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ON OPTIMAL DECISION RULES FOR SIGNS OF PARAMETERS

BY YOSEF HOCHBERG[1] AND MARC E. POSNER

New York University

The problem of deciding the signs of k parameters $(\theta_1, \dots, \theta_k) \equiv \boldsymbol{\theta}$ based on $(\hat{\theta}_1, \dots, \hat{\theta}_k) \sim N(\boldsymbol{\theta}, \boldsymbol{\Sigma})$ such that $p_{\boldsymbol{\theta}}\{\text{any error}\} \le \alpha \;\forall\; \boldsymbol{\theta}$ is discussed by Bohrer and Schervish (1980). They characterize a desirable class of procedures called *locally optimal*. For the case $k = 2$, $\boldsymbol{\Sigma} = \mathbf{I}$, and $\alpha \le \frac{1}{3}$, they present a particular rule from this class called the *double cross*. In this paper, we address the problem of selecting a best rule from among all locally optimal rules when $k = 2$ and $\boldsymbol{\Sigma} = \mathbf{I}$. When $\alpha \le \frac{1}{3}$, the double cross is shown to be an attractive choice. Other rules are obtained for higher values of α. We also examine a more general optimization criterion than the one used by Bohrer and Schervish and obtain different optimal rules for several classes of problems. The optimal rule corresponding to one of these classes has no two-decision region. A modification of the formulation is offered under which a well-known rule (with two decision regions) emerges as the unique optimal procedure.

1. Introduction. A common statistical problem in comparative experiments is the simultaneous decision of the signs of several parameters based on normally distributed estimators. As examples, the parameters of interest might be measures of the effects of several competitive drugs (relative to a control) or measures of several side effects of one drug or measures of the carcinogenic potential of various materials.

This problem was first considered by Neyman (1935) who developed goodness criteria for decision rules concerning signs. Lehmann (1957) gave a decision theoretic formulation and characterized some unbiased and optimal rules. Another early work on this subject was by Kaiser (1960). Bohrer and Schervish (1980) and Bohrer (1982) provide further results and indicate additional applications for this statistical problem.

To formalize the problem, let $\hat{\boldsymbol{\theta}} \sim N(\boldsymbol{\theta}, \boldsymbol{\Sigma})$ where diagonal elements of the correlation matrix $\boldsymbol{\Sigma}$ are all 1. Based on the vector $\hat{\boldsymbol{\theta}}$ and the known matrix $\boldsymbol{\Sigma}$, we want to decide for each component θ_i of $\boldsymbol{\theta}$ whether it is positive or negative in such a way that the probability of making at least one incorrect decision is no larger than a specified value α. Bohrer (1979) showed that if $\alpha < 0.5$, the condition

$$P_{\boldsymbol{\theta}}\{\text{no incorrect decision}\} \ge 1 - \alpha \quad \forall\; \boldsymbol{\theta} \in R^2 \tag{1}$$

requires the inclusion of a third decision about each θ_i. This decision must not be incorrect under any value of θ_i and is usually interpreted as "no decision" or "no classification." Generally, it is associated with low values of $|\hat{\theta}_i|$. By introducing

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[1] On leave from Tel Aviv University.

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the third decision, we are reducing the expected number of classifications to get a sufficiently high probability of no incorrect decision under all values of $\boldsymbol{\theta}$ (in particular the ones near $\mathbf{0}$).

We restrict our discussion to the case of two parameters with i.i.d. estimators. Some comments will be provided on the problems of extending the results to more general cases.

For the class of decision rules that satisfy (1), Bohrer and Schervish consider those procedures that are *symmetric* and *upper convex*. Symmetry of the decision rule can be expressed in terms of the following two implications:

$$\mathbf{D}(\hat{\theta}_1, \hat{\theta}_2) = (i, j) \Rightarrow \mathbf{D}(\hat{\theta}_2, \hat{\theta}_1) = (j, i), \tag{2}$$
$$\mathbf{D}(\hat{\theta}_1, \hat{\theta}_2) = (i, j) \Rightarrow \mathbf{D}(\pm\hat{\theta}_1, \pm\hat{\theta}_2) = (\pm i, \pm j),$$

where $\mathbf{D}(\hat{\boldsymbol{\theta}}) = (D_1, D_2)$ is the decision vector for the signs of θ_1 and θ_2 and each D_i takes on one of the values -1, 0, or 1 indicating the decision $\theta_i < 0$, no classification, and $\theta_i > 0$, respectively.

A rule is upper convex if whenever $\hat{\boldsymbol{\theta}}$ leads to making two classifications $\mathbf{D} = (\pm 1, \pm 1)$, then for all $c_1, c_2 \geq 1$, $(c_1\hat{\theta}_1, c_2\hat{\theta}_2)$ leads to the same two decisions.

