A STATISTICAL APPROACH TO MATHEMATICAL FORMULATION OF DEMAND-SUPPLY-PRICE RELATIONSHIPS

By
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A scientific approach to the practical problem of forecasting the prices of commodities clearly requires the development of methods of a somewhat mathematical type for analyzing the relationships between demand, supply, cost, and price. In the case of cotton and other annual crop agricultural commodities, the multiple correlation, link-relative and trend-ratio methods as applied by Moore, Schultz, B. B. Smith, Ezekiel, Holbrook and E. J. Working, and others, have demonstrated their worth. But for copper, lead, rubber, and similar commodities not on an annual crop basis, where quantity produced, or the quantity available, in a given period cannot logically be considered as the supply linked to the average price of that period, the method seems inapplicable and another type of approach is necessary. For this reason, and because of the failure of price to function as expected as a major regulator in our present money economy, it seems worth while to attempt to develop a general mathematical procedure involving cost, demand and supply functions and to analyze elasticity of supply and elasticity of demand as a mathematician naturally does. But some mathematical studies along these lines have not seemed to represent a truly scientific approach to the problem, however helpful they may be in suggesting potentially valuable ideas.

The mathematical economist of the non-statistical type sometimes seems to believe that he has contributed to the solution of economic problems if he finds an answer in the form of a mathematical equation with undetermined constants. The determination of these constants is left as a secondary step to the statistician. But in certain cases, at least, the determination of these constants would be at least as difficult as the original problem. Moreover, the assumptions made in setting up the equations, while not always stated explicitly, have not been shown to constitute a sufficiently close aproximation to actual non-exceptional conditions for the analysis to be useful. It may be suggested that, according to correct scientific procedure, careful statistical analysis of known and knowable conditions is necessary *before* worth while mathematical formulation can be attempted.

To bring out my point of view by discussion of a particular equation, Professor Evans, in his "Introduction to Mathematical Economics," uses as a cost function, that is to say, a function stating total cost for goods produced in a given unit of time, in terms of the quantity produced, u,

$$q(u) = Au^2 + Bu + C$$

This formula involves several unstated assumptions:

(a) That the same continuous formula will apply over the entire range of u which may appropriately be considered in discussing a particular problem. Actually, several points of discontinuity would be more normal. In fact the natural method of statement of this relation might be merely a number of discrete points rather than a function defined for all values of u. For instance, under the conditions of a particular problem, increase in production might be accomplished solely by increasing the number of machines engaged in that process. If so, any rate of output other than an integral multiple of the normal output of the machine might represent inefficient operation and therefore be barred from further discussion. More generally, I should expect a very high unit cost for low values of u, when production is on a job basis, with abrupt downward steps as quantity production methods are applied, and finally a nearly horizontal line.

(b) That a term of the type Au^2 is worth considering in the typical case. Some analysis seems called for as to when this term is appropriate. In mining and agriculture, some such increase of cost with increasing quantity undoubtedly does occur, but it is difficult to imagine practical cases of factory production in which unit cost increases with quantity at any such rate if advance notice of contemplated increase has been given. If production beyond present normal plant capacity is desired, such forced production might involve some temporary increase in unit cost as production increased, but in these days such a case would be unusual.

(c) That the variation of unit cost with quantity is important enough to justify singling that out as the single factor of variation, although actually my impression is that for rather broad ranges of quantity, other factors are more significant causes of variation in costs. Such factors include regularity of production rate, the weather, labor conditions, character of supervision, and management pressure reflecting price conditions for the product.

(d) That $\frac{C}{u}$ is an adequate expression for the element of unit cost which decreases as u increases. Accounting discussions of total cost usually emphasize, in addition to constant overhead, certain elements of cost which increase somewhat with the quantity produced, but not in direct proportion. There is also the very sharp reduction in unit costs when production is first initiated. As a first hypothesis as a basis for detailed statistical study, I suggest that a term of the type Ce^{-ku} might be useful, either alone or with the term $\frac{C}{u}$, to cover the elements of unit costs which decrease as u increases.

