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# JOHN W. TUKEY'S WORK ON INTERACTIVE GRAPHICS

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If there ever was a tool that could stimulate the imagination and profit from the intuition and creativity of John Tukey, it was computer graphics. John always saw graphics a being central to exploratory data analysis: "Since the aim of exploratory data analysis is to learn what seems to be, it should be no surprise that pictures play a vital role in doing it well. There is nothing better than a picture for making you think of questions you had forgotten to ask (even mentally)." Much of his work focused on static displays designed to be easily drawn by hand, but he realized that if one wanted to effectively explore multivariate data, computer graphics would be an ideal tool. PRIM-9, the first program to use interactive, dynamic graphics for viewing and dissecting multivariate data, was conceived by John during a four month visit to the Computation Research Group of the Stanford Linear Accelerator Center in early 1972. PRIM-9 opened up a fundamentally new way of exploring multivariate data. Its basic operations—Picturing, Rotation, Isolation and Masking—have stood the test of time and form the core of numerous follow-on systems.

John's experiences with PRIM-9 gave rise to a slew of other ideas for the analysis of multivariate data, many of them not tied to interactive graphics. The most well known of those is "Projection Pursuit"—automatically finding interesting low-dimensional projections of multivariate data by optimizing a projection index. John was also keenly interested in ways of detecting and modeling nonlinear structures in multivariate data which might not be manifest in projections, such as concentration of data near nonlinear lower dimensional manifolds. Many of his proposals exist only in the form of hand-written notes and appear "far out" even today.

John's work on Prim-9 and Projection Pursuit lent respectability to computationally oriented, non mathematical research in Statistics. He moved the center of gravity away from an (over)emphasis on mathematical theory to a greater balance between methodology, theory, and applications and thereby helped revitalize the discipline of Statistics.

**1. Introduction.** If there ever was a tool that could stimulate the imagination and profit from the intuition and creativity of John Tukey, it was computer graphics. John always saw graphics as being central to exploratory data analysis. As he said in Tukey and Tukey (1985):

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PRIM-9, the first program to use interactive, dynamic graphics for viewing and dissecting multivariate data, was conceived by John during a four month visit to the Computation Research Group of the Stanford Linear Accelerator Center (SLAC) in early 1972. The visit had been arranged by William Miller, the creator and first leader of the Computation Research Group and a Professor of Computer Science and later Provost of Stanford University and then President of SRI International. At the time, interactive computer graphics was still a fairly new technology, and Miller was interested in research aimed at exploring its general possibilities. He supervised the construction of a "Graphics Interpretation Facility (GIF)" at SLAC. The GIF cost several million (1970) dollars, it was cumbersome to program, its performance was very limited by today's standards, and it was despised by many members of the SLAC community because it consumed a significant part of the available computing resources whenever it was used. However, it was one of the few interactive graphics facilities at the time and thus offered unique research opportunities. To create a record of work on the GIF, Miller acquired a complete suite of equipment for shooting and producing 16 mm sound movies, including an Arriflex camera and a professional cutting machine. Thus was born "Bin-88 Productions" that produced the PRIM-9 movie featuring John as the principal actor [Fisherkeller, Friedman and Tukey (1974a)], as well as later movies by John McDonald [McDonald, Friedman and Stuetzle (1982a, b)] and Andreas Buja [Buja (1983)].

By coincidence, 1972, the year of John's sabbatical at SLAC, was also the year when JHF took over leadership of the Computation Research Group from William Miller. This began a ten year collaboration on research in graphics and multivariate analysis between John and the SLAC Computation Research Group. John paid repeated short visits and spent the Winter of 1979 at SLAC. By that time the GIF had become technically outdated, and JHF and WS (then an Assistant Professor in the Stanford Statistics Department and a staff member in the Computation Research Group) designed and built ORION-1, a follow-on system which anticipated many of the features of today's workstations. Its core was one of the first wire-wrapped SUN boards built by Andreas Bechtolsheim, who was then a computer science graduate student at Stanford and went on to found SUN Microsystems. Like today's workstations it had a bit-mapped color display (then called a "frame buffer"). ORION-1 was hard to program, but it was a stand-alone system and thus could be used without aggravating the world at large. The user interface consisted of a tracker ball and a few switches; it was this hardware for which John Tukey designed PRIM-81 [Tukey (1982)]. Only few of the ideas for PRIM-81 were ever implemented.