Bohrer and Schervish (1980) define a *locally optimal* rule as a symmetric, upper convex rule satisfying (1) that maximizes the expected number of correct decisions in the limit as $\hat{\boldsymbol{\theta}}$ goes to $\mathbf{0}$. To identify such a rule they let

$$x_0 = p\{\mathbf{D} = (0,0) | \boldsymbol{\theta} = \mathbf{0}\},$$
$$x_1 = p\{\mathbf{D} = (0,1) | \boldsymbol{\theta} = \mathbf{0}\},$$
$$x_2 = p\{\mathbf{D} = (1,1) | \boldsymbol{\theta} = \mathbf{0}\}.$$

Under the symmetry conditions of (2), x_2 also represents the probability of each of the three decision profiles $\mathbf{D} = (1, -1)$, $\mathbf{D} = (-1, 1)$, and $\mathbf{D} = (-1, -1)$. Similarly, x_1 represents the probability of each of the decision profiles $\mathbf{D} = (-1, 0)$, $\mathbf{D} = (1, 0)$, and $\mathbf{D} = (0, -1)$. The restrictions placed on the decision rules imply the following linear relations:

From (1),

$$x_0 + 2x_1 + x_2 \geq 1 - \alpha; \tag{3}$$

from (1), upper convexity, and independence,

$$x_2 \leq \alpha^2; \tag{4}$$

and from probability theory,

$$x_0 + 4x_1 + 4x_2 = 1. \tag{5}$$

As $\boldsymbol{\theta} \to \mathbf{0}$, the limit of the expected number of correct decisions is given by

$$2x_1 + 4x_2. \tag{6}$$

The resulting problem is a linear program where (6) is maximized subject to (3), (4), (5) and $x_i \geq 0 \ \forall\ i$. The solution for $\alpha \leq \frac{1}{3}$ is

$$x_0 = 1 - 2\alpha + 2\alpha^2, \qquad x_1 = \frac{\alpha}{2} - \frac{3\alpha^2}{2}, \qquad x_2 = \alpha^2. \tag{7}$$

When $\frac{1}{3} \leq \alpha \leq \frac{3}{4}$, the solution is

$$x_0 = 1 - \frac{4\alpha}{3}, \qquad x_1 = 0, \qquad x_2 = \frac{\alpha}{3}, \tag{8}$$

and for $\frac{3}{4} \leq \alpha$,

$$x_0 = 0, \qquad x_1 = 0, \qquad x_2 = \tfrac{1}{4}. \tag{9}$$

Thus, any symmetric rule satisfying upper convexity and either (7), (8), or (9) is locally optimal. The case (7) is of main interest since in practice α is usually smaller than $\frac{1}{3}$. For the case when $\alpha \leq \frac{1}{3}$, Bohrer and Schervish (1980) identified a rule in the class of locally optimal rules called the *double cross*. In this paper we set up a criterion for selecting a best procedure from the class of locally optimal procedures and for (7) end up with a formal justification for the use of the double cross. Also, we examine the effects of the modification of the optimization criterion (6) on the class of decision rules.

There are some difficulties in extending our results to the case of dependence. The main problem is characterizing those procedures that are locally optimal and then controlling the probability of at least one error under all $\boldsymbol{\theta}$. This difficulty was already noted for the $k = 2$ dependent case by Bohrer and Schervish (1980). They showed that the double cross does not control the probability of any error under all $\boldsymbol{\theta}$ if the conditions for local optimality are satisfied.

Problems also occur for higher dimensions ($k > 2$). In the case of independence, a "double cross type" rule can be shown to be locally optimal under natural conditions of symmetry and upper convexity. However, again there is difficulty in establishing the required control of the probability of any error under all $\boldsymbol{\theta}$.

2. Why the double cross? When $\alpha \leq \frac{1}{3}$ the double cross depicted in Figure 1 is a locally optimal procedure. It has a $(0,0)$ region in the shape of a cross centered at the origin that is imbedded in a larger cross formed by the union of all one-classification regions and the $(0,0)$ region. z_α is the $1 - \alpha$ quantile of a standard normal variable and $z_d > z_\alpha$ is determined so that the probability of the shaded region in Figure 1 under $\boldsymbol{\theta} = \mathbf{0}$ is $1 - \alpha$. This procedure decides that θ_i has the sign of $\hat{\theta}_i$ if $|\hat{\theta}_i| > z_d$, or if $|\hat{\theta}_i| > z_\alpha$ and $|\hat{\theta}_{3-i}| > z_\alpha$. Bohrer and Schervish (1980) proved that it controls the probability of any error under all $\boldsymbol{\theta}$. However, they did not provide further explanation as to why this rule should be singled out from among all locally optimal rules. For instance, the locally optimal rule that controls the probability of any error and minimizes the area of the $(0,0)$ region is depicted in Figure 2.

