# THE ANNALS of PROBABILITY

## AN OFFICIAL JOURNAL OF THE INSTITUTE OF MATHEMATICAL STATISTICS

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The Annals of Probability, Volume 6, Number 5, October 1978. Published bimonthly in February, April, June, August, October, and December by The Institute of Mathematical Statistics. IMS Business Office, 3401 Investment Blvd., Suite 6, Hayward, California 94545.

Mail to the Annals of Probabilty should be addressed to either the Editor, Managing Editor or the Treasurer, as described above. It should not be addressed to Waverly Press.

Printed at the WAVERLY PRESS, Inc., Baltimore, Maryland 21202 U.S.A.

Second-class postage at Hayward, California and at additional mailing offices

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Submission of Reference Papers. Copies (preferably two) of unpublished or not easily available papers cited in the manuscript should be submitted with the manuscript.

Title and Abbreviated Title. The title should be descriptive and as concise as is feasible, i.e., it should indicate the topic of the paper as clearly as possible, but every word in it should be pertinent. An abbreviated title to be used as a running head is also required, and should be given below the main title. This should normally not exceed 35 characters. For example, a title might be "A Limit Theorem for Conditioned Recurrent Random Walk Attracted to a Stable Law," with the running head "Limit Theorem for Recurrent Random Walk" or possibly "Recurrent Random Walk Attracted to a Stable Law," depending on the emphasis to be conveyed.

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