John's work on PRIM-9 gave rise to a slew of other ideas for the analysis of multivariate data, many of them not tied to interactive graphics. The most well known of these is "Projection Pursuit"—automatically finding interesting low-dimensional projections of multivariate data by optimizing a projection index. John was also keenly interested in ways of detecting and modeling nonlinear structures in multivariate data which might not be manifest in projections, such as concentration of data near nonlinear lower dimensional manifolds. Many of his proposals exist only in the form of hand-written notes and appear "far out" even today.

**2. PRIM-9.** It would be hard to give a more concise verbal description of PRIM-9 than the one provided in the introduction to Fisherkeller, Friedman and Tukey (1974b):

> PRIM-9 is an interactive computer graphics program for Picturing, Rotation, Isolation, and Masking—in up to 9 dimensions.... PRIM-9 has been developed towards ends of very different breadth and distance: first, to gain insight into what can be learned by looking at the numerical aspects of data in more than two aspects at a time and, second, to implement a tool for pictorial examination and dissection of multivariate data. The development of PRIM-9 has grown through many stages, and many of the techniques that were implemented and later discarded may turn out to be central to other data display systems. The resulting system as it is currently implemented is especially straight-forward in concept. Its emphasis is on picturing and rotation on one hand and masking and isolation on the other. Picturing means an ability to look at data from several different directions in the multidimensional space. Rotation means, as a minimum, the ability to turn the data so that it can be viewed from any direction that is chosen. Picturing and Rotation are essential abilities, valuable alone, but their usefulness is greatly enhanced when combined with Masking and Isolation. Masking is the ability to select suitable subregions of the multidimensional space for consideration. That is, only those data points that lie in the subregion are displayed. Isolation is the ability to select any subsample of the data points for consideration. That is, only those points in the selected subsample are displayed. It is important to note that masking is tied to coordinates. If we rotate the data points, different points will enter and leave the masked region. Isolation, on the other hand, is tied to data points. Under all operations of the system (except further isolation), the isolated sample remains the same. In the absence of rotation, masking and isolation are equivalent; however, in the presence of rotation the distinction is essential.
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> By interactively applying picturing, rotation, isolation, and masking to his data, the user can, in particular, perform projection pursuit. That is, he can look for those views that display to him interesting structure. He can isolate structures so found and study them separately and/or remove them from the remaining sample, simplifying the search for still further structure.

Picturing, Rotation and Isolation have remained mainstays of all the descendants of PRIM-9. The use of point cloud rotation in PRIM-9 was particularly ingenious. It was not just a visual gimmick allowing a user to perceive three dimensions instead of two. Rather, it was a clever way of addressing a fundamental problem in visualizing multivariate data:

> Being able to see projections on all coordinate pairs can be very useful. But it is not enough. To be able to get reasonably close to any two-dimensional projection means

> either a way to call for the projection that we want, or a way to move about in the multi-dimensional space. Since we usually do not know just what we want, and when we do we will find it difficult to call for it in a general way, we need a way to move about.

The solution to this problem implemented in PRIM-9 is elegant in its simplicity:

> Continuous controlled rotation, is a natural way to move about (change projection) in a multidimensional space .... In PRIM-9, directly available rotations are confined to those associated with pairs of *current* coordinates. This makes both understanding and control reasonably easy. A rotation is specified by two integers $i$ and $j$. These integers specify the current coordinate axes that participate in the rotation. ... if the axes that define the rotation coincide with the current projection axes, then the data points will simply move in circular orbits about their relative mean in the projection. If neither axis corresponds to a current projection axis, then the display will remain unchanged .... Useful rotations occur when one of the axes that participates in the rotation corresponds to a current projection axis, while the other one is one of the NDIM-2 axes orthogonal to the current display plane. In this case, the operator sees the projected points change their positions on the screen as the invisible coordinate is rotated against the visible one. When an interesting patter emerges, the operator can home in on it .... The invisible coordinate can then either be rotated against the other visible one, or the operator can change to another visible coordinate. He can then rotate these new coordinates to try to sharpen the structure even further. Continuing in this manner, the operator may manually iterate to a data orientation that provides an informative view of the multidimensional point cloud.
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> Easily recognizable continuous rotation provides an additional advantage. Its dynamic effects let one see an additional dimension, not instantaneously, but yet at the same time—in the best sense of those words. This is due to the relative motions of the points associated with parallax .... This parallax effect gives the illusion of a third dimension ....

Of course, finding interesting views is not enough. We also need the ability to describe whatever structure we find in terms of the original variables. Here again, PRIM-9 offers a simple solution.