In the following we provide some justification for choosing the double cross. Let L be the class of locally optimal procedures. When $\alpha \leq \frac{1}{3}$ the two-decision region of any procedure in L is the same as the two-decision region given in Figure 1, while the probability of the $(0,1)$ region under $\boldsymbol{\theta} = \mathbf{0}$ is $(\alpha - 3\alpha^2)/2$ (see (7)).

A possible criterion for selecting a procedure from L is to maximize the expected number of correct decisions at some $\boldsymbol{\theta} \neq \mathbf{0}$. Let $\boldsymbol{\theta} \geq \mathbf{0}$ indicate a vector $\boldsymbol{\theta}$

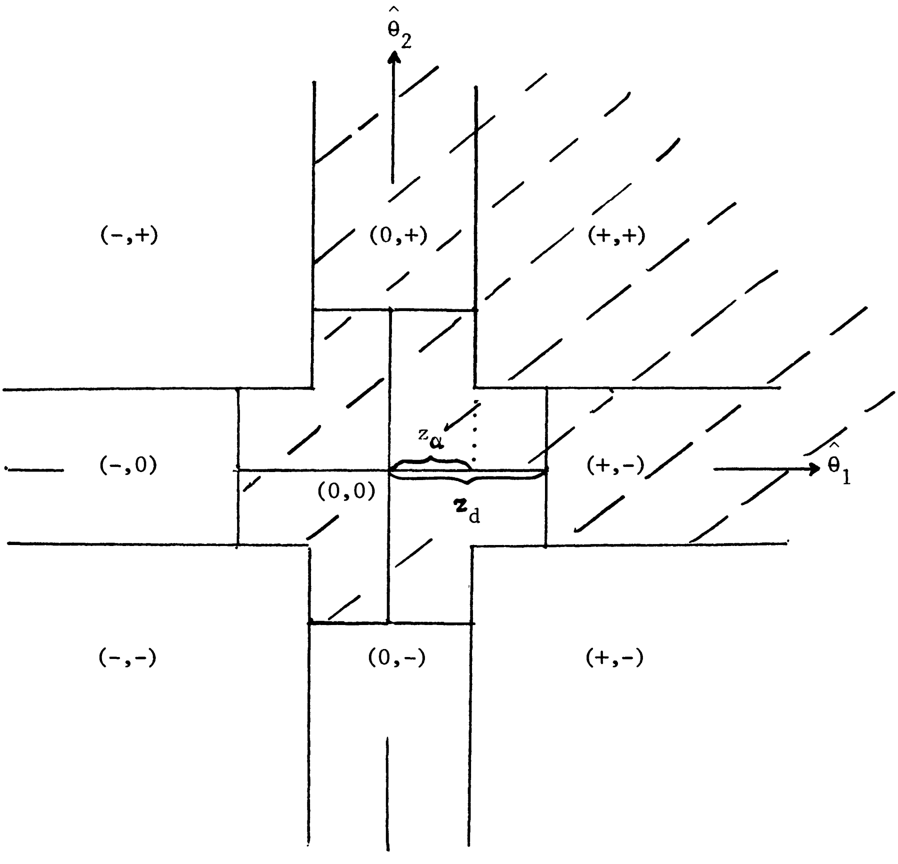

FIG. 1. *The double cross.*

with $\theta_i \geq 0$ for all i and $\theta_i > 0$ for at least one i. Also, let $T \equiv$ Positive Orthant $\cap$ $\{(0,1)$ region$\}$.

THEOREM 1. *The rule in L that maximizes the expected number of correct classifications at $\boldsymbol{\theta} \geq \mathbf{0}$ has a region T of the form*

$$T^* = \Big\{(\hat{\theta}_1, \hat{\theta}_2) \big| e^{\theta_2 \hat{\theta}_2} \cosh(\theta_1 \hat{\theta}_1) + e^{\theta_1 \hat{\theta}_2} \cosh(\theta_2 \hat{\theta}_1) \geq c(\alpha, \theta_1, \theta_2), 0 \leq \hat{\theta}_1 \leq z_\alpha, \hat{\theta}_1 \leq \hat{\theta}_2 \Big\}, \tag{10}$$

where $c(\alpha, \theta_1, \theta_2)$ is determined so that $p\{T^* | \boldsymbol{\theta} = \mathbf{0}\} = (\alpha - 3\alpha^2)/4$.