(e) That some definition of cost of a logical nature can be framed and applied to this case.

The statistical aproach to a cost function might well show that the range of applicability of a continuous formula is rather narrowly limited and that a careful statement as to attendant conditions is at least as important for reliability of the results as precise determination of the constants. Unfortunately, the results of statistical experiments along these lines are not available. In these days of stiff competition and rigorous government regulation, industrial concerns will be reluctant to permit publication of the kind of analysis of costs which is essential to what seems to be the correct scientific approach to this problem. It is hoped, however, that the preceding discussion has been specific enough to show the nature of the analysis which I think should precede the formulation of an expression of functional relationship, and to suggest the kind of discussion which a mathematical economist should include in his results.

Returning to the general criticism that the treatment of supply-demand-price relationships by certain mathematical economists is not truly scientific, let us review in broad lines the history of scientific progress in those lines where it has admittedly been successful. The steps have been about as follows:

1. Creation of a serviceable mechanism for the measurement of data. In the case of astronomy, for instance, this required the invention of the telescope and general agreement on angular measurement.

2. Careful making and recording of observations with this approved system of measurement, the observations

being in certain cases of events over which the scientist has no control and in other cases of experiments whose conditions could be modified at his convenience.

3. Derivation of empirical laws from these data.

4. Discovery of fundamental principles.

It is true that these steps are not in fact separated as completely as the outline might suggest, and that attempts at the discovery of fundamental principles often help in formulating the plan according to which observations are recorded and give the workers a motive for intensive effort. I think the statement will stand, however, that very few discoveries of fundamental principles have been made until substantial results have been secured under the (1), (2) and (3) headings.

For example, Newton's astronomical laws were discovered after Huyghens and others had made the telescope a useful instrument, Tycho Brahe had made an enormous number of observations, and Kepler had deduced empirical laws for those observations.

Again, in actuarial science, the whole structure of modern life insurance became possible only after careful vital statistics had been recorded for many years and analyzed by the empirical laws of Gompertz and Makeham.

On the other hand, when we turn to treatises on theoretical economics or to books and articles on mathematical economics, there seems to be no trace of the careful recording of observations or their analysis by empirical laws as the basis for their theoretical discussions. I admit, of course, that mathematical formulas are stated which look like empirical laws, but no references are given to any studies justifying these particular formulations. If we imagine ourselves starting the scientific procedure described above as the basis for arriving at real economic laws, we note almost at once that agreement as to the meaning of fundamental terms has not yet been secured. For instance, cost of production is one idea which is fundamental in analysis of demand-supply-price relationships, but cost accountants and economists are by no means agreed among themselves as to what the term should cover. Under the circumstances, it seems to me that the most profitable scientific approach *at present* would be to analyze various relationships which can be put on a quantitative basis, with special attention to noting all the special circumstances of the cases analyzed. For example:

(a) In many cases it would probably be possible to study the relationship between the price of a manufactured article and the price of the raw material or the raw materials used in making it. A simple case which I have actually done in my office and used in price estimating is determining the price of cotton yarn in terms of that of raw cotton. We find it advantageous to compute the cotton yarn price according to the formula, compare that with the actual price and note the relation to general business conditions or to competitive conditions within the industry. The statistical methods required for a problem of this type are obvious, but relatively little, I believe, has been done on this line. Broad comparisons of index numbers of prices of finished goods and raw materials I regard as another kettle of fish altogether.

(b) Relationship of change in price to change in stocks. Any study of actual data of the commodities, such as copper, lead, and rubber, shows that price tends to decline when stocks increase and rise when stocks decrease. A first step in the quantitative approach to price forecasting is to obtain a more precise formulation of this correspondence. It may be noted that some rather vague mathematical ideas come to the surface in discussions of these relationships. For instance, if in a given month production has decreased and consumption has increased, it is sometimes said that these are two arguments for higher prices, and prices are expected to rise. But it may happen in certain cases that even with such a decrease in production and increase in consumption the month's production still exceeds consumption and stocks are increasing. On the whole, then, the monthly figures point to lower rather than higher prices, and it is a useful duty of the mathematician to point this out.