> To help in the interpretation of a particular projection, a display of the initial coordinate axes, as projected onto the current projection plane, is easily switched in and out of view.

While Picturing, Rotation and Isolation have withstood the test of time, this is less so with Masking. Masking can be regarded as a precursor to "painting" or "scatterplot brushing" in that it allowed a user to see associations between more than two or three variables. By sliding a window across the range of a variable not currently shown on the screen, and watching points on the screen appear and disappear, a user could detect associations between the hidden variable and the visible variables. PRIM-9 allowed for the creation of more complex masks, like John's "concealed generalized episcotister." The corresponding unmasked hypervolume was a hexadecant in four dimensions, and rotating these coordinates among themselves causes a hyperconical region to rotate in the four-dimensional projection of the points. If this sounds complicated—it is. John was proud of this invention and it seemed to serve his intuition well. Nobody else could ever quite get the hang of it.

The user interacted with PRIM-9 through an array of 32 buttons and a light pen. The desktop metaphor that came to dominate user interface design, with its multiple windows, pulldown menus, icons, and mouse input, had not emerged yet—the first modern workstation, the Xerox Alto, was not unveiled till 1974. It is not surprising that much of the PRIM-9 user interface now seems antiquated. It did, however, contain some clever ideas in which John took great pride, such as his single button "whoa–back" control used to drive rotation angles. This control was characterized by two features:

1. Rotation reversal: rotation in one sense as long as the single button is held down. When the same button is released and then again depressed, rotation occurs in the opposite direction so long as it is held down.
2. Acceleration: every time the button is pressed and held down the motion starts out slowly, then accelerates until a speed limit is reached.

This combination of rotation reversal and acceleration on a single button gives the operator fast and easy approach to a desired data orientation without having to take his or her eyes off the screen. Anyone who resets the time on a digital watch can appreciate the advantage of this design compared to the naive "one direction, constant speed" alternative.

Working closely with Mary Anne Fisherkeller, John developed PRIM-9 in only four months during his stay at SLAC. This was a remarkable feat under any circumstance. It is especially remarkable when one considers that none of its particular features was in his mind when he arrived. His initial vision of how to proceed was very different than what finally emerged. As he repeatedly emphasized,

> Pictorial systems to be effective must, as PRIM-9 did, go through many stages of trial and error learning.

He felt that the most important factor leading to success was Mary Anne's ability to very quickly implement his ideas, which were changing daily, so that he could rapidly evaluate them and discard those that did not seem to be effective.

From his experience with PRIM-9, John learned that, *The details of control can make or break such a system*. This interest in operator control reached its zenith nine years later with his design of PRIM-81 [Tukey (1982)]. In this design document he addressed a problem that is familiar to all users of complex programs with graphical, as opposed to textual, user interface. It is easy to get lost—how did I get here?—and it is often desirable to go back to an earlier state, either because the current path of actions has led to a dead end, or to explore a different path that was temporarily set aside [Tukey (1982)]:

> Later parts discuss the record-keeping aspects of a successive-modification display system, particularly in their impact upon the user through required control and status displays. A record-keeping philosophy (stash philosophy) is outlined for a data-view-modification system in which (display or) potential display moves along a—probably branched—trail, with certain points being marked (as nodes). The marked points are those displays (or potential displays to which it seems most likely that the user will return—either to look again, or to start a new branch of the trail.

In addition to its innovative control and stash philosophy, other aspects of PRIM-81 deserve mention here. John was always looking for data transformations, in addition to rigid rotation, that would go well with dynamic graphical display. One of his favorites was "twisting." One views a scatter plot on two current coordinates $(x_1', x_2')$. Each data point in the plot is then separately rotated about the center by an amount that is governed by its value on a third current coordinate $x_3'$ perpendicular to the screen. By viewing the changing pattern on the screen as the twisting continues, one could choose a particular twist that most sharpened the point scatter. Twisting was probably motivated by the specific structure of the particle physics data shown in the PRIM-9 movie. Like most of the ideas for PRIM-81 it was never implemented. Nevertheless, the design of PRIM-81 clearly shows the work of a clever, original and unorthodox mind in action.

**3. Projection pursuit.** Projection pursuit [Friedman and Tukey (1974)] was intended as a adjunct to PRIM-9. Human patience tends to run out when nothing much seems to be happening, and experience with PRIM-9 showed that users tended to tire and become frustrated when progress in finding new interesting views became slow.