PROOF. A symmetric procedure in L that maximizes the expected number of correct decisions at $\boldsymbol{\theta} \geq \mathbf{0}$ has a $(0,1)$ region such that the probability of $\{(0,1)$ region$\} \cup \{(1,0)$ region$\}$ is maximal at $\boldsymbol{\theta}$. Since the $(0,1)$ region is symmetric about $\hat{\theta}_2 = 0$, if $(\hat{\theta}_1, \hat{\theta}_2) \in T$, then $(-\hat{\theta}_1, \hat{\theta}_2) \in \{(0,1)$ region$\}$. Further, $(\hat{\theta}_2, \hat{\theta}_1), (\hat{\theta}_2, -\hat{\theta}_1) \in \{(1,0)$ region$\}$. To find T^* a generalized Neyman–Pearson

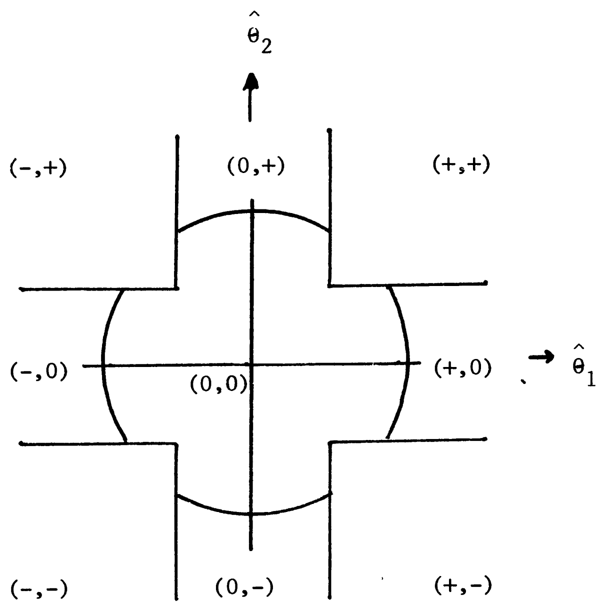

FIG. 2. *The locally optimal rule that minimizes area of (0, 0) region.*

lemma can be used. Accordingly, any point $(\hat{\theta}_1, \hat{\theta}_2)$ in the positive orthant for which the generalized likelihood ratio exceeds a critical constant $d(\alpha, \boldsymbol{\theta})$, i.e.,

$$\frac{f_{\boldsymbol{\theta}}(\hat{\theta}_1, \hat{\theta}_2) + f_{\boldsymbol{\theta}}(-\hat{\theta}_1, \hat{\theta}_2) + f_{\boldsymbol{\theta}}(\hat{\theta}_2, \hat{\theta}_1) + f_{\boldsymbol{\theta}}(\hat{\theta}_2, -\hat{\theta}_1)}{f_{\mathbf{0}}(\hat{\theta}_1, \hat{\theta}_2) + f_{\mathbf{0}}(-\hat{\theta}_1, \hat{\theta}_2) + f_{\mathbf{0}}(\hat{\theta}_2, \hat{\theta}_1) + f_{\mathbf{0}}(\hat{\theta}_2, -\hat{\theta}_1)} \geq d(\alpha, \boldsymbol{\theta}) \geq 0 \tag{11}$$

($f_{\boldsymbol{\theta}}(\hat{\theta}_1, \hat{\theta}_2)$ is the joint normal density of $(\hat{\theta}_1, \hat{\theta}_2)$ with mean $\boldsymbol{\theta}$) is in T^*. The numerator and denominator in (11) are the contributions to the likelihood of the $\{(0,1)$ region$\} \cup \{(1,0)$ region$\}$ by a point $(\hat{\theta}_1, \hat{\theta}_2)$ and the associated symmetric points $(-\hat{\theta}_1, \hat{\theta}_2), (\hat{\theta}_2, \hat{\theta}_1), (\hat{\theta}_2, -\hat{\theta}_1)$ under $\boldsymbol{\theta}$ and under $\mathbf{0}$, respectively. Substitution for $f_{\boldsymbol{\theta}}(\cdot, \cdot)$ in (11) gives

$$\frac{\exp - \frac{1}{2}(\theta_1^2 + \theta_2^2)}{4}\left[e^{\theta_2\hat{\theta}_2}(e^{\theta_1\hat{\theta}_1} + e^{-\theta_1\hat{\theta}_1}) + e^{\theta_1\hat{\theta}_2}(e^{\theta_2\hat{\theta}_1} + e^{-\theta_2\hat{\theta}_1})\right] \geq d(\alpha, \boldsymbol{\theta}).$$