An audience of mathematicians probably regards the preceding illustration as trifling. I bring it up to illustrate the fact that progress toward a more mathematical attitude in commodity forecasting must proceed step by step. A more advanced stage in the quantitative formulation is, of course, to determine the equation connecting change in price and change in stocks as reported monthly. This also has actually proved useful.

(c) Exact definition of the phrase "cost of production" as actually effective when the problem is:

1. Establishing an appropriate price under regulated monopoly conditions.

2. Determining which manufacturing or mining enterprise will survive.

3. Shutting down established sources of supply.

4. Creating new sources of supply.

As I see it, there are at least four costs of production, eacl of which is important under certain circumstances.

1. Complete economic cost, including interest on er tire investment at an appropriate rate.

2. Economic cost, excluding interest on the capital value of ownership.

3. Out-of-pocket expense, which excludes depreciation charges in excess of actual replacements in the period considered, interest on investment, and design or development expenses.

4. Economic cost plus reward for the enterpriser over and above interest at a reasonable rate on his investment.

Roughly speaking, it may be that these are in order closely related to the costs required for the problems stated above.

(d) Analysis of changes in cost at different price levels. One point which has been emphasized by the experience of the past year or two is the fact that costs are by no means kept constant when the price varies. For instance, the best information available two years ago suggested that at 18¢ New York, certain producers of rubber would begin to drop out because of costs above that figure. As a matter of fact, they seem to have been able to change their costs. Such changes are possible by several means, for instance:

1. Wage differentials varying with market price.

2. Bonuses for officials varying with profits.

3. Exploitation of best ores or plantations in times of lower prices.

4. Increased pressure for efficiency when essential.

(e) Careful analysis of the relations of cost to quantity produced with the consideration of:

1. The time ahead that the quantity is known to be required.

2. Continuous production versus intermittent production, perhaps in lots whose size has been determined to

be most economical in view of manufacturing and distributing conditions.

3. Mass production or production in smaller quantities by trained mechanics with attention to the discontinuity in costs per unit when the transition from one type of production to another occurs.

In view of the considerations suggested in (1), (2), and (3), it does not seem to me that the assumption of an algebraic formula connecting cost with quantity produced represents an adequate realistic formulation of the problem.

(f) Analysis of way elasticity of supply or demand actually works. Such an analysis seems to me essential before a definition of coefficient of elasticity is framed or made the basis for elaborate developments. Recent experience has, I think, shown:

1. Elasticity of demand does not mean shrinkage of demand with high prices to anything like the extent expected.

2. Elasticity of supply functions much more slowly than expected at the low-price end of the spectrum. The frictional factors include sympathy with employees who would lose their jobs if economically unprofitable production ceased, reserve funds which permit companies to continue operations when even out-of-pocket expenses are not being fully recovered, bank willingness to lend on commodity stocks which are not, in fact, marketable within a short time, and the cost of shutting down and reassembling a working force.

Most of the matters discussed in (a) to (f) above cannot be analyzed fully on the basis of published records. Moreover, in view of the fact that price information is part of the life blood of any particular business, it will prove difficult for outsiders to secure much of the information which would be needed to complete the analysis. A possible plan would be to place research specialists in industrial or marketing concerns to study actual data. It is not probable that the best research of this type can be done in academic halls as a sideline to teaching. It is also unlikely that corporation officers with an ax to grind can themselves complete the scientific analysis of such material. It seems clear, however, that a satisfactory solution requires a coordination of both points of view.

Summarizing the point of view I have tried to outline, I believe that the analysis of cost-price-supply-demand relationships should be relatively more inductive than it has been, especially in the type of theoretical work classified as mathematical economics. On the other hand, I think that the deductive approach is worth while, that we should try to formulate general principles in this field, and that the ultimate ideal involves a mathematical form,—though, perhaps, when it really covers price situations as they are, a mathematical form somewhat different from those required in the physical, chemical and astronomical sciences.

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