S^3 along K. Then the inequality $|n - A(\mathcal{R})| \le 1$ holds.*

(iii) *Let $\mathcal{R}$ be a region defined by $(-)$, K the knot of the billiard curve obtained from $\mathcal{R}$, and n a coefficient of finite surgery of S^3 along K. Then the inequality $|n - A(\mathcal{R})| \le 2$ holds.*

Proof. We start with families of finite surgery along torus knots and cable knots of torus knots.

Example 4.2. *Each of the followings is finite surgery:*
(1) ([21]) $M(T(a,b),n)$ *with* $n = ab \pm 1$.

(2) ([8, 13]) $M(C(T(a,b);2,2ab\pm 1),\, n)$ *with* $n = 4ab \pm 1$,
(3) ([13]) $M(C(T(2,b);3,6b\pm 1),\, n)$ *with* $n = 18b \pm 2$.

In (1) and (2) (or (3), respectively), resulting 3-manifolds are lens spaces (or prism manifolds). The case (i) in the theorem is shown by (1) above: K is the torus knot $T(a,b)$ by Proposition 2.1 and the area of $R(a\times b)_{(m,n)}$ is ab. For (2) and (3), we have that $A(\mathcal{R}) = p^2ab+p$ for $\mathcal{R} = \mathcal{R}(a,b;p,pab+1)$ and that $A(\mathcal{R}) = p^2ab-p+1$ for $\mathcal{R} = \mathcal{R}(a,b;p,pab-1)$. Thus, in these examples, the inequality $|n - A(\mathcal{R})| \leq 1$ holds.

Next, we recall more "exceptional" examples from the list in [6]. In the left-hand side of Table 1, we picked up all examples of integral finite surgery along knots of type $C(T(a,b);p,pab\pm 1)$ from Table 1 in [6], which is the complete list of 37 examples by [13]. Four examples marked by $*$ are included in Example 4.2 (3). In the right-hand side, we represent each knot by a divide of type $B(a,b)\langle B_0^{\pm}(p)\rangle$ (Method 2), which can be deformed as $X \cap \mathcal{R}$ (Method 1) and write its area $A(\mathcal{R})$.

$(+)$ $C(T(a,b);p,pab+1)$	n		$B(a,b)\langle B_0^+(p)\rangle$	$A(\mathcal{R})$
$C(T(2,3);2,13)$	27		$B(2,3)\langle B_0^+(2)\rangle$	26
$C(T(2,3);3,19)$	56	$*$	$B(2,3)\langle B_0^+(3)\rangle$	57
$C(T(2,3);4,25)$	99		$B(2,3)\langle B_0^+(4)\rangle$	100
$C(T(2,3);4,25)$	101		$B(2,3)\langle B_0^+(4)\rangle$	100
$C(T(2,3);5,31)$	154		$B(2,3)\langle B_0^+(5)\rangle$	155
$C(T(2,3);6,37)$	221		$B(2,3)\langle B_0^+(6)\rangle$	222
$C(T(2,5);2,21)$	43		$B(2,5)\langle B_0^+(2)\rangle$	42
$C(T(2,5);3,31)$	92	$*$	$B(2,5)\langle B_0^+(3)\rangle$	93
$C(T(2,5);4,41)$	163		$B(2,5)\langle B_0^+(4)\rangle$	164
$C(T(3,4);3,37)$	110		$B(3,4)\langle B_0^+(3)\rangle$	111
$C(T(3,5);3,46)$	137		$B(3,5)\langle B_0^+(3)\rangle$	138

$(-)$ $C(T(a,b);p,pab-1)$	n		$B(a,b)\langle B_0^-(p)\rangle$	$A(\mathcal{R})$
$C(T(2,3);2,11)$	21		$B(2,3)\langle B_0^-(2)\rangle$	23
$C(T(2,3);3,17)$	50		$B(2,3)\langle B_0^-(3)\rangle$	52
$C(T(2,3);3,17)$	52	$*$	$B(2,3)\langle B_0^-(3)\rangle$	52
$C(T(2,3);4,23)$	91		$B(2,3)\langle B_0^-(4)\rangle$	93
$C(T(2,3);4,23)$	93		$B(2,3)\langle B_0^-(4)\rangle$	93
$C(T(2,3);5,29)$	146		$B(2,3)\langle B_0^-(5)\rangle$	146
$C(T(2,3);6,35)$	211		$B(2,3)\langle B_0^-(6)\rangle$	211
$C(T(2,5);2,19)$	37		$B(2,5)\langle B_0^-(2)\rangle$	39
$C(T(2,5);3,29)$	88	$*$	$B(2,5)\langle B_0^-(3)\rangle$	88
$C(T(2,5);4,39)$	157		$B(2,5)\langle B_0^-(4)\rangle$	157
$C(T(3,4);3,35)$	106		$B(3,4)\langle B_0^-(3)\rangle$	106
$C(T(3,5);3,44)$	133		$B(3,5)\langle B_0^-(3)\rangle$	133

Table 1: Integral finite surgeries along cables

We have the theorem. Q.E.D.

Recent researchers' interest seems to be the finite surgery along hyperbolic knots. In the recent work [22], the author pointed out that every knot in a certain subfamily of Berge's knots ([7]) yielding lens spaces (It contains 19-surgery along the Pretzel knot of type $(-2,3,7)$, which was discovered in [12]), is a divide knot and given by a divide of $X \cap \mathcal{R}$ type. For them, it holds that $n = A(\mathcal{R})$.

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