By letting

$$c(\alpha, \theta_1, \theta_2) = \frac{4d(\alpha, \boldsymbol{\theta})}{\exp - \frac{1}{2}(\theta_1^2 + \theta_2^2)},$$

(10) follows. □

COROLLARY 1. *As* $\boldsymbol{\theta} \downarrow \mathbf{0}$, *the region defined by* (10) *converges to the* (0, 1) *region of the double cross.*

PROOF. Let $\{h(\hat{\theta}_1) | 0 \leq \hat{\theta}_1 \leq z_\alpha\}$ represent the boundary of T^* with the (0, 0) region. Therefore,

$$e^{\theta_2 h(\hat{\theta}_1)}\cosh(\theta_1\hat{\theta}_1) + e^{\theta_1 h(\hat{\theta}_1)}\cosh(\theta_2\hat{\theta}_1) = c(\alpha, \theta_1, \theta_2) \quad \text{for } 0 \leq \hat{\theta}_1 \leq z_\alpha. \tag{12}$$

Implicitly differentiating (12) with respect to $\hat{\theta}_1$ and then solving for $h'(\hat{\theta}_1)$ gives

$$h'(\hat{\theta}_1) = \frac{-\theta_1 e^{\theta_2 h(\hat{\theta}_1)} \sinh(\theta_1 \hat{\theta}_1) - \theta_2 e^{\theta_1 h(\hat{\theta}_1)} \sinh(\theta_2 \hat{\theta}_1)}{\theta_2 e^{\theta_2 h(\hat{\theta}_1)} \cosh(\theta_1 \hat{\theta}_1) + \theta_1 e^{\theta_1 h(\hat{\theta}_1)} \cosh(\theta_2 \hat{\theta}_1)}.$$

When $\boldsymbol{\theta} \to \mathbf{0}^+$, the limit of $h'(\hat{\theta}_1) \to 0$. This implies that the limiting form of $h(\cdot)$ is $h(\cdot) = \text{constant}$. □

This corollary implies that for any rule $r \in L$ where $r \neq$ double cross, there exists an $\varepsilon > 0$ such that

$$E_{\boldsymbol{\theta}}(\text{number of correct classifications under } r) < E_{\boldsymbol{\theta}}(\text{number of correct classifications under double cross}) \quad \text{for all } \|\boldsymbol{\theta}\| < \varepsilon.$$

Note that the double cross is also the rule in L that minimizes the maximal value of $|\hat{\theta}_i|$ for which no decision is made on θ_i.

Applying a similar approach to (8) for $\frac{1}{3} \leq \alpha \leq \frac{3}{4}$, gives the unique optimal procedure depicted in Figure 3. For (9), the case when $\frac{3}{4} \leq \alpha$, the optimal procedure is unique without additional restrictions to the problem.

3. Generalized optimization functions. The optimization function (6) used by Bohrer and Schervish (1980) is based on the number of correct decisions in the various decision regions. Thus, a region with one correct and one undecided classification has the same value as a region with one correct and one incorrect classification. This is somewhat unappealing as a region with one correct and one undecided classification under some circumstances might be more attractive. Also, we wonder why the doubly correct classification region should necessarily have twice the value of a one correct, one undecided region. More generally, are the relative values assigned to the different types of regions in the computation of (6) always justified?

These considerations lead us to postulate a more general objective function that is an arbitrary weighted combination of the probabilities of the various regions. Usually, the weight will increase in relation to the level of attractiveness. Due to symmetry, the probability under $\boldsymbol{\theta} = \mathbf{0}$ for each of the two classification regions is identical. Similarly, all one-classification regions have the same probability. Thus, it can be shown that for any $\alpha \in (0,1)$, and for any weighted combination of the probabilities of the regions, we can formulate an optimization problem. Since $x_0 = 1 - 4x_1 - 4x_2$ (see (5)) and $x_0 \geq 0$, the optimization problem can be formulated in terms of only x_1 and x_2 as follows:

$$\begin{aligned} &\text{maximize } ax_1 + bx_2, \\ &\text{subject to } 2x_1 + 3x_2 \leq \alpha, \\ &x_2 \leq \alpha^2, \\ &x_1 + x_2 \leq \tfrac{1}{4}, \\ &x_1, x_2 \geq 0, \end{aligned} \tag{13}$$