> As an aid to this process, the present version of PRIM-9 also provides an automatic projection pursuit algorithm. This algorithm assigns to each view a numerical index that has been found to closely correspond to the degree of data structuring in the projection. When invoked, the algorithm will search for those views of the multidimensional data that maximize this projection index. At any point in the interactive session the user may invoke the automatic projection pursuit algorithm. Starting at the current view, the algorithm will find the view corresponding to the first maximum of the projection index, uphill from the starting view. Manual projection pursuit can then continue from this new (hopefully more structured) view. The algorithm can also be invoked at the beginning of the session starting with the various original or principal axes of the data. The resulting views can then provide useful starting points for further investigation.
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> [Fisherkeller, Friedman and Tukey (1974b)]

It became apparent later that projection pursuit could be a useful data analytic procedure on its own and did not necessarily require a powerful interactive graphics system to profitably apply it. It developed a life of its own. JHF and WS extended the original idea of "exploratory projection pursuit" into a general paradigm for modeling multivariate data [Friedman and Stuetzle (1982)]. They applied the paradigm first to regression, inventing a precursor to neural networks

[Friedman and Stuetzle (1981)], and later to density estimation [Friedman, Stuetzle and Schroeder (1984)]. Huber (1985) cast exploratory projection pursuit as an estimation method and interpreted the Friedman–Tukey index as a kind of entropy measure. This led to the development of alternative projection indices that were more robust, faster to compute, or targeted at detecting different aspects of the marginal density in the projection plane. John himself had early on made suggestions in this direction. The original projection index contrasted the "small" (many small interpoint distances) with the"large" (large overall scale). He developed other indices to contrast the middle with the small, and the middle with the large. These suggestions have yet to see implementation.

Overall, John was less than totally sympathetic to the "formalization" of projection pursuit. He felt that this distracted from what he viewed as its primary goal, to uncover interesting views of the data.

**4. Other ideas in multivariate analysis.** PRIM-9/PRIM-81 and projection pursuit did not come close to exhausting John Tukey's ideas for exploring multivariate data. Mostly in collaboration with Paul Tukey, he proposed a plethora of different types of data displays that did not require elaborate interactive graphics hardware [see Tukey and Tukey (1981, 1982)]. One of his earliest suggestions he called "draftman's views," which are now known as "scatterplot matrices" and are routinely used to visualize multivariate data. Another one of John's early proposals, "casement displays," evolved into the elegant and widely used "trellis graphics" technique [Becker, Cleveland and Shyu (1986)].

John was an early and strong proponent of what he called "cognostics" [Tukey (1982), Tukey and Tukey (1985)]. These are diagnostics computed from the data that are intended for computer rather than human consumption. With high dimensional data the number of potentially useful displays is large. Human patience and attention span being limited, it is unlikely or impossible that even a small fraction of them will be viewed, thereby possibly missing some that are especially informative. The computer has no such limitations. It can examine a very large number of cognostics and rank them as to their potential usefulness. The user can then manually examine the relatively few judged by the computer to be most revealing. In analogy with projection pursuit, the idea was to assign a numeric importance "index" to each cognostic and let the computer evaluate and rank them.

A special case of cognostics was "scagnostics," where the diagnostic displays were scatterplots on every pair of variables [Tukey and Tukey (1985)]. Draftman's views (scatterplot matrices) lose their effectiveness when the number of variables is large. Using a projection index similar to that in projection pursuit, the computer could find the most "interesting" scatterplots to be presented to the user. John had proposals for a wide variety of scagnostic indices to judge the usefulness of scatterplot displays. The widespread use of cognostics and scagnostics has not yet materialized in routine data analysis. These approaches are perhaps among the potentially most useful of John's yet to be explored suggestions.

Another problem that attracted John's interest was analysis of data concentrated near nonlinear manifolds [Friedman, Tukey and Tukey (1980)]. Like "twisting" this was partly motivated by properties of particle physics data. However, John's motivation went beyond that:

> We are wholly ignorant of the extent to which such techniques will be helpful in studying, empirically, other kinds of data in four or more dimensions. We have, so far, only had multi-response techniques of a few kinds, particularly: (1) those that assumed the point cloud consists of ellipsoidal blurs; (2) those, like multiple regression, that assumed the point cloud is close to a manifold that is easily described in terms of carriers (given functions of the coordinates); (3) those, like simple-structure factor analysis, that assume the point cloud would be concentrated near a simple arrangement of hyperplanes. Without a way to look for other kinds of structure than such conventional instances, we are unlikely to know whether or not other kinds of structure are (a) never present, (b) occasional, or (c) frequent. Thus flexible methods of analyzing multiresponse data, which are guided by the particle physics picture, may—or may not—prove to be very useful in other areas of inquiry.

