

Write $y = \log_e v_r$. Then we have

$$v_r \frac{dy}{dr} = \int_{-\infty}^{\infty} |x|^r \log_e |x| dF(x),$$

$$v_r^2 \frac{d^2 y}{dr^2} = \int_{-\infty}^{\infty} |x|^r dF(x) \cdot \int_{-\infty}^{\infty} |x|^r \log_e^2 |x| dF(x) - \left\{ \int_{-\infty}^{\infty} |x|^r \log_e |x| dF(x) \right\}^2$$

≥ 0 , by Schwarz's inequality.

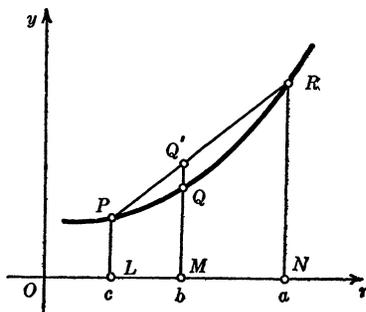


FIG. 1

It follows that the function y is convex (or exceptionally a straight line), and, on referring to the figure, it appears that

$$(1) \quad MQ \leq MQ'$$

for all chords PR . If the abscissae of the points L, M, N are c, b, a , respectively, where $c \leq b \leq a$, the inequality (1) leads at once to the relation

$$\log_e v_b \leq \frac{a-b}{a-c} \log_e v_c + \frac{b-c}{a-c} \log_e v_a.$$

Hence

$$v_b^{a-c} \leq v_c^{a-b} v_a^{b-c},$$

which is the usual form of the Liapounoff Inequality.

**REMARK ON THE NOTE "A GENERALIZATION OF
WARING'S FORMULA"**

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Before submitting for publication the note "A generalization of Waring's formula," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 218-219 the author made a diligent effort to ascertain, through correspondence with mathematicians and

actuaries both in this country and abroad, whether the generalized formula in question had been previously published, and none of the authorities communicated with knew of its prior publication. However, it has now come to his attention that the formula was published in essentially the same form by Hermite in the article "Sur la formule d'interpolation de Lagrange", *Journal für die Reine und Angewandte Mathematik* ("Crelle's Journal"), Vol. 84 (1878), pp. 70-79.