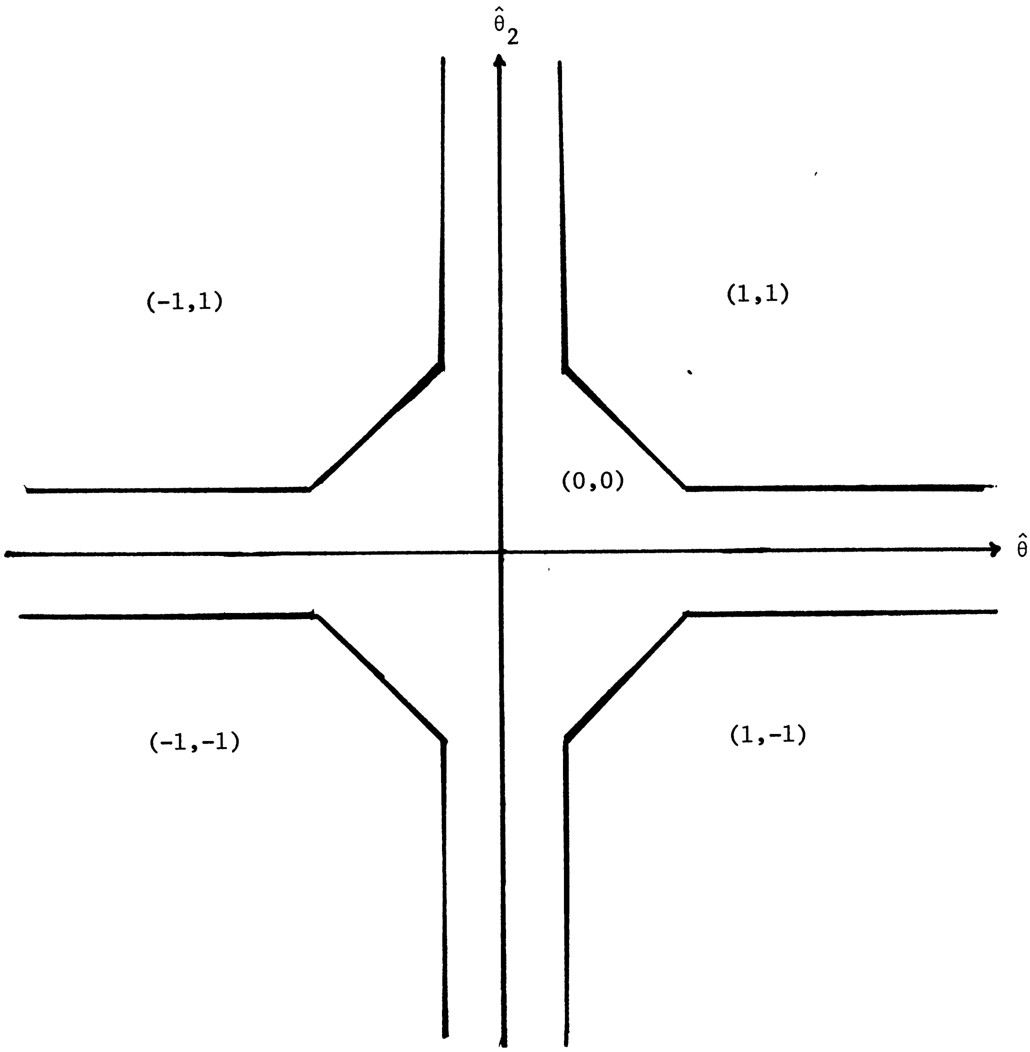

FIG. 3. *The optimal procedure for the solution (8).*

where a and b are constants obtained from the weights given to the regions. We assume that $a, b \geq 0$.

Note that the Bohrer and Schervish formulation is a special case of (13) with $a = 2,\ b = 4$. For $a = 0,\ b = 1$, the objective function becomes the (limiting) probability of making two correct decisions. When $a = 2,\ b = 3$, we are optimizing the (limiting) probability of making at least one correct decision.

Whenever $a/b < \frac{2}{3}$, we get locally optimal conditions identical to (7), (8), and (9). The discussion in Section 2 suggests good choices for the optimal rule. For $a/b > \frac{2}{3}$ and $\alpha \leq \frac{1}{2}$, the solution to (13) is $x_1 = \alpha/2,\ x_2 = 0$. By direct application of the limiting arguments given in the prior section we have the