The paper sketches two approaches for detecting and modeling nonlinear manifolds, one based on recursive partitioning, and one based on neighborhood growing. The basic idea of the first approach is to find a partition of the data space into two half spaces such that the corresponding subsets of the data are as flat as possible. The algorithm is then recursively applied to the two subsets. The idea of the second approach is to choose a seed point and grow a neighborhood in which the data are flat. New seed points are then chosen at the extremes of the neighborhood, and the process is repeated.

While his surroundings were still trying to come to grips with John's ideas for modeling nonlinear manifolds, he was already forging ahead: he wanted to model more general, nonmanifold structures, such as two intersecting nonlinear manifolds of co-dimension 1 ("monocrepes" in John's terminology) [Tukey (1979)]:

> I suggest that as a touchstone for any stepping process that it be able to deal (at least with reasonable probability) with two intersecting singly-rolled monocrepes .... This means, in particular, a good chance of passing through the intersection while staying on the same crepe.

During his 1979 visit to SLAC John produced a copious amount of handwritten notes on the subject which attest to his strong geometric intuition and his total lack of fear of venturing far into the unconventional and unknown.

**5. Conclusion.** No verbal description, however eloquent, can do justice to an interactive graphics system like PRIM-9. Even after 30 years, watching the PRIM-9 movie is still a captivating experience. Seeing John in action and hearing him provide a running commentary on his analysis of a set of particle physics data and explain some of the ideas that went into the design of PRIM-9 provides a fascinating glimpse of his personality and his thought processes.

To fully appreciate the revolutionary nature of PRIM-9 one has to view it against the backdrop of its time. When Statistics was widely taken to be synonymous with inference and hypotheses testing, PRIM-9 was a purely descriptive instrument designed for data exploration. When statistics research meant research in statistical theory, employing the tools of mathematics, the research content of PRIM-9 was in the area of computer-human interfaces, drawing on tools from computer science. When the product of statistical research was theorems published in journals, PRIM-9 was a program documented in a movie.

For those of us who have done research on multivariate graphics, revisiting PRIM-9 can be a humbling experience—many, if not most of the ideas behind today's descendants of PRIM-9 were already present in their progenitor. PRIM-9 opened up a fundamentally new way of exploring multivariate data. All the later systems were evolutionary improvements and enhancements.

PRIM-9 has had a significant impact on the practice of data analysis in that it spawned a sequence of follow-up systems culminating in XGobi [Swayne, Cook and Buja (1998)]. In the first five months of 2001 the number of distinct IP addresses that downloaded XGobi from the AT&T website alone was over 900. This does not count downloads from other sites and shipments as part of some Linux distributions.

More importantly, John's work on PRIM-9 and Projection Pursuit lent respectability to computationally oriented, nonmathematical research in Statistics. John clearly was an accomplished mathematician and a widely respected scientist, and his work could not simply be dismissed as the feeble product of an intellectual lightweight. He moved the center of gravity away from an (over)emphasis on theory to a greater balance between methodology, theory, and applications and thereby revitalized the discipline of Statistics.

Working with John was a remarkable experience. He had the rare ability to think like a mathematician or like an engineer, depending on what the problem at hand required. His creativity was amazing and not constrained by conventions or by a fear of mistakes [Tukey (1972), quoting Charles Babbage]:

> Some of my critics have amused their readers with the wildness of the schemes I have occasionally thrown out; and I myself have sometimes smiled along with them. Perhaps it were wiser for present reputation to offer nothing but profoundly meditated plans, but I do not think knowledge will be most advanced by that course; such sparks may kindle the energies of other minds more favorably circumstanced for pursuing the inquiries.

Given his track record and status, John could obviously have built himself an empire, had he chosen to do so. However, he chose not to. He found joy in ideas and in solving technical problems, and not in the trappings of power and prestige.

On a personal level interacting with John confirmed the belief that truly great men (or women) see no need to be pretentious or go around beating their own drum. He was extremely modest in his personal habits, friendly, unassuming, and respectful toward anybody he interacted with, independent of their status. He was generous with ideas and advice. It is these personal traits as well as his professional accomplishments that have made John a role model for many of us whose lives he has touched.

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