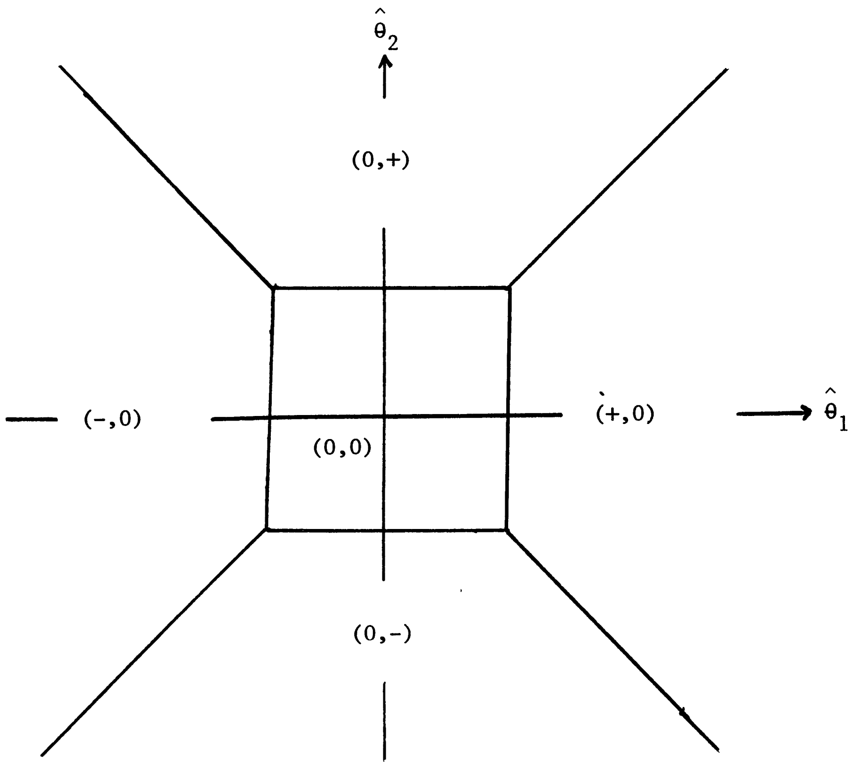

FIG. 4. *An optimal rule when $a / b \geq \frac{2}{3}$ and $\alpha \leq \frac{1}{2}$.*

unique rule that is depicted in Figure 4. For $1 > a/b > \frac{2}{3}$ and $\frac{1}{2} \leq \alpha \leq \frac{3}{4}$, the solution is $x_1 = \frac{3}{4} - \alpha$, $x_2 = \alpha - \frac{1}{2}$ and the double cross can be used as the optimal rule. When $\frac{3}{4} \leq \alpha$ the solution $x_1 = 0$, $x_2 = \frac{1}{4}$ yields a unique optimal rule.

Procedures with no two-decision region (such as the one shown in Figure 4) seem unappealing. It is hard to accept a rule that does not classify θ_i for large values of $\hat{\theta}_i$. Nevertheless, such procedures are optimal for the given set of constraints and objective function. One approach to circumvent this situation is by adding some restrictions to L. Notice that upper convexity is only defined for the two-decision region. It seems logical to extend the upper convexity requirement to the one-decision region. This implies that if $\hat{\boldsymbol{\theta}}$ leads to $D_i = \pm 1$, then for all $c_1, c_2 \geq 1$, $(c_1\hat{\theta}_1, c_2\hat{\theta}_2)$ leads to the same D_i. Under this generalized upper convexity requirement we have the following result.

THEOREM 2. *There is a unique symmetric and generalized upper convex rule that maximizes* (13) *for* $1 > a/b > \frac{2}{3}$ *and* $\alpha < \frac{3}{4}$, *which satisfies* (1). *For this rule, the* $(0,0)$ *region is a square* $|\hat{\theta}_i| \leq K$ *and the two-decision regions are of the form* $\{(\hat{\theta}_1, \hat{\theta}_2) | |\hat{\theta}_i| > K, \ i = 1,2\}$ *where* K *is the* $(1 - \alpha)^{1/2}$ *percentile of a standard normal variate.*

PROOF. The $\{(0,0) \text{ region}\} \cap \{(1,1) \text{ region}\} \neq \{\phi\}$. Otherwise, there exists a point $(\hat{\theta}_1, \hat{\theta}_2) \in \{(0,1) \text{ region}\}$ such that for some $c > 1$, $(c\hat{\theta}_1, \hat{\theta}_2) \in \{(1,0) \text{ region}\}$. This would violate generalized upper convexity for D_2. Suppose that the intersection of the $(0,0)$ and $(1,1)$ decision regions consists of more than 1 point. Since $x_1 + x_2 = \frac{1}{1}$ is not a binding constraint, there exists an $\varepsilon > 0$ such that x_2 can be reduced by ε, x_1 increased by $\frac{3}{2}\varepsilon$, and the solution remains feasible. As this new solution has a higher value, the original one is not optimal for (13). Consequently, every optimal rule must have the $(0,0)$ and $(1,1)$ regions intersect at precisely one point.

To satisfy upper convexity, the $(1,1)$ decision region must contain the set $S = \{(\hat{\theta}_1, \hat{\theta}_2) | \min\{\hat{\theta}_1, \hat{\theta}_2\} > K\}$ for some $K > 0$. If S is a proper subset of the $(1,1)$ region, x_2 can be reduced and x_1 increased as above. Thus, in an optimal rule the $(1,1)$ decision region is of the form $\{(\hat{\theta}_1, \hat{\theta}_2) | \min\{\hat{\theta}_1, \hat{\theta}_2\} > K > 0\}$.

Suppose the $(0,0)$ region is not a square. By rearranging the areas (keeping x_1 and x_2 constant), we can find an equivalent rule where the $(0,0)$ and $(1,1)$ regions intersect at more than one point. By a prior argument, x_1 and x_2 are not optimal for (13). As a result, the $(0,0)$ region is square.

Now, for some $K > 0$, the optimal solution can be written as $x_1(K) = [\Phi(K) - \Phi(-K)][1 - \Phi(K)]$, $x_2 = [1 - \Phi(K)]^2$. Let $y = \Phi(K)$. The objective

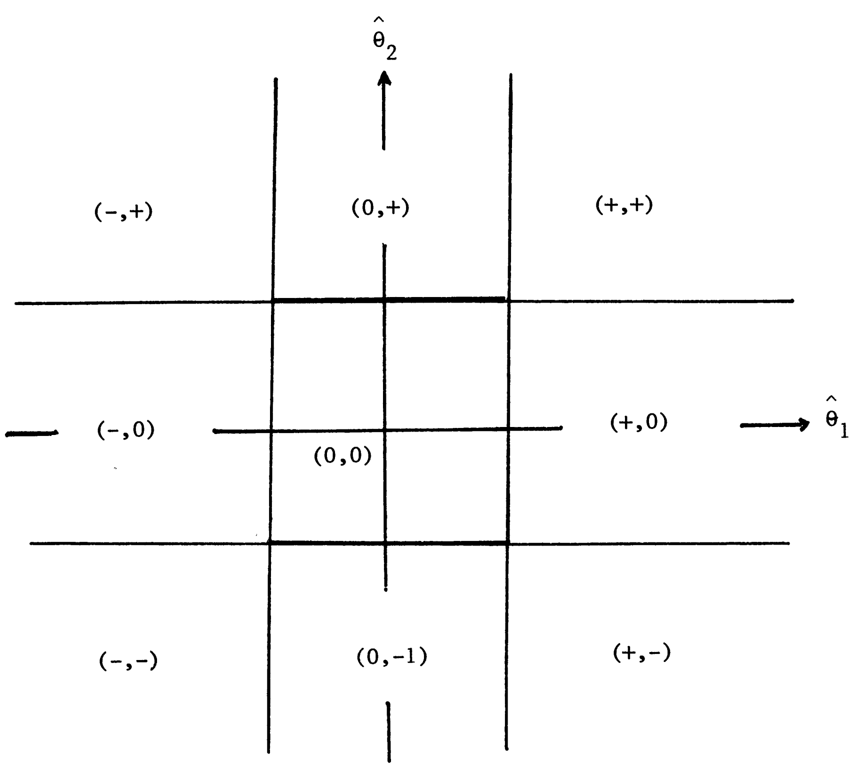

FIG. 5. *The Sp cross.*

function of (13) becomes $\psi(y) = a(2y - 1)(1 - y) + b(1 - y)^2$. Since $0 < K < \infty$, $0.50 < y < 1$. $\psi'(y) < 0$ for $y \in (0.50, 1)$ if $a/b \leq 1$. Hence, $\psi(y)$ is monotone decreasing and attains the maximum value on the boundary of the constraint set. Since the constraints $2x_1 + 3x_2 \leq \alpha$ and $x_2 \leq \alpha^2$ are equivalent to $y \geq (1 - \alpha)^{1/2}$ and $y \geq 1 - \alpha$, respectively, the optimal solution has $\Phi(K) = (1 - \alpha)^{1/2}$. This completes the proof. □

The optimal rule according to Theorem 2 is depicted in Figure 5. This rule is a well recognized procedure that is discussed by Bohrer (1979) and Bohrer and Schervish (1980). It has been named the Sp cross and was originally discussed by Spjøtvoll (1972) for a slightly different problem.

We have omitted the case of $a/b = \frac{2}{3}$ from our discussion. When $\alpha < \frac{3}{4}$, the optimal solution to this case will not be unique. Thus, examination of additional criteria will be required to fix x_1 and x_2. We leave this to future research.

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GRADUATE SCHOOL OF BUSINESS ADMINISTRATION
NEW YORK UNIVERSITY
NEW YORK, NEW YORK 10006